A
Compendious System
of
Natural Philosophy:

With NOTES,

Containing the

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occasional REMARKS.

In FOUR PARTS.

VOL. II.

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PART III. continued.

Containing

DISSERTATIONS on the following Subjects,
Viz.

Of the *Cause* of the *Reflection* of *Light*.

Of *Microscopes* and *Telescopes*.

AND

Of the Phænomenon of the *Rainbow*.

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Containing the MATHEMATICAL DEMONSTRATIONS, and some occasional REMARKS.

PART III. continued.

CONTAINING

CATOPTRICS,

AND THE

Doctrine of LIGHT and COLOURS.

To this are added

DISSERTATIONS on the following Subjects, *Viz.*

Of the *Cause* of the *Reflection* of *Light.*
Of *Microscopes* and *Telescopes.* And
Of the Phænomenon of the *Rainbow.*

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CHAP. VIII.

Of the Manner *wherein* Light *is* reflected.

THE *Refraction* of Light has been considered, and explained: its *Reflection* is now to be inquired into *(a)*. And first for the Manner wherein it is performed.

When a Ray of Light falls upon an *opake* Body, *Part* or *all* of it *is reflected*; if any enters, it is suffocated and lost within the Body: When it falls upon a *transparent* Body, *Part* of it *is reflected*, and *Part* enters; of what enters some is also suffocated and lost in the Body, the rest, when it arrives at the *other Side*, is also *in part reflected* there, and *in part transmitted*, unless its Inclination to *that Side* exceeds a *certain* Degree; which if it does, it is *all reflected* there. And the Power whereby a Ray is reflected at *this other Side* of a Body (which for Distinction sake I shall hereafter call *the second Surface*) is stronger than that by which it would be reflected by the same Surface, were it about to enter the Body there with an equal Degree of Obliquity.

(a) The Subject of this *Inquiry* is distinguished from that of the former, by the Name of *Catoptrics*, as tending to explain the Manner in which Objects appear, when seen by *reflected* Light.

The *Degree of Inclination* necessary to cause a *total Reflection* of a Ray at the second Surface of a *Medium*, is that which requires that the refracted Angle (was the Ray supposed to pass out there) should be equal to, or greater than a right one; and consequently it depends on the refractive Power of the *Medium* through which the Ray passes; and is therefore different in different *Media*. When a Ray passes through Glass surrounded with Air, and is inclined to its second Surface under an Angle of 42 *Degrees or more*, it will be *wholly reflected* there (*b*). For as 11 is to 17,

(*b*) From hence it follows, that when a Ray of Light arrives at the second Surface of a transparent substance with as great or greater Degree of Obliquity, than that which is necessary to make a total Reflection, it will there be all returned back to the first; and if it proceeds towards that with as great an Obliquity as it did towards the other (which it will do if the Surfaces of the *Medium* be parallel to each other) it will there be all reflected again, &c. and will therefore never get out, but pass from Side to Side, till it be wholly suffocated and lost within the Body.

From hence may arise an obvious Inquiry, how it comes to pass that Light, falling very obliquely upon a Glass-Window from without, should be transmitted into the Room? In Answer to this, it must be considered, that however obliquely a Ray falls upon the first Surface of any *Medium* whose Sides are parallel (as those of the Glass in a Window are) it will suffer such a Degree of Refraction in entring there, that it shall fall upon the second with a less Obliquity than that which is necessary to cause a total Reflection. For Instance let the *Medium* be Glass, as supposed in the present Case, then as 17 is to 11 (the *Ratio* of Refraction out of Air into Glass) so is the Sine of the largest Angle of Incidence with which a Ray can fall upon any Surface, to the Sine of a less Angle than that of total Reflection. And therefore, if the Sides of a Glass be parallel, the Obliquity, with which a Ray falls upon the first Surface, cannot be so great, but hat it shall pass the second without suffering a total Reflection there.

(the

(the *Ratio* of Refraction out of Glaſs into Air) ſo is the Sine of an Angle of 42 Degrees to a fourth Number, that will exceed the Sine of a right Angle.

When Light paſſes out of a denſer into a rarer *Medium*, the nearer the ſecond *Medium* approaches the firſt in Denſity (or more properly in its refractive Power) the leſs of it will be reflected in paſſing from one to the other: and when their refracting Powers are equal, all of it will paſs into the ſecond *Medium*.

Whether Light be reflected from the firſt or ſecond Surface of a Body, the Law it obſerves is this, viz. *That the Angle of Reflection of each Ray ſhall be equal to the Angle of Incidence of the ſame.*

By the *Angle of Reflection* is meant the Angle comprehended between a Perpendicular to the Surface at the Point where the Reflection is made, and the reflected Ray.

These are all the Circumſtances attending the Reflection of Light neceſſary to be taken Notice of at preſent. There are others, but they reſpect the *Doctrine of Light and Colours* not yet explained; we ſhall therefore paſs them by till we treat of that Subject, and in the mean Time proceed to conſider the Reflection of Light from plain and ſpherical Surfaces.

Chap. IX.

Of the Reflection *of* Light *from* plain *and* ſpherical Surfaces.

IT was obſerved in the foregoing Chapter, that the Law of Reflection is ſuch, that the Angle of Reflection of each Ray ſhall be equal to the Angle of Incidence of the ſame. From whence the ſeven following Propoſitions relating to the Reflection of Light from plain and ſpherical Surfaces may be deduced.

I. Rays of Light reflected from a plain Surface have the ſame Degree of Inclination to each other that their reſpective incident ones have.

For the Angle of Reflection of each Ray being equal to that of its reſpective incident one, it is evident that each reflected Ray will have the ſame Degree of Inclination to that Portion of the Surface from whence it is reflected, that its incident one has; but it is here ſuppoſed that all thoſe Portions of Surface from whence the Rays are reflected are ſituated in the ſame Plain; conſequently the reflected Rays will have the ſame Degree of Inclination to each other that their incident ones have, from whatever Part of the Surface they are reflected.

See

See this and the following Propositions expressed more determinately, and demonstrated in the Note below (a).

II. Parallel Rays, reflected from a concave Surface, are render'd converging.

To

(a) Proposition I.

Of the Reflection of Rays from a plain Surface.

When Rays fall upon a plain Surface, if they diverge, the Focus *of the reflected Rays will be at the same Distance behind the Surface, that the* Radiant *Point is before it: if they converge, it will be at the same Distance before the surface, that the imaginary* Focus *of the incident Rays is behind it.*

This Proposition admits of two Cases.

Case I. *Of diverging Rays.*

Dem. Let AB, AC (Fig. 45.) be two diverging Rays incident in the plain Surface DE, the one perpendicularly, the other obliquely; the perpendicular one AB will be reflected to A proceeding as from some Point in the Line AB produced; the oblique one AC will be reflected into some Line as CF such, that the Point G, where the Line FC produced intersects the Line AB produced also, shall be at an equal Distance from the Surface DE with the *Radiant* A. For the Perpendicular CH being drawn, ACH and HCF will be the Angles of Incidence and Reflection, which being equal, their Complements ACB and FCE are so too: but the Angle BCG is equal to FCE as being vertical to it; therefore in the Triangles ABC and GBC the Angles at C are equal, the Side BC is common, and the Angles at B are also equal to each other, as being right ones; therefore the Lines AB and BG, which respect the equal Angles at C, are also equal, and consequently the Point G, the *Focus* of the incident Rays AB, AC, is at the same Distance behind the Surface, that the Point A is before it. Q. E. D.

Case II. *Of converging Rays.*

This is the Converse of the former Case. For supposing FC and AB to be two converging incident Rays, CA and BA will be the reflected ones (the Angles of Incidence in the former

Case

To illustrate this, let AF, CD, EB, (Fig. 43.) represent three parallel Rays falling upon the concave surface FB, whose Center is C. To the Points F and B draw the Lines CF, CB; these being drawn from the Center will be perpendicular to the surface at those Points. The

Case being now the Angles of Reflection, and *vice versâ)* having the Point A for their *Focus*; but this, from what was demonstrated above, is at an equal Distance from the reflecting Surface with the Point G, which in this Case is the imaginary *Focus* of the incident Rays, FC, and AB.

Observat. I. *It is not here, as in the Refraction of Rays in passing through a plain Surface, where some of the refracted Rays proceed as from one Point, and some as from another (See Observat. I. Chap. 3. in the Notes) but they all proceed after Reflection as from one and the same Point, however obliquely they may fall upon the Surface; for what is here demonstrated of the Ray AC holds equally of any other, as AI, AK, &c.*

The Case of parallel Rays incident on a plain Surface, is included in this Proposition; for in that Case we are to suppose the *Radiant* to be at an infinite Distance from the Surface, and then by the Proposition, the *Focus* of the reflected Rays will be so too: that is, *the Rays will be parallel after Reflection, as they were before.*

Proposition II.

Of the Reflection of parallel Rays from a spherical Surface.

When parallel Rays are incident upon a spherical Surface, the Focus *of the reflected Rays will be the middle Point between the Center of Convexity and the Surface.*

This Proposition admits of two Cases.

Case I. *Of parallel Rays falling upon a convex surface.*

Dem. Let AB, DH, (Fig. 46) represent two parallel Rays incident on the convex Surface BH, the one perpendicularly, the other obliquely; and let C be the Center of Convexity: suppose HE to be the reflected Ray of the oblique Incident one DH proceeding as from F a Point in the Line AB produced. Through the Point H draw the Line CI, which will be perpendicular to the Surface at that Point, and the Angles DHI and IHE, *being the Angles and Incidence and Reflection*, will be equal. To

The incident Ray CD also passing through the Center will be perpendicular to the Surface, and therefore will return after Reflection in the same Line; but the oblique Rays AF and

To the former of these, the Angle HCF is equal, *the Lines AC and DH being parallel,* and to the latter the Angle CHF *as being vertical*; wherefore the Triangle CFH is *Isosceles*; and consequently the Sides CF and FH are equal: But supposing BH to *vanish*, FH is equal to FB, and therefore upon this Supposition FC and FB are equal, that is, the *Focus* of the reflected Rays is the middle Point between the Center of Convexity and the Surface. Q. E. D.

Case II. *Of parallel Rays falling upon a concave Surface.*

Dem. Let AB, DH (Fig. 47.) be two parallel Rays incident, the one perpendicularly, the other obliquely, on the concave Surface BH, whose Center of Concavity is C. Let BF and HF be the reflected Rays meeting each other in F: this will be the middle Point between B and C. For drawing through C the Perpendicular CH, the Angles DHC and FHC, *being the Angles of Incidence and Reflection,* will be equal, to the former of which the Angle HCF is equal, *as alternate*; and therefore the Triangle CFH is *Isosceles.* Wherefore CF and FH are equal: but if we suppose BH to *vanish*, FB and FH are also equal, and therefore CF is equal to FB; that is, the focal Distance of the reflected Rays is the middle Point between the Center and the Surface. Q. E. D.

Observat. II. *It is here observable, that the farther the Line DH, either in Figure the* 46 *or* 47, *is taken from AB, the nearer the Point F falls to the Surface. For the farther the Point H recedes from B, the larger the Triangle CFH will become; and consequently since it is always an* Isosceles *one, and the Base CH, being the* Radius, *is every where of the same Length, the equal Legs CF and FH will lengthen; but CF cannot grow longer unless the Point F approach towards the surface. And the farther H is removed from B, the faster F approaches to it.*

This is the Reason, that whenever parallel Rays are considered as reflected from a spherical Surface, the Distance of the oblique one from the perpendicular one is taken so small with respect to the focal Distance of that Surface, that without any physical *Error, it may be supposed to* vanish.

From

and EB will be reflected into the Lines FM and BM situated on the contrary side their respective Perpendiculars CF and CB. They will therefore proceed converging after Reflection towards some Point as M, in the Line CD: which Point, by what is demonstrated in the Note last referred to, will be in the Middle between C and D.

III.

From hence it follows, that if a Number of parallel Rays as AB, CD, EG, &c. fall upon a convex Surface, as expressed Figure the 48, *and if BA, DK, the reflected Rays of the incident ones AB, CD, proceed as from the Point F, those of the incident ones CD, EG,* viz. *DK, GL, will proceed as from N, those of the incident ones EG, HI, as from O, &c. because the farther the incident ones CD, EG &c. are from AB, the nearer to the Surface are the Points F, f, f, in the Line BF, from which they proceed after Reflection; so that properly the* Foci *of the reflected Rays BA, DK, GL, &c. are not in the Line AB produced, but in a curve Line passing through the Points F, N, O, &c.*

The same is applicable to the Case of parallel Rays reflected from a concave Surface, as expressed by the pricked Lines on the other half of the Figure, where PQ, RS, TV, are the incident Rays; QF Sf, Vf, the reflected ones interfecting each other in the Points X, Y and F; so that the Foci *of those Rays are not in the Line FB, but in a Curve passing through those Points.*

Had the Surface BH in Figure 46, *or* 47, *been form'd by the Revolution of a* Parabola *about its* Axis *having its* Focus *in the Point F, all the Rays reflected from the convex Surface would have proceeded as from the Point F, and those reflected from the concave would have fallen upon it, however distant their incident ones AB, DH, might have been from each other. For in the* Parabola, *all Lines drawn parallel to the* Axis *make Angles with the Tangents to the Points where they cut the* Parabola *(that is, with the Surface of the* Parabola*) equal to those which are made with the same Tangents by Lines drawn from thence to the* Focus, *(De* L'Hospital *Sections Coniques. Liv.* I. *Prop.* 5.) *therefore, if the incident Rays describe those parallel Lines, the reflected ones will necessarily describe those other, and so will all proceed as from, or meet in, the same Point.*

Pro-

III. Converging Rays falling on the like Surface are made to converge more.

For, every thing remaining as above, let GF, HB, be the incident Rays. Now because these Rays have larger Angles of Incidence than the parallel ones AF and EB in the foregoing Case, their Angles of Reflection will also be larger than theirs; they will therefore converge after Reflection, suppose in the Lines FN, and BN, having their Point of Con-

Proposition III:

Of the Reflection of diverging and converging Rays from a spherical Surface.

When Rays fall upon and spherical Surface, if they diverge, the Distance of the Focus *of the reflected Rays from the Surface is to the Distance of the* Radiant *Point from the same (or, if they converge, to that of the imaginary* Focus *of the incident Rays) as the Distance of the* Focus *of the reflected Rays from the Center is to the Distance of the* Radiant *Point (or imaginary* Focus *of the incident Rays) from the same.*

This Proposition admits of ten Cases.

Case I. *Of diverging Rays falling upon a convex Surface.*

Dem. Let RB, RD (Fig. 49) represent two diverging Rays flowing from the Point R as from a *Radiant*, and falling the one perpendicularly, the other obliquely, on the convex Surface BD, whose Center is C. Let DE be the reflected Ray of the incident one RD, produce ED to F, and through R draw the Line RH parallel to FE till it meets CD produced in H. Then will the Angle RHD be equal to EDH the Angle of Reflection, *as being alternate to it*, and therefore equal also to RDH which is the Angle of Incidence; wherefore the Triangle DRH is *Isosceles*, and consequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are similar, (or to express it in *Euclid's* Way, the Sides of the Triangle RHC are cut proportionably. 2. Elem. 6.) and therefore FD is

 to

Concourse N farther from C than the Point M; that to which the parallel Rays AF and EB converged to in the foregoing Case: and their precise Degree of Convergency as determined in the Note, will be greater than that wherein they converged before Reflection.

IV. Diverging Rays, falling upon the like Surface, are after Reflection parallel, diverging, or converging. If they diverge from the *Focus* of parallel Rays, they then become parallel; if from

to RH, or its equal RD, as CF to CR; but BD *vanishing*, FD and RD differ not from FB and RB, wherefore FB is to RB also, as CF to CR; that is, the Distance of the *Focus* from the Surface is to the Distance of the *Radiant* Point from the same, as the Distance of the *Focus* from the Center is to the Distance of the *Radiant* from thence. Q. E. D.

Case II. *Of converging Rays falling upon a concave Surface.*

Dem. Let KD and CB be the converging incident Rays having their imaginary *Focus* in the Point R, which was the *Radiant* in the foregoing Case. Then as RD was in that Case reflected into DE, KD will in this be reflected into DF; for, since the Angles of Incidence in both Cases are equal, *as they are by being vertical*, the Angles of Reflection will be so too; so that F will be the *Focus* of the reflected Rays: but it was there demonstrated that FB is to RB as CF to CR, that is, the Distance of the *Focus* from the Surface is to the Distance (in this Case) of the imaginary *Focus* of the incident Rays, as the Distance of the *Focus* from the Center is to the Distance of the imaginary *Focus* of the incident Rays from the same. Q. E. D.

Case III. *Of converging Rays falling upon a convex Surface, and tending to a Point between the* Focus *of Parallel Rays and the Center.*

Dem. Let BD (Fig. 50.) represent a convex Surface whose Center is C, and *Focus* of parallel Rays is P; and let AB, KD, be two converging Rays incident upon it, and having their imaginary *Focus* at R, a Point between P and C. Now because KD tends

from a Point nearer to the Surface than that, they will diverge, but in a less Degree than before Reflection; if from a Point between that and the Center, they will converge after Reflection and that to some Point on the contrary Side the Center, but situated farther from it than the Point they diverged from: if the incident Rays diverge from a Point beyond the Center, the reflected ones will converge to one on the other Side of it, but nearer to it than the

tends to a Point between the *Focus* of parallel Rays and the Center, the reflected Ray DE will diverge from some Point on the other Side the Center, suppose F; as explained above in the Text under Proposition the VIIth. Through D draw the Perpendicular CD and produce it to H, then will KDH and HDE be the Angles of Incidence and Reflection, which being equal, their vertical ones RDC and CDF will be so too, and therefore the *Vertex* of the Triangle RDF is bisected by the Line DC: wherefore (3 El. 6.) FD and DR, or, BD vanishing, FB and BR are to each other as FC to CR; that is, the Distance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident ones, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

Case IV. *Of diverging Rays falling upon a concave Surface and proceeding from a Point between the* Focus *of parallel Rays and the Center.*

Dem. Let RB, RD, be the diverging Rays incident upon the concave Surface BD, having their *Radiant* in the Point R, the imaginary *Focus* of the incident Rays in the foregoing Case. Then as KD was in that Case reflected into DE, RD will now be reflected into DF. But is was there demonstrated that FB and RB are to each other as CF to CR; that is, the Distance of the *Focus* is to that of the *Radiant*, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

The Angles of Incidence and Reflection being equal, it is evident, that if in any Case, the reflected Ray be made the incident one, the

the Point they diverged from; and if they diverge from the Center, they will be reflected thither again.

1. Let them diverge in the Lines MF, MB, proceeding from M the *Focus* of parallel Rays; then as the parallel Rays AF and EB were reflected into the Lines FM and BM (Proposit. 2.) these Rays will now on the contrary be reflected into them.

2. Let them diverge from N a Point nearer to the Surface than the *Focus* of parallel Rays, they will then be reflected into the diverging Lines

the incident will become the reflected one: and therefore the four following Cases may be considered respectively as the Converse of the four foregoing; for in each of them the incident Rays are supposed to coincide with the reflected ones in the other. Or they may be demonstrated independently of them as follows.

Case V. *Of converging Rays falling upon a convex Surface, and tending to a Point nearer the Surface than the* Focus *of parallel Rays.*

Dem. Let ED, RB (Fig. 49.) be the converging Rays incident upon the convex Surface BD whose Center is C, and *Focus* of parallel Rays is at P; and let the imaginary *Focus* of the incident Rays be at F, a Point between P and B, and let DR be the reflected Ray. From C and R draw the Lines CH, RH, the one passing through D, the other parallel to FE. Then will the Angle RHD be equal to HDE the Angle of Incidence, *as alternate to it*, and therefore equal to HDR, the Angle of Reflection; wherefore the Triangle HDR is *Isosceles*, and consequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are similar, and therefore RH, or RD, is to FD as CR to CF; but BD vanishing, RD and FD coincide with RB and FB, wherefore RB is to FB as CR to CF; that is, the Distance of the *Focus* from the Surface is to the Distance of the imaginary *Focus* of the incident Rays, as the Distance of the *Focus* from the Center is to the Distance of the imaginary *Focus* of the incident Rays from the same. Q. E. D.

Case

Lines FG and BH which the incident Rays GF and HB deſcribed, that were ſhewn to be reflected into them in the foregoing Propoſition: but the Degree wherein they diverge, as demonſtrated in the Note, will be leſs than that wherein they diverged before Reflection.

3. Let them proceed diverging from X a Point between the *Focus* of parallel Rays and the Center, they then make leſs Angles of Incidence than the Rays MF and MB which became parallel by Reflection, they will conſequently have leſs Angles of Reflection, and proceed therefore converging towards ſome Point as Y; which Point will always fall on the contrary Side the Center, becauſe a reflected

Caſe VI. *Of diverging Rays falling upon a concave Surface, and proceeding from a Point between the* Focus *of parallel Rays and the Surface.*

Dem. Let FD and FB repreſent two diverging Rays flowing from the Point F as a *Radiant*, which was the imaginary *Focus* of the incident Rays in the foregoing Caſe. Then as ED was in that Caſe reflected into DR, FD will be reflected into DK, (for the Reaſon mention'd in Caſe the ſecond) ſo that the reflected Ray will proceed as from the Point R: but it was demonſtrated in the Caſe immediately before-going, that RB is to FB as CR to CF; that is, the Diſtance of the *Focus* from the Surface is to that of the *Radiant* from the ſame, as the Diſtance of the former from the Center is to that of the latter from the ſame. Q. E. D.

Caſe VII. *Of converging Rays falling upon a convex Surface, and tending towards a Point beyond the Center.*

Dem. Let AB, ED, (Fig. 50.) be the incident Rays tending to F, a Point beyond the Center C, and let DK be the reflected Ray of the incident one ED. Then becauſe the incident Ray ED

flected Ray always falls on the contrary Side the Perpendicular with respect to that on which its incident one falls; and, as demonstrated in the Note, it will be farther distant from the Center than X.

4. If the incident ones diverge from Y, they will after Reflection converge to X, those which were the incident Rays in the former Case being the reflected ones in this.

And lastly, if the incident Rays proceed from the Center, they fall in with their respective Perpendiculars, and for that Reason are reflected thither again.

V. Parallel Rays reflected from a convex Surface are rendered diverging.

To shew this, let AB, GD, EF, (Fig. 44.) be three parallel Rays falling upon the convex Sur-

ED tends to a Point beyond the Center, the reflected Ray DK will proceed as from one on the contrary Side, suppose R; as explained in the Text under Proposition the VIIth. Through D draw the Perpendicular CD and produce it to H. Then will EDH and HDK be the Angles of Incidence and Reflection, which being equal, their vertical ones CDF and CDR will be so too: consequently the *Vertex* of the Triangle FDR is bisected by the Line CD: wherefore (3 Elem. 6.) RD is to DF or BD vanishing, RB is to BF as RC to CF; that is, the Distance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident Rays, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

Case VIII. *Of diverging Rays falling upon a concave Surface, and proceeding from a Point beyond the Center.*

Dem. Let FB, FD, be the incident Rays having their *Radiant* in F, the imaginary *Focus* of the incident Rays in the foregoing Case. Then as ED was in that Case reflected into DK, FD will now be reflected into DR; so that R will the *Focus* of the re-

Surface BF whose Center of Convexity is C, and let one of them, viz. GD, be perpendicular to the Surface: through B, D, and F, the Points of Reflection, draw the Lines CV, CG, and CT, which because they pass through the Center will be perpendicular to the Surface at those Points. The incident Ray GD being perpendicular to the Surface will return after Reflection in the same Line, but the oblique ones AB and EF in the Lines BK and FL situated on the contrary Side their respective Perpendiculars BV and FT. They will therefore diverge after Reflection as from some Point M in the Line GD produced; which Point, as demonstrated in the Note, will be in the middle between D and C.

VI.

reflected Rays. But it was demonstrated in the foregoing Case, that RB is to FB as RC to CF; that is, the Distance of the *Focus* of the reflected Rays from the Surface is to the Distance of the *Radiant* from the same, as the Distance of the *Focus* of the reflected Rays from the Center is to the Distance of the *Radiant* from thence. Q. E. D.

The two remaining Cases may be considered, as the Converse of those under Proposition the second of this Note, because the incident Rays in these are the reflected ones in them; or they may be demonstrated in the same Manner with the foregoing, as follows.

Case IX. *Converging Rays falling upon a convex Surface, and tending to the* Focus *of parallel Rays, become parallel after Reflection.*

Dem. Let ED, RB, (Fig. 49.) represent two converging Rays incident on the convex Surface BD, and tending towards F, which we will *now* suppose to be the *Focus* of parallel Rays; and let DR be the reflected Ray, and C the Center of Convexity of the reflecting Surface. Through C draw the Line CD, and produce it

VI. Diverging Rays reflected from the like Surface are rendered more diverging.

For, every thing remaining as above, let GB, GF, be the incident Rays. These having larger Angles of Incidence than the parallel ones AB and EF in the preceding Case, their Angles of Reflection will also be larger than theirs; they will therefore diverge after Reflection, suppose in the Lines BP and FQ, as from some Point N farther from C than the Point M; and the Degree wherein they will diverge, as determined in the Notes, will exceed that wherein they diverged before Reflection.

VII.

it to H, drawing RH parallel to ED produced to F. Now it has been demonstrated (Case 5.) where the incident Rays are supposed to tend to the Point F, that RB is to FB as RC to CF; but F in this Case being supposed to be the *Focus* of parallel Rays, it is the middle Point between C and B (by Proposition 2d) and therefore FB and FC are equal, and consequently the two other Terms in the Proportion, viz. RB and RC must be so too; which can only be upon a Supposition that R is at an infinite Distance from B; that is, that the reflected Rays BR and DR be parallel. Q. E. D.

Case X. *Diverging Rays falling upon a concave Surface, and proceeding from the* Focus *of parallel Rays, become parallel after Reflection.*

Dem. Let RD, RB (Fig. 50.) be two diverging Rays incident upon the concave Surface BD, as supposed in the Case the fourth; where it was demonstrated that FB is to RB as CF to CR. But in the present Case RB and CR are equal, because R is supposed to be the *Focus* of parallel Rays; therefore FB and FC are so too. Which cannot be unless F be taken at an infinite Distance from B; that is, unless the reflected Rays BF and DF be parallel. Q. E. D.

Observat.

VII. Converging Rays reflected from the like Surface, are parallel, converging or diverging. If they tend towards the *Focus* of parallel Rays, they then become parallel; if to a Point nearer the Surface than that they converge, but in a leſs Degree than before Reflection; if to a Point between that and the Center, they ſhall diverge after Reflection, as from ſome Point on the contrary Side the Center, but ſituated farther from it than the Point they converged to; if the incident Rays converge to a Point beyond the Center, the reflected ones will diverge as from one on the contrary Side of it, but nearer to it than the Point the incident ones converged to: and if the incident Rays converge towards the Center, the reflected ones will proceed as from thence.

1. Let

Obſervat. 3. *It is here obſervable that in the Caſe of diverging Rays falling upon a convex Surface (ſee Fig. 49.) the farther the Point D is taken from B, the nearer the Point F, the* Focus *of the reflected Rays, approaches to B, while the* Radiant *R remains the ſame. For it is evident from the Curvature of a Circle that the Point D (See Fig. 51.) may be taken ſo far from B, that the reflected Ray DE ſhall proceed as from F, G, H, or even from B, or from any Point between B and R, and the farther it is taken from B, the* faſter *the Point, from which it proceeds, approaches towards R: as will eaſily appear if we draw ſeveral incident Rays with their reſpective reflected ones, in ſuch Manner that the Angles of Reflection may be all equal to their reſpective Angles of Incidence, as is done in the Figure. The like is applicable to any of the other*

Caſes

1. Let them converge in the Lines KB and LF tending towards M the *Focus* of parallel Rays; then as the parallel Rays AB, EF were reflected into the Lines BK and FL (*Propofit.* 5.) those Rays will now on the contrary be reflected into them.

2. Let them converge in the Lines PB, QF, tending towards N a Point nearer the Surface than the *Focus* of parallel Rays, they will then be reflected into the converging Lines BG and FG, in which the Rays GB, GF proceeded that were shewn to be reflected into them

Cases of diverging or converging Rays incident upon a spherical Surface. This is the Reason that, when Rays are considered as reflected from a spherical Surface, the Distance of the oblique Rays from the perpendicular one is taken so small, that it may be supposed to vanish.

From hence it follows, that if a Number of diverging Rays are incident upon the convex Surface BD at the several Points B, D, D, &c. *they shall not proceed after Reflection as from any one Point in the Line RB produced, but as from a curve Line passing through the several Points F, f, f,* &c. *The same is applicable in all the other Cases.*

Had the Curvature BD (Fig. 49.) *been* Hyperbolical *having its* Foci *in R and F, than R being the* Radiant (*or the imaginary* Focus *of incident Rays) F would have been the* Focus *of the reflected ones, and* vice versâ, *however distant the Points B and D might be taken from each other. In like Manner had the Curve BD (Fig.* 50.) *been* Elliptical *having its* Foci *in F and R, the one of these being made the* Radiant (*or imaginary* Focus *of incident Rays) the other would have been the* Focus *of the reflected ones, and* vice versâ. *For both in the* Hyperbola *and* Ellipsis, *Lines drawn from each of their* Foci *through any Point make equal Angles with the Tangent to that Point*; (*De* L'Hospital *Sections Coniques, Liv.* II. *Prop.* 8. & *Liv.* III. *Prop.* 11.) *Therefore, if the incident Rays proceed to or from one of their* Foci, *the reflected ones,*

them in the Proposition immediately foregoing; but the Degree wherein they will converge, as demonstrated in the Note, will be less than that wherein they converged before Reflection.

3. Let them converge in the Lines RB and SF proceeding towards X, a Point between the *Focus* of parallel Rays and the Center; their Angles of Incidence will then be less than those of the Rays KB and LF which became parallel after Reflection, their Angles of

ones will all proceed, as from or to the other. So that in order that diverging or converging Rays may be accurately reflected to or from a Point, the reflecting Surface must be formed by the Revolution of an Hyperbola *about its longer* Axis, *when the incident Rays are such that their* Radiant, *or imaginary* Focus *of incident Rays, shall fall on one Side the Surface, and the* Focus *of the reflected ones on the other: When they are both to fall on the* same *Side, it must be formed by the Revolution of an* Ellipsis *about its longer* Axis. *However upon Account of the great Facility with which spherical Surfaces are formed in Comparison of that with which Surfaces, formed by the Revolution of any of the* Cornic *Sections about their* Axes, *are made, the latter are very rarely used. Add to this another Inconvenience,* viz. *that, the* Foci *of these Curves being Mathematical Points, it is but one Point of the Surface of an Object that can be placed in any of them at a Time, so that it is only* in Theory *that Surfaces formed by the Revolution of these Curves about their* Axes *render Reflection perfect.*

Now because the focal Distance of Rays reflected from a spherical Surface cannot be found by the *Analogy* laid down in the third Proposition of this Note, without making use of the Quantity sought; I shall here give an Instance whereby the Method of doing it in all others will readily appear.

Prob. Let it be required to find the focal Distance of diverging Rays incident upon a convex Surface, whose *Radius* of Convexity is

of Reflection will therefore be less, on which Account they must necessarily diverge, suppose in the Lines BH and FI, from some Point as Y; which Point for the Reason given under Proposition the fourth will fall on the contrary Side the Center with respect to X, and, as demonstrated in the Note, will be farther from it than that.

4. If the incident Rays tend towards Y, the reflected ones will diverge as from X, those, which

is 5 Parts, and the Distance of the *Radiant* from the Surface is 20.

Sol. Call the focal Distance sought x, then will the Distance of the *Focus* from the Center be $5-x$, and that of the *Radiant* from the same 25; therefore by Proposition the third, we have the following Proportion, *viz.* $x : 20 :: 5 - x : 25$, and, multiplying Extreams together and Means together, we have $25x = 100 - 20x$, which after due Reduction gives $X = \frac{100}{45}$.

If in any Case it should happen, that the Value of x should be a negative Quantity, the focal Point must then be taken on the contrary Side the Surface to that on which it was supposed that it would fall in stating the *Problem*.

If Letters instead of Figures had been made use of in the foregoing Solution a general *Theorem* might have been raised, to have determined the focal Distance of reflected Rays in all Cases whatever. See this done by *Dr. Brown* in his Supplement to *Gregory*'s Optics, pag. 112. Edit. Second.

Because it was observed (Chap. III. in the Notes) that different incident Rays, though tending to or from one Point, would after *Refraction* proceed to or from different Points, a Method was there inserted of determining the *distinct* Point, which each separate Ray entring a spherical Surface converges to or diverges from after *Refraction*: the same has been observed here with regard to Rays *reflected* from a spherical Surface, (See *Observat.* 2 & 3.) but

which were the incident ones in one Case, being the reflected ones in the other.

And Lastly, if the incident Rays converge towards the Center, they fall in with their respective Perpendiculars; on which Account they proceed after Reflection, as from thence.

but the Method of determining the *distinct* Point, to or from which any given incident Ray proceeds after *Reflection*, is much more simple. It is only necessary to draw the reflected Ray such, that the Angle of Reflection may be equal to the Angle of Incidence, which will determine the Point it proceeds to or from in any Case whatever.

CHAP.

CHAP. X.

Of the Appearance *of Bodies seen by* Light *reflected from plain and spherical Surfaces.*

IN the Beginning of the seventh Chapter, in which was explained the Appearance of Bodies seen through *refracting* Substances of various Forms, we laid down some Observations respecting the apparent Situation of Bodies seen by *refracted* Light: all which equally respect the apparent Situation of Bodies seen by *Reflection*; to them therefore we refer the Reader. But besides those, there is one peculiar to the Subject of this Chapter, *viz.* That each Point in the Representation of an Object made by Reflection appears situated somewhere in an infinite right Line that passes through its correspondent Point in the Object, and is perpendicular to the reflecting Surface.

The Truth of this appears sufficiently from the several Propositions laid down in the foregoing Chapter, in each of which Rays flowing from any *Radiant* are shewn to proceed after Reflection to or from some Point in a Line that passes through the said *Radiant*, and is perpendicular to the reflecting Surface. For Instance (Fig. 43.) Rays flowing from Y are

are collected in X, a Point in the Perpendicular CD, that produced passes through Y; again (Fig. 44.) Rays flowing from G proceed after Reflection, as from N, a Point in the Perpendicular CD, that produced passes through G; and so for the rest [a].

I. When an Object is seen by Reflection from a *plain* Surface, the Image of it appears at the *same Distance* behind the Surface that the Object is placed before it, of the *same Magnitude* therewith, and directly *opposite* to it.

To explain this, let AB (Fig 52.) represent an Object seen by Reflection from the plain Surface SV, and let the Rays AF, AG be so inclined to the Surface that they shall enter an Eye at H after Reflection; and let AE be

[a] This Observation, except where an Object is seen by Reflection from a plain Surface, relates only to those Cases where the Representation is made by means of such Rays, as fall upon the reflecting Surface with a very small Degree of Obliquity; because such, as fall at a considerable Distance from the Perpendicular, proceed not after Reflection as from any Point in that Perpendicular (See the second and third Observations in the foregoing Chapter) but as from other Points situated in a certain Curve, as there explained; upon which Account these Rays are neglected as forming a confused and deformed Representation. And therefore it is to be remembered, that however the Situation of the Eye, with respect to the Object and reflecting Surface, may be represented in the following Figures, it is to be supposed as situated in such Manner with respect to the Object, that Rays, flowing from thence and entring it after Reflection, may be such only as fall with a very small Degree of Obliquity upon the Surface: that is, the Eye must be supposed to be placed almost directly behind the Object, or between it and the reflecting Surface. The Reason why it is not always so placed, is only to avoid Confusion in the Figures.

perpen-

perpendicular to the Surface: then by the Observation just laid down, the Point A will appear in some Part of the Line AE produced, suppose I; that is, the oblique Rays AF and AG will proceed after Reflection as from that Point, and further, because the reflected Rays FH, GK, will have the same Degree of Inclination to each other, that their incident ones have (as was shewn in Proposition the first of the foregoing Chapter) that Point must necessarily be at the same Distance from the Surface that the Point A is; the Representation therefore of the Point A, will be at the same Distance behind the Surface, that the Point itself is before it, and directly opposite to it: consequently since the like may be shewn of the Point B, or any other, the whole Image IM will appear at the *same Distance* behind the Surface that the Object is before it, and directly *opposite* to it; and because the Lines AI, BM, which are perpendicular to the plain Surface, are for that Reason parallel to each other, it will also be of the *same Magnitude* therewith. As was to be shewn [b].

II. When an Object is seen by Reflection from a *convex* Surface, its Image appears *nearer* to the Surface, and *less* than the Object.

Let

[b] If the Object be placed before a common Looking-glass, and viewed obliquely, three, four, or more Images of it will appear behind the Glass.

To

PART III. PLATE VII Pag. 120.

[Fig.] 43. P. 102.

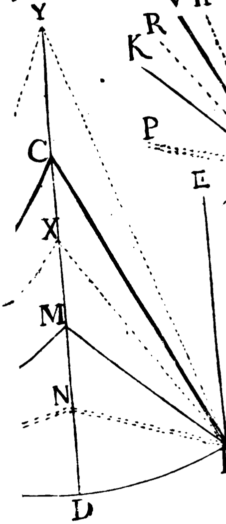
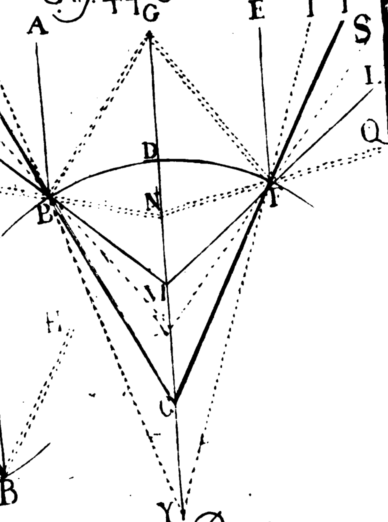


Fig. 44. P. 110.



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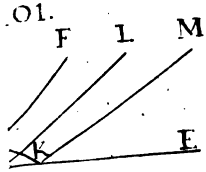


Fig. 46. P. 102.

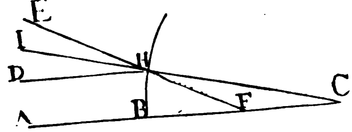
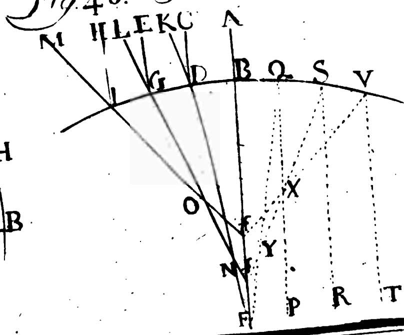
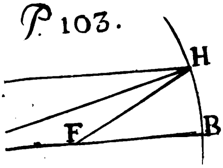


Fig. 48. P. 104.



P. 103.



Let AB (Fig. 54.) represent the Object, SV a reflecting Surface, whose Center of Convexity is C: and let the Rays AF, AG, be so inclined to the Surface, that after Reflection thereat, they shall enter the Eye at H: and let AE be perpendicular to the Surface: then will the oblique Rays AF, AG, proceed after Reflection as from some Point in the Line AE produced, (for the Reason mentioned under Proposition the last) suppose from I; which Point, because the reflected Rays will diverge more

To explain this, let ABCD (Fig. 53.) represent the Glass, and let EF be the *Axis* of a Pencil of Rays flowing from E, a Point in an Object situated there. The Rays of this Pencil will in Part be reflected at F, suppose into the Line FG, (see the Manner in which Light is reflected Chap. 18.) What remains will (after Refraction at F which we don't consider here) pass on to H; from whence (on Account of the Quickfilver which is spread over the second Surface of Glasses of this Kind to prevent any of the Rays from being transmitted there) they will be strongly reflected to K, where Part of them will emerge and enter an Eye at L: by this means one Representation of the said Point will be formed in the Line LK produced, suppose in M. Again, another Pencil whose *Axis* is EN, first reflected at N, then at O, and afterwards at P, will form a second Representation of the same Point at Q. And thirdly, another Pencil whose *Axis* is ER, after Reflection at the several Points R, S, H, T, V, successively, will exhibit a third Representation of the same Point at X; and so on *in infinitum*. The same being true of each Point in the Object, the whole will be represented in the like Manner; but the Representations will be faint, in Proportion to the Number of Reflections the Rays suffer and the Length of their Progress within the Glass. We may add to these another Representation of the same Object in the Line LO produced, made by such of the Rays as fall upon O, and are from thence reflected to the Eye at L.

This may be tried by placing a Candle before the Glass as at E, and viewing it obliquely, as from L.

than the incident ones (Prop. the ſixth of the foregoing Chapter) muſt be nearer to the Surface than the Point A. And ſince the ſame is true alſo of the Rays which flow from B, or any other Point, the Repreſentation IM will be *nearer* to the Surface than the Object; and becauſe it is terminated by the Perpendiculars AE and BF which incline to each other, as concurring at the Center, it will alſo appear *leſs*.

III. When an Object is ſeen by Reflection from a *concave* Surface, the Repreſentation of it is *various*, both with regard to its Magnitude and Situation, according as the Diſtance of the Object from the reflecting Surface is *greater* or *leſs*.

1. When the Object is nearer to the Surface, than its *Focus* of parallel Rays, the Image falls on the *oppoſite* Side the Surface, is more *diſtant* from it, and *larger* than the Object.

Thus, let AB, (Fig. 55.) be the Object, SV the reflecting Surface, F the *Focus* of parallel Rays, C its Center. Through A and B the Extremities of the Object draw the Lines, CE, CR, which will be perpendicular to the Surface, and let the Rays AR, AG, be incident upon ſuch Points of it that they ſhall be reflected into an Eye at H. Now becauſe the *Radiant* Points A and B are nearer the Surface than F the *Focus* of parallel Rays, the reflected Rays will diverge (Chap. IX. Prop. 4.) and will

will therefore proceed as from ſome Points on the oppoſite Side the Surface; which Points, by the Obſervation laid down at the Beginning of this Chapter, will be in the Perpendiculars AE, BR, produced, ſuppoſe in I and M: but they will diverge in a leſs Degree than their incident ones (ſee the Propoſition juſt referred to) and therefore the ſaid Points will be farther from the Surface than the Points A and B. The Image therefore will be on the *oppoſite* Side the Surface with reſpect to the Object, it will be *more diſtant* than it, and conſequently, being terminated by the Perpendiculars CI and CM, it will alſo be *larger*.

2. When the Object is placed in the *Focus* of parallel Rays, the reflected Rays enter the Eye parallel (Chap. IX. Prop. 4.) in which Caſe the Image ought to appear at an *infinite* Diſtance behind the reflecting Surface; but the Repreſentation of it, for the like Reaſons that were given in the foregoing Caſe, being *large* and *diſtinct*, we judge it not much farther from the Surface than the Image [c]

3. When the Object is placed between the *Focus* of parallel Rays and the Center, the Image falls on the *oppoſite* Side the Center, is *larger* than the Object, and in an *inverted Poſition*.

Thus let AB (Fig. 56.) repreſent the Object, SV the reflecting Surface, F its *Focus* of pa-

[c] See what has been ſaid concerning the apparent Situation of Objects ſeen by parallel Rays, in Chapter VII.

rallel Rays, and C its Center. Through A and B the Extremities of the Object draw the Lines CE and CN which will be perpendicular to the Surface; and let AR, AG, be a Pencil of Rays flowing from A. These Rays proceeding from a Point beyond the *Focus* of parallel Rays will after Reflection converge towards some Point on the opposite Side the Center (Chap. IX. Prop. 4. Case 3.) which will fall upon the Perpendicular EC produced; but at a greater Distance from C than the Radiant A from which they diverged, (by the Proposition and Case just referred to). For the same Reason, Rays flowing from B will converge to a Point in the Perpendicular NC produced, which shall be farther from C than the Point B; from whence it is evident, that the Image IM is *larger* than the Object AB, that it falls on the *contrary* Side the Center, and that their Positions are *inverted* with respect to each other.

4. If the Object be placed beyond the Center of Convexity the Image is then formed *between the Center and the* Focus *of parallel Rays*, is *less* than the Object, and its Position is *inverted.*

This Proposition is the Converse of the foregoing: for as in that Case Rays proceeding from A were reflected to I, and from B to M; so Rays flowing from I and M will be reflected to A and B; if therefore an Object

Object be supposed to be situated beyond the Center in IM, the Image of it will be formed in AB, *between that and the* Focus *of parallel Rays,* will be *less* than the Object, and *inverted.*

5. If the Middle of the Object be placed in the Center of Convexity of the reflecting Surface, the Object and its Image will be *coincident*; but the Image will be *inverted* with respect to the Object.

That the Place of the Image and the Object should be the same in this Case needs little Explication; for the Middle of the Object being in the Center, Rays flowing from thence will fall perpendicularly upon the Surface, and therefore necessarily return thither again; so that the Middle of the Image will be *coincident* with the Middle of the Object. But that the Image should be inverted is perhaps not so clear. To explain this, let AB (Fig. 57.) be the Object having its middle Point C in the Center of the reflecting Surface SV; through the Center and the Point R draw the Line CR which will be perpendicular to the reflecting Surface, join the Points AR and BR, and let AR represent a Ray flowing from A, this will be reflected into RB, for C being the middle Point between A and B the Angles ARC and CRB are equal; and a Ray from B will likewise be reflected to A; and therefore the Position

ſition of the Image will be *inverted* with reſpect to that of the Object [d].

6. If in any of the three laſt Caſes, in each of which the Image is formed on the ſame Side the reflecting Surface with the Object, the Eye be ſituated farther from the Surface than the Place where the Image falls, the Rays of each Pencil, croſſing each other in the ſeveral Points of the Image, will enter the Eye as from a real Object ſituated there; ſo that the Image will appear *pendulous* in the Air between the Eye and the reflecting Surface, and in the Poſition wherein it is formed, *viz. inverted* with reſpect to the Object, in the ſame Manner that an Image formed by *refracted* Light appears to an Eye placed beyond it; which was fully explained under the fourth Propoſition of the ſeventh Chapter, and therefore needs not be repeated here.

But as to what relates to the Appearance of the Object when the Eye is placed nearer to the Surface than the Image, that was not there fully inquired into. That Point ſhall therefore now be more ſtrictly examined under the following Caſe, which equally relates to *refracted* and *reflected* Light.

[d] In this Propoſition it is to be ſuppoſed that the Object AB is ſo ſituated with reſpect to the reflecting Surface, that the Angle ACR may be right; for otherwiſe the Angles ARC and BRC will not be equal, and Part of the Image will therefore fall upon the Object and part off.

7. If

7. If the Eye be ſituated between the reflecting Surface and the Place of the Image, the Object is then ſeen *beyond* the Surface; and the farther the Eye recedes from the Surface towards the Place of the Image, the *more confuſed*, *larger*, and *nearer* the Object appears.

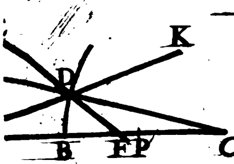
To explain this, let AB (Fig. 58.) repreſent the Object, IM its Image, one of whoſe Points is formed by the Concurrence of the reflected Rays DM, EM, *&c.* which before Reflection came from B; the other, I, by the Concurrence of DI, EI, *&c.* which came from A: and let *ab* be the Pupil of an Eye, ſituated between the Surface DP and the Image. This Pupil will admit the Rays Ha, Kb, which, becauſe they are tending towards I, are ſuch as came from A, and therefore the Point A will appear diffuſed over the Space RS. In like Manner the Pupil will alſo receive into it the reflected Rays Ka and Lb, which, becauſe they are tending towards M, by Suppoſition came from B; and therefore the Point B will be ſeen ſpread as it were over the Space TV, and the Object will ſeem to fill the Space RV; but the Repreſentation of it will be *confuſed*, becauſe the intermediate Points of the Object, being equally inlarged in Appearance, there will not be Room for them between the Points S and T, but they will coincide in part one with another; for Inſtance, the Appearance of that Point in the Object, whoſe Repreſentation falls upon *c* in the

the Image, will fill the Space *mn*, and so of the rest. Now if the same Pupil be removed into the Situation *ef*, the reflected Rays E*e* and G*f* will then enter the Eye, and therefore one Extremity of the Object will appear to cover the Space XY; and because the Rays O*f* and L*e* will also enter it in their Progress towards M, the Point B, from whence they came, will appear to cover ZV; the Object therefore will appear *larger* and *more confused* than before. And when the Eye recedes quite to the Image, it sees but one single Point of the Object, and that appears diffused all over the reflecting Surface: for Instance, if the Eye recedes to the Point M, then Rays flowing from the Point B enter it upon whatever Part of the Surface they fall: and so for the rest. The Object also appears *nearer* to the Surface, the farther the Eye recedes from it towards the Place of the Image, probably because as the Appearance of the Object becomes more and more confused, its Place is not so easily distinguished from that of the reflecting Surface itself, till at last when it is quite confused (as it is when the Eye is arrived at M) they both appear as one, the Surface assuming the Colour of the Object [e].

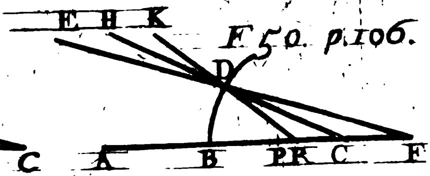
As

[e] As to the precise apparent Magnitude of an Object seen after this Manner, it is such that the Angle it appears under shall be equal to that which the Image of the same Object would appear under, were we to suppose it seen from the same Place: that is, the apparent Object (for such I must call it to distinguish

p. 105.



F 50. p. 106.



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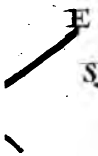
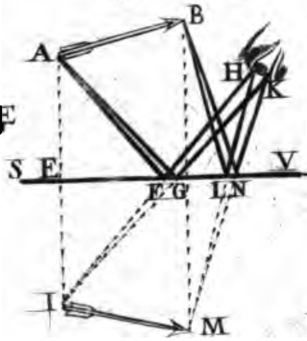
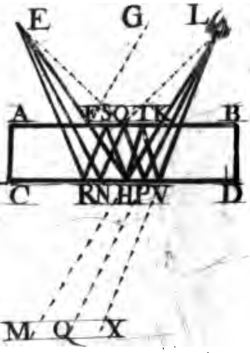


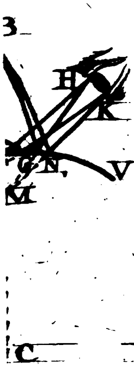
F 52. p. 119.



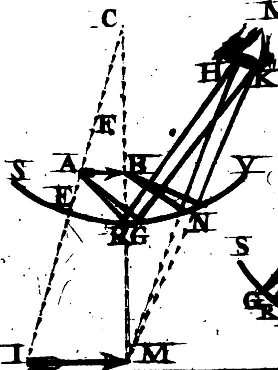
F 53. p. 120.



b. 121.



F. 55. p. 122.



F 56. p. 123.

As each Point in the Representation of an Object made by Reflection is situated somewhere in an infinite right Line that passes through its correspondent Point in the Object, and is perpendicular to the reflecting Surface, as was shewn in the Beginning of this Chapter; we may from hence deduce a most easy and expeditious Method of determining both the

distinguish it from the Image of the same Object) and the Image subtend equal Angles at the Eye.

Dem. Here we must suppose the Pupil of the Eye to be a Point only, because the Magnitude of that causes some small Alteration in the apparent Magnitude of the Object; as we shall see by and by. Let then the Point *a* represent the Pupil, then will the extreme Rays that can enter it be H*a* and K*a*, the Object therefore will appear under the Angle H*a*K, which is equal to its vertical one M*a*I under which the Image IM would appear, were it to be seen from *a*. Again if the Eye be placed in *f*, the Object appears under the Angle G*f*O equal to I*f*M which the Image subtends at the same Place, and therefore the apparent Object and Image of it subtend equal Angles at the Eye. Q. E. D.

Now if we suppose the Pupil to have any sensible Magnitude, such suppose that its Diameter may be *ab*, then the Object seen by the Eye in that Situation will appear under the Angle H*x*L, which is larger than the Angle H*a*K under which it appear'd before; because the Angle at *x* is nearer than the Angle at *a*, to the Line IM, which is a Subtense common to them both.

From this Proposition it follows, that, were the Eye close to the Surface at K, the real and apparent Object would be seen under equal Angles (for the real Object appears from that Place under the same Angle that the Image does, as will be shewn at the End of this Chapter) therefore when the Eye is nearer to the Image than that Point, the Image will subtend a larger Angle at it than the Object does; and consequently since the Image and apparent Object subtend equal Angles at the Eye, the apparent Object must necessarily be seen under a *larger* Angle than the Object itself, wherever the Eye be placed between the Surface and the Image.

Magnitude and Situation of the Image in all Cases whatever. Thus,

Through the Extremities of the Object AB, and the Center C, (Fig: 59, 60, or 61.) draw the Lines AC, BC, and produce them as the Case requires; these Lines will be perpendicular to the reflecting Surface, and therefore the Extremities of the Image will fall upon them. Through F the middle Point of the Object and the Center, draw the Line FC and produce it till it passes through the reflecting Surface, this will also be perpendicular to the Surface. Through G, the Point where this Line cuts the Surface, draw the Lines AG and BG and produce them this way or that, till they cross the former Perpendiculars; and where they cross, there I and M the Extremities of the Image will fall. For supposing AG to be a Ray proceeding from the Point A and falling upon G, it will be reflected to B; because FA is equal to FB, and FG is perpendicular to the reflecting Surface; and therefore the Representation of the Point A will be in BG produced as well as in AC, consequently it will fall on the Point I where they cross each other. Likewise the Ray BG will for the same Reason be reflected to A, and therefore the Representation of the Point B will be in AG produced as well as in some Part of BC, that is, in M where

whence they croſs. From whence the Propoſition is clear.

If it happens that the Lines will not croſs which way ſoever they are produced, as in (Fig. 62.) then is the Object in the *Focus* of parallel Rays of that Surface, and has no Image formed in any Place whatever. For in this Caſe the Rays AH, AG, flowing from the Point A, become parallel after Reflection in the Lines HC, GB, and therefore do not flow as to or from any Point: in like Manner Rays flowing from B are reflected into the parallel Lines KB and GA; ſo that no Repreſentation can be formed by ſuch Reflection.

From hence we learn another Circumſtance relating to the Magnitude of the Image made by Reflection; *viz.* that it ſubtends the ſame Angle at the *Vertex* of the reflecting Surface that the Object does. This appears by Inſpection of the 59, 60, or 61ſt Figure, in each of which the Angle IGM, which the Image ſubtends at G the *Vertex* of the reflecting Surface, is equal to the Angle AGB, which the Object ſubtends at the ſame Place; for in the two firſt of thoſe Figures they are vertical, in the third they are the ſame. And

Farther, the Angle ICM, which the Image ſubtends at the Center, is alſo equal to the Angle ACB which the Object ſubtends at the ſame Place; for in the two firſt Figures

they are the ſame, in the laſt they are vertical to each other.

From whence it is evident, that the Object and its Image are to each other in Diameter, either as their reſpective Diſtances from the *Vertex* of the reflecting Surface, or as their Diſtances from the Center of the ſame.

IV. As Objects are multiplied by being ſeen through tranſparent *Media*, whoſe Surfaces are properly diſpoſed, as was explained (Chap. VII. Prop. 4.) ſo they may alſo by reflecting Surfaces. Thus,

1. If two reflecting Surfaces be diſpoſed at right Angles, as the Surfaces AB, BC, (Fig. 63.) an Object at D may be ſeen by an Eye at E, after one Reflection at F, in the Line EF produced; after two Reflections, the firſt at G, the ſecond at H, in the Line EH produced; and alſo after one Reflection made at A, in the Line EA produced.

2. If the Surfaces be parallel, as AB, CD, (Fig. 64.) and the Object be placed at E and the Eye at F, the Object will appear multiplied an *infinite* Number of Times: Thus, it may be ſeen in the Line FG produced, after one Reflection at G; in the Line FH produced, after two Reflections, the firſt at I, the ſecond at H; and alſo in FP produced, after ſeveral ſucceſſive Reflections of the Ray EL, at the Points L, M, N, O, and P: and ſo on *in infinitum*. But the greater the Number of Re-

flections

flections are, the weaker the Representation will be.

There are reflecting Surfaces made, some cylindrically convex, others cylindrically concave; both which Kinds are designed to render the Image of an Object deformed, or the contrary, by augmenting or diminishing it in one Dimension and not in another; for the former Sort are convex one way, *viz.* round their *Axes*, and plain the other, *viz.* lengthwise; the other are concave round their *Axes*, and plain the other way: upon which Account Surfaces of these Kinds necessarily have such Effects.

When a very deformed Picture, or rather no Picture at all, but a seemingly irregular and accidental Position of Colours is placed before one of these Surfaces, and seen by Reflection from thence, a beautiful and well proportion'd Image shall appear therein. But to explain *Phænomena* of this Kind is not the Intent of this *Compendium*. Enough has been said to explain the Principles of *Catoptrics*. Pass we on now to the third and last Division of *Optics*, viz. *the Doctrine of Colours.*

CHAP. XI.

Of the different Refrangibility *in the* Rays *of* Light; *of the* Colours *the* distinct Species *of them are disposed to excite*; *and of the Cause of that* Variety *of* Colours *which is observable in Bodies.*

IN treating of the Refraction of Light in the Beginning of this Part, we supposed that all Light, in passing out of one *Medium* into another of different Density, is equally refracted in the same or like Circumstances. This is the Notion the Philosophers before Sir *Isaac Newton*'s Time had of it; but that indefatigable and circumspect Author has discovered that it is not so, but that *there are different* Species *of Light*; *and that each* Species *is disposed both to suffer a different Degree of Refrangibility in passing out of one* Medium *into another*, *and to excite in us the* Idea *of a different Colour from the rest*; *and that Bodies appear of that Colour which arises from the Composition of the Colours the several* Species *they reflect are disposed to excite.*

There are Abundance of Experiments brought by Sir *Isaac Newton* and others for the Con-

firmation of this Doctrine; I shall only select the following ones, which will sufficiently illustrate the Proposition, and evince the Truth of it. And

First, There are different *Species* of Light, and each *Species* is disposed to suffer a different Degree of Refrangibility, and to excite the *Idea* of a different Colour.

To shew this, let a Room be darkened, and the Sun permitted to shine into it through a small Hole in the Window-shutter, and be made to fall upon a Glass *Prism* (by which is meant a Piece of Glass of a triangular Form, such as is represented in Figure 65.) then will the Sun's Light in passing through this *Prism* suffer different Degrees of Refraction, and by that means be parted into different Rays, which Rays being received upon a clean white Paper will exhibit the following Colours *viz. Red, Orange, Yellow, Green, Blue, Indigo,* and a *Violet Purple.* Thus, let AB (Fig. 66.) represent the Window-shutter, C the Hole in it, DEF the Prism, ZY a Ray of Light coming from the Sun, which passes through the Hole and falls upon the Prism at Y, and if the *Prism* were removed would go on to X, but in entring its first Surface EF shall be refracted into the Course YW, falling upon the second in W, where in going out into the Air it shall be refracted again. Let the Light now, after it has passed the *Prism*, be received upon a

Sheet

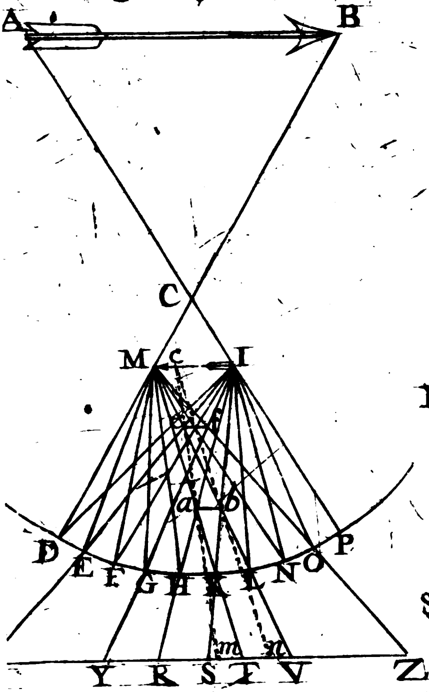
Sheet of white Paper GHIK held at a proper Diſtance, and it will exhibit upon the Paper a Picture or Image at LM of an oblong Figure, whoſe Ends are ſemicircular and Sides ſtrait. And it ſhall be variegated with Colours after the following Manner. From the Extremity M to ſome Length ſuppoſe to the Line *no*, it ſhall be of an intenſe *Red*; from *no* to *pg* it ſhall be of an *Orange* Colour; from *pg* to *rs* it ſhall be *Yellow*; from thence to *tu* it ſhall be *Green*; from thence to *wx* *Blue*; from thence to yz *Indigo*; and from thence to the End *Violet*. And if the whole Image be divided lengthwiſe into 360 equal Parts, the *Red* ſhall take up 45 of them, the *Orange* 27, the *Yellow* 48, the *Green* 60, the *Blue* 60, the *Indigo* 40, and the *Violet* 80[f].

To

[f] Sir *Iſaac Newton* in his *Optics* has ſhewn, how from the Refraction of the moſt refrangible and leaſt refrangible Rays, to find the Refraction of all the intermediate ones. His Rule is this, if the Sine of Incidence be to the Sine of Refraction in the leaſt refrangible Rays as AV to BC (Fig. 67.) and to the Sine of Refraction in the moſt refrangible as AV to BD; and if CE be taken equal to CD, and then ED be ſo divided in F, G, H, I, K, L, that ED, EF, EG, EH, EI, EK, EL, EC, may be proportional to the eight Lengths of muſical Chords, which ſhall ſound the Notes in an Octave, ED being the Length of the *Key*, EF the Length of the *Tone* above that Key, EG the Length of the *leſſer Third*, EH of the *Fourth*, EI of the *Fifth*, EK of the *greater Sixth*, EL of the *Seventh*, and EC of the *Octave* above that Key; that is, if the Lines ED, EF, EG, EH, EI, EK, EL, and EC bear the ſame Proportion to each other as the Numbers

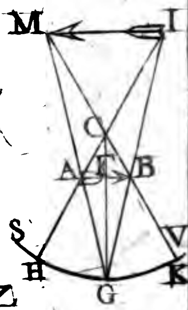
1,

F. 58. p. 127.

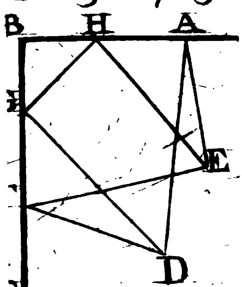


F. 59.

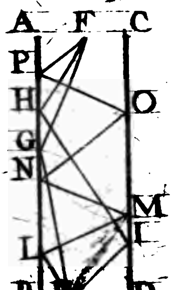
F. 61.



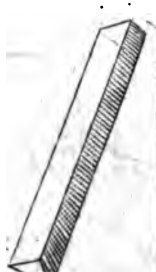
F. 63. p. 132.



F. 64.



F. 65.



To render this Proof complete, we must now shew, that these Dispositions of the Rays of Light, to produce some one Colour, and some another, which manifest themselves after being refracted, are not wrought by any Action of the Prism upon them, but are originally inherent in those Rays; and that the Prism only affords each *Species* an Occasion of shewing its distinct Quality, by separating them one from the other, which before, while they were blended together in the unrefracted Light of the Sun, lay concealed.

This will be proved by the following Experiment: Things remaining as in the foregoing one,

1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, respectively, then shall BD and BF be the Limits of the Sines of Refraction of the *violet* Rays; that is, the *violet* coloured Rays shall not all of them have precisely the same Sine of Refraction, but none of them shall have a greater Sine than BD nor a less than BF, though there be *violet* coloured Rays which answer to any Sine of Refraction that can be taken between these two. In the same manner BF and BG are the Limits of the Sines of Refraction of the *Indigo*; BG and BH are the Limits belonging to the *Blue*; BH and BI, the Limits pertaining to the *Green*; BI and BK, the Limits for the *Yellow*; BK and BL, the Limits for the *orange* coloured Rays; and lastly BL and BC, those of the Sines of Refraction belonging to the *Red*.

And particularly, when Light passes out of Glass into Air; if the Sine of its Angle of Incidence be 50, the Sine of the Angle of Refraction of the *Red* will be between 77 and $77\frac{1}{8}$, of the *Orange* coloured between $77\frac{1}{8}$ and $77\frac{1}{5}$, of the *Yellow* between $77\frac{1}{5}$ and $77\frac{1}{3}$, of the *Green* between $77\frac{1}{3}$ and $77\frac{1}{2}$, of the *Blue* between $77\frac{1}{2}$, and $77\frac{2}{3}$, of the *Indigo* between $77\frac{2}{3}$ and $77\frac{7}{9}$ and of the *violet* coloured Rays between $77\frac{7}{9}$ and 78.

let another Prism, as NO (Fig. 68.) be placed either immediately, or at some Distance after the first, in a perpendicular Situation with respect to the former, so that it may refract the Rays issuing from the first, sideways. Now, if this Prism could separate the Light which falls upon it into coloured Rays, as the other did, it would divide the Image breadthwise into Colours, as before it was divided lengthwise; but no such Thing is observable: For the Image shall only be thrown out of the perpendicular Situation LM into the oblique one PQ; the upper Parts, which were more refracted in the former Case, being more refracted in this, and therefore made to recede farther sideways from their former Situation L, than the lower ones are from M. And farther, each Colour shall be uniform from Side to Side in the oblique Image, as well as in the perpendicular one.

If there be any Objection against the Sufficiency of this Proof, it must be, that the Rays, when they fall upon the second Prism, are not all in like Circumstances, with regard to their Inclination to its Surface; I shall therefore, to obviate that Objection, add one more Experiment which seems to be peculiarly adapted to that Purpose. It is as follows:

Two Boards AB, CD, (Fig. 69.) being erected in a darkened Room at a proper Distance, one of them AB, being near the Window-shutter EF, a Space being only left for the

Prism

Priſm GHI to be placed between them; ſo that Part of the Rays, which enter the Hole M, may, after paſſing through the Priſm, be tranſmitted through a ſmaller Hole K made in the Board AB, and paſſing on from thence go out at another Hole L, made in the Board CD, of the ſame Size as the Hole K, and ſmall enough to tranſmit the Rays of one Colour only at a Time: Let another Priſm PQR be placed behind the Board CD to receive the Rays paſſing through the Holes K and L, and after Refraction by that Priſm, let the Rays fall upon the white Surface ST. Suppoſe, firſt, the *violet* Light to paſs through the Holes, and to be refracted by the Priſm PQR to *s*, which, if that Priſm were not there, would have paſſed on to W. If the Priſm GHI be turned about ſlowly, ſo that the incident Ray ZY may fall more obliquely upon it, while the Boards and the other Priſm remain fixed, in a little Time another Colour, ſuppoſe *Indigo*, which we may ſuppoſe before to have proceeded to *i*, will paſs through the Holes K and L, and, if the Priſm PQR were away, would proceed like the former Rays to the ſame Point W. Now the Refraction of this Priſm will not carry theſe Rays to *s* as it did the other, but to ſome Place leſs diſtant from W, as to *t*. But it is manifeſt that the Holes K and L being in the ſame Situation in each Caſe, both Sorts of Rays enter the Priſm PQR under the ſame

Circumstances, for they are equally inclined to its Surface RP, and enter it at the same Point thereof; which shews that the one *Species* is more diverted out of its Course by Refraction than the other is, when the Circumstances of Incidence are the same in each. Farther, if the Prism GHI be turned about till the Rays which exhibit *Blue* pass through the Hole L, these will fall upon the Surface ST below *t*, as at *u*, and therefore are subject to a less Degree of Refraction than such as produce *Indigo*. And thus by proceeding it will be found that the *Green* is less refracted than the *Blue*, and so of the remaining Colours, according to the Order in which they are represented in an Image formed by a single Prism g.

g There are Abundance of Experiments made by the Author of this Doctrine and others for the Confirmation of it, as was observed above. To give them all at full Length would tire the Reader. As that if a Body be painted one half Red, and the other Blue, and then viewed through a Prism; the apparent Place of the one half shall be different from that of the other: and if it be painted with a Mixture of these two Colours it shall appear confused and deformed. Both which evidently shew that the Rays, which each of these Colours reflect, suffer different Degrees of Refraction in passing through the Prism. And if two Bodies be painted, the one Red and the other Blue, and the Rays which flow from them be made to pass through a convex *Lens*, the *Focus*, made by the Concurrence of the Rays which flow from that which is painted with Blue, shall fall at a less Distance behind the *Lens*, than that which is made by those which come from the Red one. See the Experiment made by Dr. *Desaguliers* in a very accurate Manner (Philosoph. Transf. Nº. 426.) in Opposition to Senr. *Ritzetti*, who disputes the Conclusiveness of Sir *Isaac Newton*'s Experiments.

And alſo each *Species* of Rays is diſpoſed to excite in us the *Idea* of a different Colour.

This is ſufficiently clear from what has been already ſaid, and is farther confirmed by what follows, *viz.* That whatever Species of Rays are thrown upon any Body, they make that Body appear of their own Colour. Thus *Minium* in *red* Light appears of its own Colour; but in *yellow* Light it appears yellow; and in *green* Light it appears green; in *blue*, blue; and in *violet-purple* coloured Light it appears of a purple Colour: in like manner Verdigreaſe will put on the Appearance of that Colour in which it is placed. But each of theſe Bodies appears moſt luminous and bright when enlightened with its own Colour, and dimmeſt in ſuch as are moſt remote from that. 'Tis certain therefore each Ray is diſpoſed to excite its own Colour, which is neither to be alter'd by Refraction nor Reflection.

Thus much in Confirmation of the firſt Part of the Propoſition, *viz.* That there are different *Species* of Light, that each *Species* is diſpoſed to ſuffer a different Degree of Refrangibility, and to excite in us the *Idea* of a different Colour. We proceed now to the ſecond Part of the Propoſition, *viz.*

2. That Bodies appear of that Colour, which reſults from a Compoſition of thoſe Colours,

which

which the several *Species* they reflect are disposed to excite.

We have just now seen that each Ray, whatever be the Colour of the Body it is reflected from, is able to excite no other *Idea* than that of its own Colour; and that coloured Bodies reflect not all the different Sorts of Rays that fall upon them in equal Plenty; but some Sorts, *viz.* those of their own Colour, much more copiously than others. We will now proceed to shew, that the other Colours may be produced from a Mixture of those seven, which Rays of Light when separated by a Prism are disposed to exhibit, *viz. Red, Orange, Yellow, Green, Blue, Indigo,* and *Violet.* From whence it will be rational to conclude, that Bodies appear of that Colour which arises from the Mixture of those which they reflect.

1. All the *prismatic* Colours (*viz.* those which are made by the Prism) mixed together appear white, a little inclining to Yellow, such as is that of the Sun's Light.

To shew this, let a convex *Lens* be placed between the Prism and the Paper which receives the Image, in order that the Rays separated by it may be collected into a *Focus*; and let the *Focus* fall upon the Paper, then will the Spot where it falls appear white. And that the Whiteness of this focal Point is owing to the Union of those Colours appears from hence, that if we remove the Paper from the focal

focal Point, and suffer the Rays to cross each other in the *Focus*, and if when they have proceeded to some Distance beyond, they be then received upon the Paper, the same coloured Image will be exhibited, but inverted, because the Rays cross each other in the *Focus*; an evident Proof that the Whiteness of the Spot was owing to Nothing but the Mixture of the Rays constituting the several Colours of the Image. But if the Rays of any particular Colour be intercepted before they are collected in the said Spot, it then appears not only of a different Colour from what it did before, but different from any of the *prismatic* Colours taken separately.

Or if the Circumference of a Wheel be painted with the *prismatic* Colours taken in the same Proportion with respect to each other, in whith they are exhibited in the Image made by the Prism, and the Wheel be turned swiftly about, the Circumference of that Wheel shall appear *white*: if they are taken in other Proportions, the Colour of the Wheel when turned about will vary accordingly. From whence this Part of the Proposition is also abundantly clear.

No Composition of these Colours will produce Black: That being no Colour, but the Defect, or Absence of all Colour whatever.

That *Species* of Light, which is disposed to suffer a greater Degree of Refraction, requires pro-

proportionally less Obliquity at the second Surface of any *Medium* to occasion a *total Reflection* of it there; so that it is possible that a Ray of Light may pass through a *Medium* with such Obliquity, that only that Part of it which is disposed to exhibit a *violet* Colour shall be reflected at the second Surface, and all the rest transmitted there. This indeed is a necessary Consequence of what was observed concerning the Reflection of Light at the second Surface of any *Medium*; (Chap. 8.) *viz.* that the Reflection of a Ray is total, when the Obliquity of the incident Ray is such, that the Angle of Refraction ought to be equal to, or to exceed a right one. I say this is a Consequence of that, because the Angle of the Refraction of the *violet* coloured Light is larger than the Angle of Refraction of any other, though their Angles of Incidence be equal. And accordingly thus it happens, as appears by the following Experiment.

Let AB (Fig. 70.) represent the Window-shutter of a darkened Room; C an Hole to let in a Ray of the Sun, DEF, GHI, two Prisms so applied together that the Sides EF and GI be contiguous, and the Sides DF and GH parallel: In this Situation Light will pass through them without any Separation into Colours; for the opposite Sides being parallel, if the Rays are refracted one Way where they

go

PART III. PLATE X. Pag. 144.

Fig. 66. P. 135.

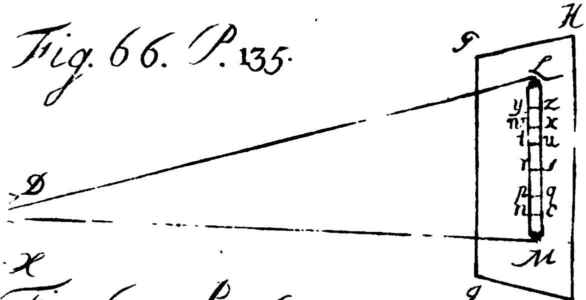


Fig. 67. P. 136

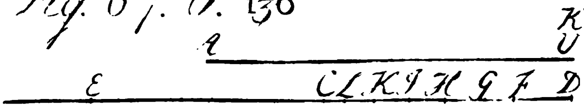


Fig. 68. P. 138.

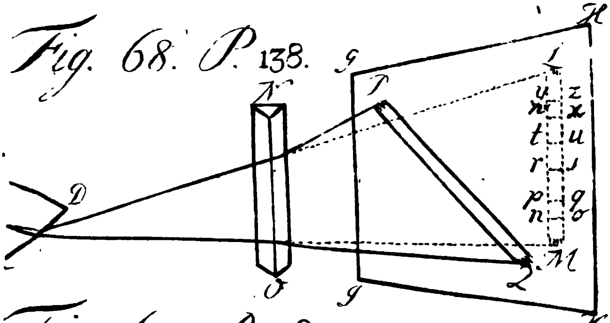
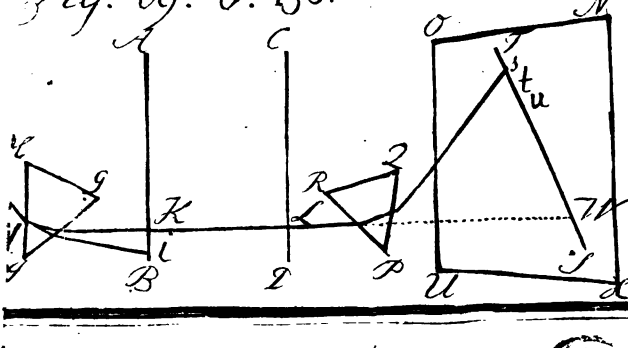


Fig. 69. P. 138.



go in, they will be as much refracted the contrary Way where they go out (See Page 51.) But if it be afterwards received by a third Prism KLM, it will be divided so as to form upon any white Body NOYU the usual Colours, *Violet* at *l*, *Indigo* at *m*, *Blue* at *n*, and *Red* at *r*. Now let it be supposed that the Surfaces EF and GI are not quite close together, but that the Rays, in passing from one to the other, pass through a *Medium* (*viz.* the Air) of different Density from that of the Prisms: and that the Ray ZC is not so much inclined to the second Surface of the first Prism as to cause a total Reflection of any one *Species* there; then will Part only of each *Species* be reflected and Part transmitted, agreeably to what was observed (Chapter 8.) concerning the Manner of Reflection. Let now the reflected Rays be received by a fourth Prism TXV; these, after passing through it, will paint upon a white Surface RS the Colours of the Prism, *viz.* *Red* at *s*, *Orange* at *t*, *Yellow* at *v*, and *Violet* at *z*. Let now the Prisms DEF, GHI, be slowly turned about, keeping still the same Situation with respect to each other, until the Obliquity of the Rays ZC to the Surface EF be so far increased, that there shall begin to be a total Reflection of them there. In which Case it is observable, that first of all the *violet* Light will be *totally* reflected, and will therefore disappear

at *l*, appearing instead thereof at *z*, and increasing the *violet* Light which fell there before. And when the Rays ZC become more oblique by the Prisms being turned farther about, the *Indigo* shall be totally reflected, disappearing at *m*, but falling upon *y*, and making the *violet* there more intense. And by turning the Prisms still farther about, all the remaining Colours will be successively removed from the Surface PQ to RS.

CHAP.

CHAP. XII.

Of the Qualifications *in Bodies, which dispose them to reflect the Rays of* different Colours.

WE are now to inquire what it is that gives Bodies this Power of reflecting, some one sort of Rays most copiously, and some another. And this is probably no other than the different Magnitude of the Particles whereof they are composed; as will appear from the following Observations.

If Water be prepared with Soap so as to render it sufficiently tenacious, and then blown up into a Bubble; it is observable, that as the Bubble grows thinner, and thinner (as it will do by reason of the Water's continually running down from the Top of it, till it breaks) different Colours will arise one after another at the Top of the Bubble, spreading themselves into Rings, and descending till they vanish at the Bottom in the same Order they arose at the Top. Thus, in an Experiment of this Kind, tried by Sir *Isaac Newton*, the Colours arose in this Order; first *Red*, then *Blue*; to which succeeded *Red* a second Time, and *Blue*

Blue immediately followed; after that *Red* a third Time, succeeded by *Blue*; to which followed a fourth *Red*, but succeeded by *Green*; after this a more numerous Order of Colours, first *Red*, then *Yellow*, next *Green* and after that *Blue*, and at last *Purple*; then again, *Red*, *Yellow*, *Green*, *Blue*, *Violet* followed each other; and the last Order of Colours that arose was *Red*, *Yellow*, *White*, *Blue*; to which succeeded a *dark* Spot that afforded scarce any Light, though it was observed to cause some very obscure Reflection, for the Image of the Sun or a Candle might be faintly discerned in it; and this last Spot spread itself more and more, till the Bubble broke.

Now 'tis apparent that the only Reason, why those different Colours succeeded each other at the Top of the Bubble in the abovementioned Manner, was because its Thickness in that Part continually varied, till it broke. It remained therefore to examine what was the Thickness of the Bubble at the Top, at the Time it exhibited each particular Colour. And this was effected by the following Contrivance, *viz.* by taking the Object Glass of a long Telescope, such having but a very small Degree of Convexity, and placing it upon a flat Glass: these Glasses by reason of the Convexity of the former would touch but in one Point, and the Distance between them, where they did not touch, would be exceedingly small, but larger the farther we

we consider it from the Point of Contact. Now Water being put between these Glasses the same Colours appeared as in the Bubble, in the Form of Circles or Rings surrounding the Point where the Glasses touched, which Point appeared black, like the Top of the Bubble when it was thinnest. Next to this Spot lay a blue Circle, and next without that a white one, and so on in the same, but contrary Order to that in which the Colours arose on the Top of the Bubble.

Now the Distance between the Glasses, that is, the Thickness of the Body of Water between them, where it exhibited any one Colour of a particular Order, was equal to the Thickness of the Bubble at the Time the same Colour appeared upon it. For though the *Medium* the Light must pass through to come at the Water is in one Case Glass, and in the other, Air; that makes no Difference in the *Species* of the Colour reflected from the Water: for Pieces of *Muscovy* Glass, made thin enough to appear coloured, would have their Colours faded, but not the *Species* of them altered by being made wet with Water. But it was found that transparent Bodies of different Density would not, under the same Thicknesses, exhibit the same Colours: for if the forementioned Glasses were laid upon each other without any Water between them, the Air between them would then afford the same Colours as the Water,

but

but more expanded, so that each Ring had a larger Diameter, though they bore all the same Proportion to each other; so that the Thickness of the Air proper to reflect each Colour was in the same Proportion larger, than the Thickness of the Water adapted to reflect the same.

Farther, all the Light which is not reflected by the thin Substances, whether of Air or Water contained between the Glasses, is transmitted through them; for when viewed from the other Side, they exhibit also coloured Rings as before, but in a contrary Order; for the middle Spot, which in the other View appears black for Want of reflected Light, now looks perfectly white; next without this Spot the Light appears tinged with a yellowish *Red*; where the *White* appeared before, it now seems *Black*; and so of the rest.

It is farther observable, that the forementioned thin Plates, whether of Air or Water, do not appear of the same Colour when viewed obliquely, as when seen direct: for if the Rings and Colours between a convex and plain Glass be viewed first in a direct Manner, and then under different Degrees of Obliquity, the Rings will be observed to dilate themselves as the Obliquity is increased. But a Plate of Air between the Glasses alters its Colour much sooner than the Water in the Bubble which is surrounded with Air. For in the Water when

when viewed obliquely the ſame Colour might be ſeen at more than twelve Times the Thickneſs it appeared at under a direct View; but when the Air was viewed under ſuch an Obliquity that the Thickneſs of the Plate, where it was obſerved, was but half as much again as when it was viewed directly, a different Colour appeared.

Laſtly, the ſame Colour reflected from a denſer Subſtance reduced to a thin Plate, and ſurrounded by a rarer, will be more briſk, than the ſame Colour, when reflected from a thin Plate formed of the rarer Subſtance, and ſurrounded by the denſer, as was found by blowing Glaſs very thin, which exhibited in the open Air more vivid Colours, than the Air does between two Glaſſes.

As to the Thickneſs of the Plate of Air by which the ſeveral Colours were reflected, it was found by carefully meaſuring the Diſtances of the Rings from the Point where the Glaſſes touched, that the Diſtance between the Glaſſes where the firſt Order of Colours was reflected, was from $\frac{1}{178000}$ to $\frac{2}{178000}$ Part of an Inch; that where the ſecond, was from $\frac{3}{178000}$ to $\frac{4}{178000}$; that where the third, from $\frac{5}{1780}$ to $\frac{6}{178000}$, and ſo on in a *Series* of the odd Numbers: and that the Diſtance of the Glaſſes, where the firſt Order of Colours that was tranſmitted paſſed through, was from 0 to $\frac{1}{178000}$ Part of an

an Inch; that where the ſecond, was from $\frac{2}{178000}$ to $\frac{3}{178000}$; that where the third, from $\frac{4}{178000}$ to $\frac{5}{178000}$, and on ſo on in a *Series* of the even Numbers. And the Thickneſs of a Plate of Water, where it reflected or tranſmitted the ſame Colours, was $\frac{3}{4}$ of the Thickneſs of the Plate of Air.

Now we learn from Experiments made with the Microſcope, that the leaſt Parts of almoſt all Bodies are tranſparent; or the ſame may be experienced in the following Manner: Take a very thin Plate of the opakeſt Body, and the Room being darkened apply it to a ſmall Hole in the Window-ſhutter, and it will ſufficiently diſcover its Tranſparency. This Experiment cannot be ſo well performed with a white Body, becauſe of the ſtrong reflective Power in ſuch; but even thoſe, when diſſolved in *Aqua Fortis* or other proper *Menſtruum*, do alſo become tranſparent. Wherefore if we ſhould ſuppoſe any Body reduced to a Thinneſs proper to produce any particular Colour and then broken into Fragments, in all Probability they would exhibit the ſame Colour, and an Heap of ſuch Fragments would conſtitute a Body of that Colour: ſo that the Cauſe, why ſome Bodies reflect one Sort of Rays moſt copiouſly, and ſome another, is probably no other than the different Magnitude of their conſtituent

Par-

Particles [h]. This will be farther confirmed by Particulars. The Colours in the same Part of a Peacock's Tail vary as the Tail changes its Posture, with respect to the Eye; just so the thin Plates of Air or Water appear of a different Colour in the same Plate when view'd directly, from what they do when seen obliquely, as was observed above. The Colours of Silks, Cloths, and other Substances, which Water or Oil can intimately penetrate, become faint and dull by being wet with such Fluids, and recover their Brightness when dry; just as we observed, that Plates of *Muscovy* Glass grew faint and dim by wetting. All which Particulars, and many more that might be produced, give abundant Proof of the present Point [i].

[h] This Sir *Isaac Newton* thinks a probable Ground for making Conjecture concerning the Magnitude of the constituent Particles of Bodies. The Green of Vegetables he takes to be of the third Order, as likewise the Blue of Syrup of Violets. The azure Colour of the Sky, he thinks is of the first Order, as also the most intense and luminous White; but if it is less strong, he then conjectures it to be a Mixture of the Colours of all Orders. Of the latter Sort he takes the Colour of Linen, Paper, and such like Substances to be; but white Metals to be of the former Sort. For producing *Black*, the Particles must be smaller than for exhibiting any of the Colours, *viz.* of a Size answering to the Thickness of the Bubble where it reflected little or no Light, and for that Reason appeared colourless.

[i] See Sir *Isaac Newton's* Optics, *passim*.

CHAP. XIII.

Of the Cause of Opacity *and* Transparency *in Bodies.*

WE obſerved above (in Chap. VIII.) that when two *Mediums* or tranſparent Subſtances of equal Denſity are contiguous, or as near to each other as the Glaſſes where the Light was wholly tranſmitted, a Ray of Light will paſs from one to the other without ſuffering either Reflection or Refraction; but if they differ in Denſity, the Light will undergo both; Part of it being reflected and Part refracted. Juſt ſo it is with a Ray of Light in paſſing through the different Particles of the ſame Body. For Inſtance, if when the Ray has paſſed through any one Particle of a Body, it finds another contiguous to it, it will enter that Particle without Interruption; but if at its Emerſion out of that Particle, it enters a Pore ſufficiently large, Part of it will be tranſmitted and Part reflected. Thus will the Light every time it enters a Pore, unleſs it be an exceeding ſmall one, be in part reflected: So that nothing more ſeems neceſſary to render a Body opake, than that the Particles, of which it is compoſed, touch but in few Points, and that the

Pores

Pores of it be very numerous and large; ſo that the Light, which enters it, may by numerous Refractions and Reflections be ſtifled and loſt within it. On the contrary, if the Particles of a Body touch each other in many Points, and its Pores be few and ſmall, or filled with a Subſtance of nearly an equal Denſity with the Particles of the Body, that Body will be tranſparent.

In Confirmation of this, we may obſerve, that opake Bodies become tranſparent by filling their Pores with a Subſtance nearly of the ſame Denſity with that of their ſolid Parts: As when Paper is made wet with Water or Oil; when Linnen Cloth is dipped in Water, oiled, or varniſhed, or the *Oculus Mundi* Stone ſteeped in Water. Beſides, as filling the Pores of an opake Body renders it tranſparent, ſo on the other hand evacuating the Pores of a Body that is tranſparent, or ſeparating the Parts of it from one another, makes it opake; as Salt or wet Paper by being dried, and Glaſs by being powder'd, loſe their Tranſparency, or Water beat up into Froth.

Beſides which Inſtances, abundance more might be brought in Confirmation of what is here laid down; but theſe are ſufficient.

But becauſe it may be Matter of Surprize, that Bodies ſhould be ſufficiently porous to tranſmit Light in that Plenty we obſerve they

do, and yet be hard or solid; I shall conclude the Subject of this Chapter and of the Doctrine of Light and Colours, with shewing the Consistency of such a Supposition. In order to which, let us imagine a Body whose constituent Particles are of such a Form that, when laid together, the Vacancies between them may be equal in Bigness to the Particles; how this may be done, and yet the Body be hard, is easy to comprehend. Now the solid Parts of a Body thus formed will be but half its Bulk; and if we suppose each constituent Particle of this Body to be formed of less Particles with Vacuities between them, equal to each Particle as before, the solid Parts of this Body will then be but a fourth Part of its Bulk; and if each of these lesser Particles again be formed in the same Manner, the solid Parts of the Body shall be but one Eighth of its Bulk: and thus if the Composition be continued according to the same Rule, the solid Parts of the Body may be made to bear as small a Proportion to its whole Magnitude as shall be desired, notwithstanding which, the Body, by means of the Contiguity of the Parts, shall be capable of being hard in any Degree. Thus any the least Portion of Matter may be supposed to be wrought into a Body of any designed Dimensions how great soever, and yet the Pores of that Body none of them greater than

the

the ſmalleſt Magnitude propoſed at Pleaſure; and yet the Parts of the Body ſhall ſo touch, that the Body itſelf ſhall be hard or ſolid. Which ſhews that the whole Globe of Earth, nay, all the known Bodies of the Univerſe, for any thing that appears to us to the contrary, may be compoſed of no greater Quantity of Matter than what might be reduced into a Globe of an Inch Diameter, or into a *Nut-ſhell*.

DISSERTATION II.

Of the Cause *of* Reflection *of* Light.

THE Opinions of Philosophers relating to the Cause of this difficult *Phænomenon*, are principally four, which I shall here lay down and examine particularly; after which I shall give my own Thoughts, concerning it. And,

1. It was the Opinion of Philosophers before Sir *Isaac Newton* discovered the contrary, that Light is reflected by impinging upon the solid Parts of Bodies; but that it is not so, is clear for the following Reasons.

And first it is not reflected at the first Surface of a Body by impinging against it.

For it is evident, that in order to the due and regular Reflection of Light, that is, that the reflected Rays should not be dispersed and scattered one from another, there ought to be no Rasures or Uneavinefs in the reflecting Surface large enough to bear a sensible Proportion to the Magnitude of a Ray of Light: because if the Surface abounds with such, the reflected Rays will rather be scattered like a Parcel of Pebbles thrown upon a rough Pavement, than reflected with that Regularity with which Light is observed to be from a well

well polished Surface. Now those Surfaces, which to our Senses appear perfectly smooth and well polished, are far from being so; for to polish is no other than to grind off the larger Eminences and Protuberances of the Metal with the rough and sharp Particles of Sand, Emery or Putty, which must of Necessity leave behind them an Infinity of Rasures and Scratches, which though inconsiderable with regard to the former Roughnesses, and too minute to be discerned by us, must nevertheless bear a large Proportion to, if not vastly exceed the Magnitude of the Particles of Light.

Secondly, it is not reflected at the second Surface, by impinging against any solid Particles.

That it is not reflected by impinging upon the solid Particles which constitute this second Surface, is sufficiently clear from the foregoing Argument; the second Surfaces of Bodies being as uncapable of a perfect Polish as the first; and it is farther confirmed from hence, *viz.* that the Quantity of Light reflected differs according to the different Density of the *Medium* behind the Body: And that it is not reflected by impinging upon the Particles which constitute the Surface of the *Medium* behind it, is evident, because the strongest Reflection of all at the second Surface of a Body, is when there is a *Vacuum* behind it. This therefore wants no farther Proof. (See the Manner

ner in which Light is reflected in Chapter the 8th.)

II. It has been thought by some [k]; that it is reflected at the first Surface of a Body, by a repulsive Force equally diffused over it, and at the second, by an attractive Force.

1. If there be a repulsive Force diffused over the Surface of Bodies, that repels Rays of Light at all Times, then, since by increasing the Obliquity of a Ray we diminish its perpendicular Force (which is that only, whereby it must make its Way through this repulsive Force) however weakly that Force may be supposed to act, Rays of Light may be made to fall with so great a Degree of Obliquity on the reflecting Surface, that there shall be a *total* Reflection of them there [l], and not one Particle of Light be able to make its Way through, which is contrary to Observation; the Reflection of Light at the first Surface of a transparent Body being never total in any Obliquity

[k] See *Muschenbroek*, Element. Physic. Cap. 35.

[l] *Dem.* Let AB (Fig. 71.) represent the reflecting Surface, ABCD the repellent Power diffused over it, EF a Ray of Light incident upon it at the Point F, and let the Line EF by its Length express the Force with which the Ray moves. This Force is resolvable into the Forces EG and EH, or, which is the same thing HF and GF, which latter is the sole Force by which the Ray endeavours to pierce through the repulsive Power. But this Force may be diminished *in infinitum* by augmenting the Obliquity of the Ray EF, and therefore it may be made less than that of the repelling Power, in which Case the Ray will necessarily be reflected: and since the

Obliquity whatever. The *Hypothesis* therefore in this Particular must be false [m].

2. As to the Reflection at the second Surface by the attractive Force of the Body; this may be considered in two Respects; first, when the Reflection is *total*; secondly, when it is *partial*.

And first, in Cases where the Reflection is *total*, the Cause of it is undoubtedly that same attractive Force by which Light would be refracted in passing out of the same Body. This is manifest from that Analogy which is observable between the Reflection of Light at this second Surface, and its Refraction there. For otherwise, what can be the Reason that the total Reflection should begin just when the Obliquity of the incident Ray, at its Ar-

the same is demonstrable of any Ray, (let it move with a greater or less Force than the Ray EF) the Obliquity of the Rays may be so great, that there shall be a total Reflection of them. Q. E. D.

m And yet from an Experiment made by Sir *Isaac Newton*, (See his Optics Book III.) it appears that there is a repulsive Force between Light and some Bodies. The Experiment is as follows. If over an Hole in a Window-shutter be fixed a thin Piece of Lead or the like, in which there is an Hole about the fortieth Part of an Inch in Diameter; and if when the Sun shines through that Hole, an Hair be held in its Ray at some Distance from the Hole; the Progress of the Rays after they have passed by the Hair will be as expressed in the 72d Figure, where A is a Section of an Hair, BC an Hole in a Window-shutter; BL, UF, DE, &c. Rays passing through it, in the Middle of which is placed the Hair A; and let RP be a Paper held at some Distance behind the Hair. Things being thus disposed, the Ray BL shall fall upon O, U F upon Q, DE upon R, GH upon P, &c. as represented in the Figure.

rival at the ſecond Surface, is ſuch, that the refracted Angle ought to be a right one; or when the Ray, were it not to return in Reflection, ought to paſs on parallel to the Surface, without going from it? For in this Caſe it is evident, that it ought to be returned by this very Power, and in ſuch Manner that the Angle of Reflection ſhall be equal to the Angle of Incidence: juſt as a Stone thrown obliquely from the Earth, after it is ſo far turned out of its Courſe by the Attraction of the Earth, as to begin to move horizontally, or parallel to the Surface of the Earth, is then by the ſame Power made to return in a Curve ſimilar to that which it deſcribed in its Departure from the Earth, and ſo falls with the ſame Degree of Obliquity that it was thrown with.

But ſecondly, as to the Reflection at the ſecond Surface, when it is *partial*; an attractive Force *uniformly* ſpread over it, as the Maintainers of this *Hypotheſis* conceive it to be, can never be the Cauſe thereof. Becauſe, it is inconceivable that the ſame Force, acting in the ſame Circumſtances in every Reſpect can ſometimes reflect the *Violet* coloured Rays and tranſmit the *Red*, and at other times reflect the *Red* and tranſmit the *Violet*.

This Argument concludes equally againſt a repulſive Force *uniformly* diffuſed over the firſt

first Surface of a Body, and reflecting Light there; becaufe fome Bodies reflect the *Violet* and tranfmit the *Red*, others reflect the *Red* and tranfmit the *Violet* at their firft Surface; which cannot poffibly be upon this Suppofition, the Rays of whichever of thefe Colours we fuppofe to be the ftrongeft.

III. Some, being apprehenfive of the Infufficiency of a repulfive and attractive Force diffufed over the Surfaces of Bodies and acting uniformly, have fuppofed, that by the Action of Light upon the Surfaces of Bodies the Matter of thefe Forces is put into an *undulatory* Motion, and that where the Surface of it is fubfiding, Light is tranfmitted, and in thofe Places where it is rifing, Light is reflected [n]. But this feems not to advance us one Jot farther; for in thofe Cafes, fuppofe where *Red* is reflected and *Violet* tranfmitted, how comes it to pafs that the *Red* impinges only on thofe Parts when the Waves are rifing, and the *Violet* when they are fubfiding?

IV. The next *Hypothefis*, that I fhall take notice of, is that remarkable one of Sir *Ifaac Newton*'s *Fits of eafy Reflection and Tranfmiffion*, which I fhall now explain and examine.

That Author, as far as I can apprehend his Meaning in this Particular, is of Opinion, that

[n] See *Mufchenbrock* Element. Phyfic. Cap. 35.

Light, in its Passage from the luminous Body, is supposed to be *alternately* reflected by and transmitted through any refracting Surface it may meet with; that these Dispositions (which he calls *Fits of easy Reflection and easy Transmission*) return successively at equal Intervals: and that they are communicated to it at its first Emission out of the luminous Body it proceeds from, probably by some very subtle and elastic Substance diffused through the Universe, and that in the following Manner. As Bodies falling upon Water, or passing through the Air cause Undulations in each, so the Rays of Light may excite Vibrations in this elastic Substance. The Quickness of which Vibrations depending on the Elasticity of the *Medium* (as the Quickness of the Vibrations in the Air, which propagate Sound, depend solely on the Elasticity of the Air, and not upon the Quickness of those in the sounding Body) the Motion of the Particles of it may be quicker than that of the Rays, and therefore when a Ray, at the Instant it impinges upon any Surface, is in that Part of the Vibration of this elastic Substance which conspires with its Motion, it may be easily transmitted, and when it is in that Part of its Vibration which is contrary to its Motion, it may be reflected. He farther supposes, that when Light falls upon the first Surface of a Body, none is reflected there,

but

but all that happens to it there is, that every Ray that is not in a Fit of eafy Tranfmiffion is there put into one, fo that when they come at the *other* Side (for this elaftic Subftance eafily pervading the Pores of Bodies, is capable of the fame Vibrations within the Body as without it) the Rays of one Colour fhall be in a Fit of eafy Tranfmiffion, and thofe of another in a Fit of eafy Reflection, according to the Thicknefs of the Body, the Intervals of the Fits being different in Rays of a different Kind. This very well accounts for the different Colours of the Bubble and thin Plate of Air and Water (mention'd in Chap. XII.) as is obvious enough; and likewife for the Reflection of Light at the fecond Surface of a thicker Body; for the Light reflected from thence is alfo obferved to be coloured, and to form Rings according to the different Thicknefs of the Body, when not intermix'd and confounded with other Light, as will appear from the following Eperiment. If a Piece of Glafs be ground concave on one Side, and convex on the other, both its Concavity and Convexity having one common Center, and if a Ray of Light be made to pafs through a fmall Hole in a Piece of Paper held in that common Center, and be permitted to fall on the Glafs; befides thofe Rays which are regularly reflected back to the Hole again, there will be others reflected

to

to the Paper, and form coloured Rings surrounding the Hole, not unlike those occasioned by the Reflection of Light from thin Plates. The same will happen if the Rays be reflected from a metalline *Speculum*, but the Light will not be *coloured*; which shews that the Colours arise from that Light which is reflected from the Back-side, and that in the following Manner: Beside that Light which is regularly reflected from the farther Surface of the Glass, there is some reflected irregularly, which passing from the back Surface under different Obliquities, does as it were pass through Glasses of different Thicknesses, and therefore is in part reflected back again when it comes to the first Surface, and is in part transmitted through it, the transmitted Light, when received upon the white Paper exhibiting the Rings of Colours abovementioned °.

As to the Light which is supposed to be reflected at the first Surface, his Opinion seems to be, that it is not *there* reflected, as I observed above, but that it really enters the Surface, and is reflected from the Back-side of the first *Series* of Particles that lie therein; so that according as these Particles are larger or smaller, the Rays of Light which at their Entrance into them

° This Experiment succeeds better, when the Back-part of the Glass is Quick-silver'd over.

(for

(for they are tranfparent, whether the Body they compofe be fo or not, *See Page* 154) are thereby put into Fits of eafy Tranfmiffion; at their Emerfion at the other Side are fome in a Fit of eafy Tranfmiffion, others in a Fit of eafy Reflection, according as the Interval of their Fits are large or fmall. So that the Particles of a Body may be of fuch a Size that they fhall reflect the *Red* and tranfmit the *Violet*; or that they may reflect the *Violet* and tranfmit the *Red*; or, in general, that the ftrongeft and moft forcible Rays may be tranfmitted, while the weaker are reflected; or the weaker may be tranfmitted, while the ftronger are reflected.

Thus I have endeavour'd to clear up the Account Sir ISAAC NEWTON has left us of his own Sentiments concerning this Matter. But after all, I cannot fay, that I think his Solution the true one. It is too much clogged with Suppofitions; neither is it confonant to that Simplicity, Uniformity, and Regularity, with which Nature is every where obferved to act. The Time will come, when that fimple Principle of *Attraction* which folves every thing elfe, where the neceffary *Data* are not wanting, will be found alone fufficient to account for this perplexing *Phænomenon*. Would any one that has a *Genius* for a Work of this Kind, and *Opportunity* to make the

the neceſſary Experiments, aſſume thoſe Sir *Iſaac Newton* has left, and add others, as his Judgment ſhould direct him; he would ſoon be able to give us as eaſy and ſimple a Solution of the *Reflection of Light*, as we already have of any other *Phænomenon* whatever [p].

[p] Perhaps it may be of Service to one that ſhall undertake this, to acquaint him of a Fact relating to this Matter, which every Philoſopher is not appriſed of, *viz.* That a Piece of Iron, when heated, aſſumes all the Colours of the Rainbow before it becomes red-hot.

DISSERTATION III.

Of Microſcopes *and* Teleſcopes.

THAT the Teleſcope is of modern Invention is moſt certain; neither does it appear that Microſcopes or *Optic* Glaſſes of any Kind were known to the Ancients, though there are two Paſſages brought to ſhew that ſuch Glaſſes were of Uſe among them. The one is quoted by *Pancirollus* from *Plautus*, *Cedo Vitrum, neceſſe eſt* Conſpicillo *uti*; the other is taken from *Pliny*, *C. Julius Medicus, dum inungit* Specillum, *&c.* The former of theſe Quotations is a Fiction, no ſuch Paſſage being to be found in the Writings of *Plautus* [q]; and the Word *Specillum* in *Pliny* is not to be underſtood of an Optic-glaſs of any Kind, but of a Probe or other Inſtrument made uſe of by the Surgeons of that Time [r].

It is contended, that *Alexander de Spina*, a Native of *Piſa*, was the firſt that made the Uſe of Glaſſes known to the World; but our Countryman, Frier *Bacon*, who died one and twenty Years before him [ſ], was, in

[q] Vid. Lettere Memorabili del Abbate Michele Giuſtiani. Parte terza, Let. 16.

[r] See *Molineux*'s Dioptrics, Part II. Ch. 6.

[ſ] In the Year 1292.

all Probability, acquainted with them firſt, for he wrote a Book of *Perſpective*, in which, he plainly ſhews that he did not only underſtand the Nature of convex and concave Glaſſes, but the Uſe of them when combined in Teleſcopes [t]; though he no where in that Treatiſe diſcovers the Manner in which they are to be put together.

The Teleſcope with the concave Eye-glaſs was firſt invented by a Mechanic of *Middleburg* in *Zeland*, called *Z. Johannides*, about the Year 1590, though *J. Lipperhoy*, another *Dutchman*, is Candidate for the ſame Diſcovery [u]. From whence, this Sort of Teleſcope is called *Tubus Batavus* [x].

Franciſcus Fontana, a *Neapolitan*, contends, that he was the firſt Contriver of the Teleſcope compoſed of two convex Glaſſes, which is now the common aſtronomical Teleſcope [y]; and *Rheita* pretends to be the firſt that rendered that Teleſcope fit for terreſtrial Uſes, by adding two Eye-glaſſes to it [z]. This Kind of Teleſcope is called *dioptrical.*

t See his *Perſpective*, Part III. and his *Epiſtola ad Pariſienſem*, Cap. 5.

u Vid. *Borellus* de vero Teleſcopii Inventore.

x This is by ſome called *Galileo*'s Teleſcope, as invented by him; but *Galileo* acknowledges, that it was upon hearing that the *Dutchman* had contrived one, that he effected his.

y See his *Obſervationes cæleſtium terreſtriumq; Rerum*.

z Ocul. *Enoch* & *Eliæ*. Lib. IV. Hiſt. Acad. Reg. Lib. V. Sect. 1. Cap. 7.

The

The *Catadioptrical* or reflecting Teleſcope was invented by Sir *Iſaac Newton*; of which we ſhall give a particular Deſcription when we have explained the former Sort, and ſhewn the Defects of them.

Microſcopes are of two Kinds, Simple and Compound. The firſt Sort conſiſts of one Glaſs; the other of two or more.

The Simple Microſcope is no other than a convex *Lens*, through which, as we have ſhewn (Chap. VIII.) Objects appear magnified.

An Object ſeen through this Microſcope appears magnified nearly in that Proportion which the Diſtance, at which an Object would be ſeen diſtinctly with the naked Eye, bears to the focal Diſtance of the Microſcope.

Thus, let AB (Fig. 73.) repreſent the Microſcope, CD an Object placed at the focal Diſtance of parallel Rays, or ſomething nearer, that the Rays of the ſame Pencil may be parallel to each other, or rather diverging in a ſmall Degree, when they enter the Eye (this Circumſtance being requiſite to diſtinct Viſion): And let the Microſcope be ſo ſmall, that all the Rays which paſs through it from the Object may enter the Pupil of the Eye EF at the ſame time, when placed cloſe to it

as in the Figure (for, unless it be so small, it will scarce magnify sufficiently to obtain the Name of a Microscope.) Things being thus disposed, the Angle under which this Object appears will be GIH, or CID; but this is nearly the same it would have appeared under, had there been no Microscope interposed [a]. Notwithstanding which, the Object is properly enough said to appear magnified by this Microscope, because, without that, it could not have been seen distinctly at so small a Distance from the Eye, but must have been situated eight or ten Inches from it; and therefore, since Objects appear under a larger Angle the nearer they are to the naked Eye, this Object appears larger, or is magnified by means of the Microscope, in Proportion as it is seen distinctly at a less Distance with it than without it; that is, nearly as the focal Distance of the Microscope is to that at which Objects are seen distinctly with the naked Eye [b].

[a] For had there been no Microscope interposed, the Angle CID would have been in the Middle of the Pupil (see the Note in Page 61.) and therefore something less, as being farther from the Object; but this is an *accidental* Circumstance depending on the Thickness of the *Lens*, and its Distance from the Center of the Pupil, and therefore not considered in the Theory.

[b] An Object will also appear distinct though it be situated at a very small Distance from the Eye, by being viewed thro' a small Hole in a Piece of Paper, the Reason of which was explained in Note Page 64. But then this Hole must be made so very small, that, unless the Object be strongly illuminated, it will appear very obscurely through it.

The

The Form of a Compound Microſcope is expreſſed in the 74th Figure, where AB repreſents a ſmall convex *Lens*, whoſe focal Diſtance is ſuch, that Rays flowing from the Point C may be collected in D; and EF is a larger *Lens* whoſe *Focus* of parallel Rays coincides with the Point D; and FG repreſents an Eye ſo ſituated that Rays proceeding from an Object at KL may enter the Pupil of it, after having paſſed through both Glaſſes. Things being diſpoſed in this Manner, the Object KL will appear *magnified* and alſo *diſtinct*.

For firſt, let RCS repreſent a Pencil of Rays flowing from the Point C, theſe will meet their *Axis* again in the Point D by Suppoſition, and croſſing there will enter the *Lens* EF diverging from its *Focus* of parallel Rays, and will therefore enter the Pupil of the Eye in Directions parallel to each other, and concur upon the *Retina* at Q; the Object will therefore appear *diſtinct*.

Secondly, A Pencil of Rays flowing from another Point of the Object, as L, will meet their *Axis* in M, and diverging from thence will, after being refracted by the Lens EF, become parallel with Reſpect to each other; but with Reſpect to the former they will converge, becauſe, with Regard to them, they diverged before they paſſed through the *Lens* EF from I, a Point more diſtant than its

Focus

Focus of parallel Rays. They will conſequently croſs them at ſome Diſtance from it, ſuppoſe at H, where the Pupil of an Eye being placed to receive them, the Point L will be repreſented at O. And for the like Reaſon, the Point K being repreſented at P, the Object will be ſeen under the Angle PHO or EHF, which, as will be demonſtrated in Note (*e*), is much larger than that under which it would have appeared to the naked Eye [c].

That Glaſs AB, which is ſituated next the Object, is called the *Object-glaſs*; that which is placed next the Eye, the *Eye-glaſs* [d].

The Proportion of magnifying, in a Microſcope of this Kind, is nearly in a *Ratio* compounded of the Proportion which the Diſtance of the Image from the Object-glaſs bears to its Diſtance from the Eye-glaſs; and of that which the Diſtance of the Ob-

[c] It may be remarked here, that when we view an Object through an Inſtrument of this Kind, we are then in reality looking at the Image of that Object through a ſingle Microſcope. Thus, it is MN the Image of the Object KL, formed by the Concurrence of the Rays of each Pencil in their reſpective *Foci*, which we ſee through the *Lens* or ſingle Microſcope EF; ſo that the Addition of the Glaſs AB is only that we may have an Image of the Object to look at, larger than the Object itſelf.

[d] In ſome Microſcopes there is a third Glaſs placed between the Object-glaſs and the Image, and is called a *Middle-glaſs*. This is placed there only to bring the Rays to a *Focus* the ſooner, in order that the Image may fall nearer the Object-glaſs than it otherwiſe would do.

ject from the Eye bears to its Diftance from the Object-glafs [e].

After what has been faid concerning the Structure of the Compound Microfcope, and

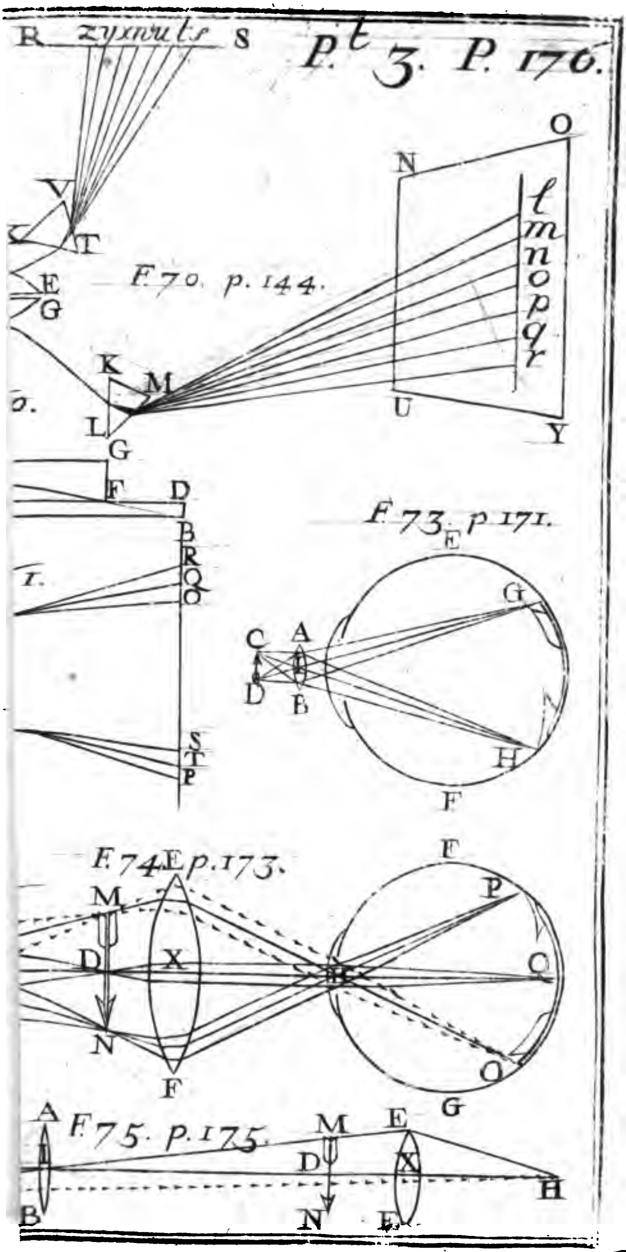
[e] *Dem.* To avoid the Confufion which might arife from the Multiplicity of Lines in the 74th Figure, let only CXH and LEH, *viz.* the *Axes* of the Rays which proceed from the Points C and L, be reprefented as in Fig. 75, and draw the Line LH, then will CHL be the Angle under which half the Object would be feen by the naked Eye at H; but EHX is the Angle under which the fame Half appears when viewed through the Microfcope. Now this Angle is to the former in a *Ratio* compounded of the Angle EHX to EIX, and of the fame EIX, or (which is equal to it becaufe vertical) CIL to CHL; *becaufe the* Ratio *that any two Quantities bear to each other, is compounded of the* Ratio *which the firft bears to any other, and of the* Ratio *which that other bears to the fecond.* But the firft of thefe *Ratio*'s, *viz.* EHX to EIX, is as IX to XH, or which is the fame Thing, as ID to DX, (for ID : IX : : IX : IH ; fee *Hugens* Opera Pofthuma, Prop. 20: and therefore ID : IX—ID : : IX : IH—IX, that is, ID : DX : : IX : XH.) and therefore the firft of the *Ratio*'s is as the Diftance of the Image from the Object-glafs to its Diftance from the Eye-glafs: and the other *Ratio*, *viz.* CIL to CHL, is as CH to CI, that is, as the Diftance of the Object from the Eye to its Diftance from the Object-glafs. Therefore, &c. QED.

But it is proper to compare the Angle EHX with that under which the Object would appear to the naked Eye at a Diftance proper for diftinct Vifion; becaufe, when a Perfon views an Object by the Help of a Microfcope, he is often obliged to place his Eye at a Diftance from the Object, very different from that at which he would choofe to place it, were he to look at it with the naked Eye; and then, inftead of the Diftance of the Object from the Eye in the foregoing Propofition, we muft fubftitute the Diftance of diftinct Vifion; in which Cafe it will ftand thus: The Proportion of magnifying is nearly in a *Ratio* compounded of the Proportion which the Diftance of the Image from the Object-glafs bears to its Diftance from the Eye-glafs, and of that which the Diftance of diftinct Vifion bears to the Diftance of the Object from the Object-glafs.

the

the Manner in which the Rays paſs through it to the Eye, the Nature of the common *aſtronomical* Teleſcope will eaſily be underſtood; for it differs from the Microſcope only in that the Object is placed at ſo great a Diſtance from it, that the Rays of the ſame Pencil, flowing from ence, may be conſidered as falling parallel to one another upon the Object-glaſs; and therefore, the Image made by that Glaſs is looked upon as coincident with its *Focus* of parallel Rays.

The 76th Figure will render this very plain, in which ABC is the Object emitting the ſeveral Pencils of Rays ADF, BDF, &c. but ſuppoſed to be at ſo great a Diſtance from the Object-glaſs DF that the Rays of the ſame Pencil may be conſidered as parallel to each other, they are therefore ſuppoſed to be collected into their reſpective *Foci* at the Points G, H, I, ſituated at the focal Diſtance of the Object-glaſs DF. Here they form an Image, and croſſing each other proceed diverging to the Eye-glaſs KM; which being placed at its own focal Diſtance from the Points G, H, I, the Rays of each Pencil, after paſſing through that Glaſs, will become parallel among themſelves, but the Pencils themſelves will converge conſiderably with Reſpect to one another, even ſo as to croſs at P, very little farther from the Glaſs KM than its *Focus*; becauſe, when they entered the Glaſs, their


Pl. 3. P. 170.
R zyxwuts S
V
T
N O
l m n o p q r
U Y
E G
F.70. p.144.
K M
L G
F D
B
R Q O
S T P
F.73. p.171.
E G C A D B H F
F.74. E p.173.
F P M D X C N O F G
A F.75. p.175.
M E D X H B N F

their *Axes* were almost parallel, as coming through the Object-glass at the Point E, to whose Distance the Breadth of the Eye-glass KM in a long Telescope bears very small Proportion. So that the Place of the Eye will be nearly at the focal Distance of the Eye-glass, and the Rays of each respective Pencil being parallel among themselves, and their Axes crossing each other in a larger Angle than they would do if the Object were to be seen by the naked Eye, as we shall demonstrate in the Notes; Vision will be *distinct*; and the Object will appear *magnified*.

The Power of magnifying in this Telescope is as the focal Length of the Object-glass to the focal Length of the Eye-glass [f].

It is evident from the Figure, that the visible *Area*, or Space which can be seen at one View when we look through this Telescope,

[f] *Dem.* In order to prove this, we may consider the Angle AEC as that under which the Object would be seen by the naked Eye; for in considering the Distance of the Object, the Length of the Telescope EP may be omitted, as bearing no Proportion to it. Now the Angle, under which the Object is seen by means of the Telescope, is KPM, which is to the other AEC, or its Equal KEM, as the Distance LE to LP, or as HE to HL (for as was observed in the Article of the Power of magnifying in the Microscope, EH . EL : : EL : EP, and therefore EH : EL—EH : : EL : EP—EL, that is EH : HL : : EL : LP) The Angle therefore, under which an Object appears to an Eye assisted by a Telescope of this Kind, is to that under which it would be seen without it, as the focal Length of the Object-glass to the focal-Length of the Eye-glass.

depends on the Breadth of the Eye-glaſs, and not of the Object-glaſs; for if the Eye-glaſs be too ſmall to receive the Rays GM, IK, the Extremities of the Object could not have been ſeen at all: a larger Breadth of the Object-glaſs conduces only to the rendering each Point of the Image more luminous by receiving a larger Pencil of Rays from each Point of the Object.

It is in this Teleſcope as was remarked of the compound Microſcope in the Notes, Page (174), that what we ſee, when we look through it, is not the Object itſelf, but only an Image of it at GI: Now that Image being inverted with reſpect to the Object, by reaſon that the *Axes* of the Pencils that flow from the Object croſs each other at E, Objects ſeen through a Teleſcope of this Kind neceſſarily appear *inverted*.

This is a Circumſtance not at all regarded by Aſtronomers, but for viewing Objects upon the Earth, it is convenient that the Teleſcope ſhould repreſent them in their natural Poſture; to which Uſe the Teleſcope with three Eye-glaſſes, as repreſented Fig. 77. is peculiarly adapted, and the Progreſs of the Rays through it from the Object to the Eye is as follows:

AB is the Object ſending out the ſeveral Pencils ACD, BCD, &c. which, paſſing thro' the Object-glaſs CD, are collected into their reſpective

refpective *Foci* in EF, where they form an inverted Image, from hence they proceed to the firft Eye-glafs HI, whofe *Focus* being at G, the Rays of each Pencil are rendered parallel among themfelves, and their *Axes*, which were nearly parallel before, are made to converge and crofs each other at K: the fecond Eye-glafs LM, being fo placed that its *Focus* fhall fall upon K, renders the *Axes* of the Pencils which diverge from thence parallel, and caufes the Rays of each which were parallel among themfelves to meet again at its *Focus* NO on the other Side, where they form a fecond Image inverted with refpect to the former, but direct with refpect to the Object. Now this Image, being feen by the Eye at XY through the Eye-glafs QR, affords a direct Reprefentation of the Object, and under the fame Angle that the firft Image EF would have appeared, had the Eye been placed at K, fuppofing the Eye-glaffes to be of equal Convexity; and therefore the Object is feen equally magnified in this, as in the former Telefcope, that is, as the focal Diftance of the Object-glafs to that of any one of the Eye-glaffes, and appears *direct*.

If a Telefcope exceeds 20 Feet, it is of no Ufe in viewing Objects upon the Surface of the Earth; for if it magnifies above 90 or 100 times, as thofe of that Length ufually do, the Vapours, which continually float near the

Earth in great Plenty, will be ſo magnified as to render Viſion obſcure.

The Teleſcope with the concave Eye-glaſs is conſtructed as follows:

AB (Fig. 78.) is an Object ſending forth the Pencils of Rays ADE, CDE, *&c.* which, after paſſing through the Object-glaſs, DE, tend towards FG (where we will ſuppoſe the *Focus* of it to be) in order to form an inverted Image there as before; but in their Way to it are made to paſs through the concave Glaſs HI, ſo placed, that its *Focus* may fall upon S, and conſequently the Rays of the ſeveral Pencils which were converging towards thoſe reſpective focal Points F, S, G, will be rendered parallel among themſelves; but the *Axes* of thoſe Pencils croſſing each other at K, and diverging from thence, will be rendered more diverging, ſuppoſe in the Directions LM, NO. Now theſe Rays, entering the Pupil of an Eye, will form a large and diſtinct Image PQ upon the *Retina* which will be *inverted* with reſpect to the Object, becauſe the *Axes* of the Pencils croſs in K; and the Angle the Object will appear under will be ſuch as the Lines ML, ON, form when produced back through the Eye-glaſs.

'Tis evident, that the leſs the Pupil of the Eye is, the leſs is the viſible *Area* ſeen through a Teleſcope of this Kind; for a leſs

Pupil

Pupil would exclude fuch Pencils as proceed from the Extremities of the Object AB, as is evident from the Figure. This is an Inconvenience that renders this Telefcope unfit for many Ufes, and is only to be remedy'd by the Telefcope with the convex Eye-glaffes, where the Rays which form the extreme Parts of the Image are brought together in order to enter the Pupil of the Eye, as explained above.

It is apparent alfo, that the nearer the Eye is placed to the Eye-glafs of this Telefcope, the larger is the *Area* feen through it; for, being placed clofe to the Glafs, as in the Figure, it admits Rays that come from A and B, the Extremities of the Object, which it could not if it was placed farther off.

The Degree of magnifying in this Telefcope is in the fame Proportion with that in the other, *viz.* as the focal Diftance of the Object-glafs is to the focal Diftance of the Eye-glafs.

For there is no other Difference but this, *viz.* that as the extreme Pencils in that Telefcope were made to converge and form the Angle KPM, thefe are now made to diverge and form the Angle MXO, which Angles, if the concave Glafs in one has an equal refractive Power with the convex one in the other, will be equal, and therefore each Kind will

will exhibit the Object magnified in the ſame Degree.

There is a Defect in all theſe Kinds of Teleſcopes, not to be remedied by any Means whatever, which was thought only to ariſe from hence, *viz.* that *ſpherical* Glaſſes do not collect Rays to one and the ſame Point, as was obſerved (Chapter III. in the Notes) but it was happily diſcovered by Sir *Iſaac Newton*, that the Imperfection of this Sort of Teleſcope, ſo far as it ariſes from the ſpherical Form of the Glaſſes, bears almoſt no Proportion to that which is owing to the different Refrangibility of Light. This Diverſity in the Refraction of Rays is about a twenty-eighth Part of the Whole, ſo that the Object-glaſs of a Teleſcope cannot collect the Rays which flow from any one Point in the Object into a leſs Room than a circular Space whoſe Diameter is the fifty-ſixth Part of the Breadth of the Glaſs g. Therefore, ſince each Point

g To illuſtrate this, let AB, Fig. 79. repreſent a convex *Lens*, and let CDF be a Pencil of Rays flowing from the Point D, and let H be the Point at which the leaſt refrangible Rays are collected to a *Focus*, and I, that where the moſt refrangible concur; then, if IH be the twenty-eighth Part of EH, IK will be a proportionable Part of EC (the Triangle HIK and HEC being ſimilar): Conſequently LK will be the twenty-eighth Part of FC. But MN will be the leaſt Space into which the Rays will be collected, as appears by their Progreſs repreſented in the Figure. Now MN is but about half of KL, and therefore it is but about the

Point of the Object will be reprefented in fo large a Space, and the Centers of thofe Spaces will be contiguous, becaufe the Points in the Object the Rays flow from are fo, it is evident that the Image of an Object made by fuch a Glafs muft be a moft confufed Reprefentation, though it does not appear fo when viewed through an Eye-glafs that magnifies in a moderate Degree; confequently the Degree of magnifying in the Eye-glafs muft not be too great with refpect to that of the Object-glafs, left the Confufion become fenfible.

Notwithftanding this Imperfection, a dioptrical Telefcope may be made to magnify in any given Degree, provided it be of fufficient Length; for the greater the focal Diftance of the Object-glafs is, the lefs may be the *Proportion* which the focal Diftance of the Eye-glafs may bear to that of the Object-glafs, without rendering the Image obfcure. Thus, an Object glafs, whofe focal Diftance is about four Feet, will admit of an Eye-glafs whofe focal Diftance fhall be little more than one Inch, and confequently will magnify almoft forty-eight times: but an Object-glafs of forty Foot *Focus* will admit of an Eye-glafs of only four Inch *Focus*,

the fifty-fixth Part of CF, fo that the Diameter of the Space, into which the Rays are collected, will be about the fifty-fixth Part of the Breadth of the Glafs, Which was to be fhewn.

and

and will therefore magnify 120 times; and an Object-glaſs of an hundred Foot *Focus* will admit of an Eye-glaſs of little more than ſix Inch *Focus*, and will therefore magnify almoſt 200 times [h].

But Teleſcopes of ſuch prodigious Length being very incommodious and unfit for Practice, the *catadioptrical* or reflecting Teleſcope

[h] The Reaſon of this Diſproportion in their ſeveral Degrees of magnifying is to be explained in the following Manner; Since the Diameters of the Spaces, into which Rays flowing from the ſeveral Points of an Object are collected, are as the Breadth of the Object-glaſs, it is evident that the Degree of Confuſedneſs in the Image is as the Breadth of that Glaſs (for the Degree of Confuſedneſs will only be as the Diameters or Breadths of thoſe Spaces, and not as the Spaces themſelves.) Now the focal Length of the Eye-glaſs, that is, its Power of magnifying, muſt be as that Degree; for, if it exceeds it, it will render the Confuſedneſs ſenſible; and therefore it muſt be as the Breadth or Diameter of the Object-glaſs. The Diameter of the Object-glaſs, which is as the Square Root of its Aperture or Magnitude, muſt be as the ſquare Root of the Power of magnifying in the Teleſcope, for unleſs the Aperture itſelf be as the Power of magnifying, the Image will want Light; the ſquare Root of the Power of magnifying will be as the ſquare Root of the focal Diſtance of the Object-glaſs; and therefore the focal Diſtance of the Eye-glaſs muſt be only as the ſquare Root of that of the Object-glaſs. So that in making Uſe of an Object-glaſs of a longer *Focus*, ſuppoſe than one that is given, you are not obliged to apply an Eye-glaſs of a proportionably longer *Focus* than what would ſuit the given Object-glaſs, but ſuch an one only whoſe focal Diſtance ſhall be to the focal Diſtance of that which will ſuit the given Object-glaſs, as the ſquare Root of the focal Length of the Object-glaſs, you make Uſe of, is to the ſquare Root of the focal Length of the given one. And this is the Reaſon that longer Teleſcopes are capable of magnifying in a greater Degree than ſhorter ones, without rendering the Object confuſed or coloured.

as it is commonly called, invented by Sir *Ifaac Newton*, is infinitely preferable to them, for one of thefe, fix Feet in Length, fhall magnify as much as one of the other of an hundred. The Form of the Tube, and the Progrefs of the Rays through it, are as defcribed in Figure the 80th, where ABCD is the Tube, BC a concave reflecting Metal, EF a plain reflecting Metal fixed to the Tube by Means of the Stem HI. MN reprefents a diftant Object emitting Pencils of Rays from each Point, two only of which are here reprefented, and thofe cut off before they reach the Metal, to prevent Confufion in the Figure. Now it is evident from what has been explained above (Chap. X. Propofit. 3. Cafe 4.) that thefe Rays, were they not intercepted in their Way, would return after Reflection at the concave Surface BC, and form an inverted Image at OP, fuppofing that to be the Place of the *Focus* of reflected Rays. But in this Cafe the reflected Rays are intercepted in their Return to that Place by the plain Metal, and are thereby thrown fide-ways, and, inftead of forming the Image OP, are made to form the Image QR; which, becaufe the Rays have as yet fuffered no Refraction, is not liable to the Imperfection which arifes from the different Refrangibility of the Rays of Light, nor to

any other except what may ariſe from an imperfect Poliſh, or the Want of the Form of one of the *conic* Sections in the Reflector BC; and therefore may be viewed by an Eye at T with a very ſmall *Lens* or Eye-glaſs KL, without appearing either coloured or confuſed.

It being inconvenient to find the Object with a Teleſcope of this Form, a ſmall dioptrical Teleſcope with two Hairs or Wires run through the Tube in the common *Focus* of the two Glaſſes, and croſſing each other at right Angles, is generally fixed upon it in ſuch Manner that the *Axis* of one Teleſcope ſhall be parallel to that of the other, ſo that when the Object appears in one at the Interſection of the Hairs, the other may be duly poſited for viewing the ſame Object through its Side.

But this Method of finding an Object is very incommodious for viewing terreſtrial Objects, and therefore the ſame Kind of Teleſcope has been contrived and effected in the following Manner:

ABCDEFGH (Fig. 81.) is the Tube, BG the concave reflecting Metal, with an Hole in it at IK. LM is another reflecting Concave fix'd to the Tube by means of the Stem NO, the common *Focus* of the two Metals being at P. Things being thus diſpoſed, let QR repreſent

reprefent an Object emitting feveral Pencils of Rays, two of which are reprefented in the Figure. Thefe, after Reflection, will form the two Extremities of the inverted Image ST (as explained Chap. X. Prop. 3. Cafe 4.) where the feveral Rays of the fame Pencil crofs each other, and, being afterwards reflected by the concave Surface LM, become parallel among themfelves, but the Pencils themfelves are made to converge, and, croffing each other at V, pafs through the *Lens* CF, which having its focal Diftance about V, makes the Pencils parallel, and at the fame Time renders the Rays of each Pencil converging, fo as to form an erect Image WX, which is feen by the Eye at Y through another *Lens* at DE.

This Kind of Telefcope is called the *Gregorian*, as being attempted by *J. Gregory*, though in vain. (See his *Optica promota*, Propofit. 59.) It is now grown common, and is excellently well adapted for the viewing terreftrial Objects, becaufe a Tube of this Kind, of two or Three Feet in Length, will magnify fufficiently for that Purpofe.

If the Reader would fee a particular Hiftory of the Invention of the feveral Sorts of reflecting Telefcopes, he may confult the *Appendix* to the laft Edition of *Gregory*'s

Optics, where he will find a full Account of it, together with the Letters that paſſed between the Inventors themſelves upon that Occaſion. And for ſuch Authors as have explained the Nature of Microſcopes and Teleſcopes in general, conſult Mr. *Johnſon's Quæſtiones Philoſoph.* Q. 34 & 35.

DIS-

DISSERTATION IV.

Of the Rainbow.

BEfore the different Refrangibility of Light was discovered, all Attempts to account for the Appearance of the Rainbow proved ineffectual; for it is no other than the Diversity of Refrangibility to which that *Phænomenon* is to be ascribed: as will appear from the following Explication of it; in which, because it is a *Phænomenon* not easily apprehended by Beginners, I hope to be excused, if I am more than ordinarily particular. To begin then:

The Rainbow is never seen, but when the Sun shines upon Drops of Rain falling on that Side of the Spectator which is opposite to the Sun.

To illustrate this, let A (Fig. 82.) represent the Eye of a Spectator, and let B, C, D, E, be a Series of Drops of Rain falling from a Cloud, on which let the Sun be supposed to shine from the Parts about S, &c. then will there be exhibited the Appearance of a Rainbow in the Cloud; and it will be formed as follows. Let SB, SC, SD, &c. represent the Sun's Rays, which (because of the Sun's great Distance) we will suppose parallel;

parallel; and let the Ray SC fall upon the Drop C at the Point C: then will ſo much of it as enters the Drop be refracted towards the Perpendicular, and proceed on, ſuppoſe to F, where Part of it will be tranſmitted, and Part reflected, ſuppoſe to G [i]: Of that which is reflected to G, ſome will be there reflected and ſome tranſmitted; that which is tranſmitted will, on account of the Diverſity of Refrangibility to which Light is ſubject [k], be ſeparated by Refraction, and made to exhibit the ſeveral *priſmatic* Colours, *viz. Red*, *Orange*, *Yellow*, *&c.* And if the *Red* Light proceeds from the Drop in the Line GR, the *Orange*, ſuffering a greater Degree of Refraction, will proceed in one ſituated above this, ſuppoſe in GO, and the *Yellow* in GY, *&c.* and the *Violet* in GV; therefore, to an Eye placed any where in the Line GR, the Drop C will exhibit a *Red* Colour, that is, the Cloud will appear *Red* in that Place. To an Eye placed any where in the Line GO, the ſame Drop would exhibit the *Idea* of *Orange* Colour, and ſo on through all the Colours of the *Priſm*.

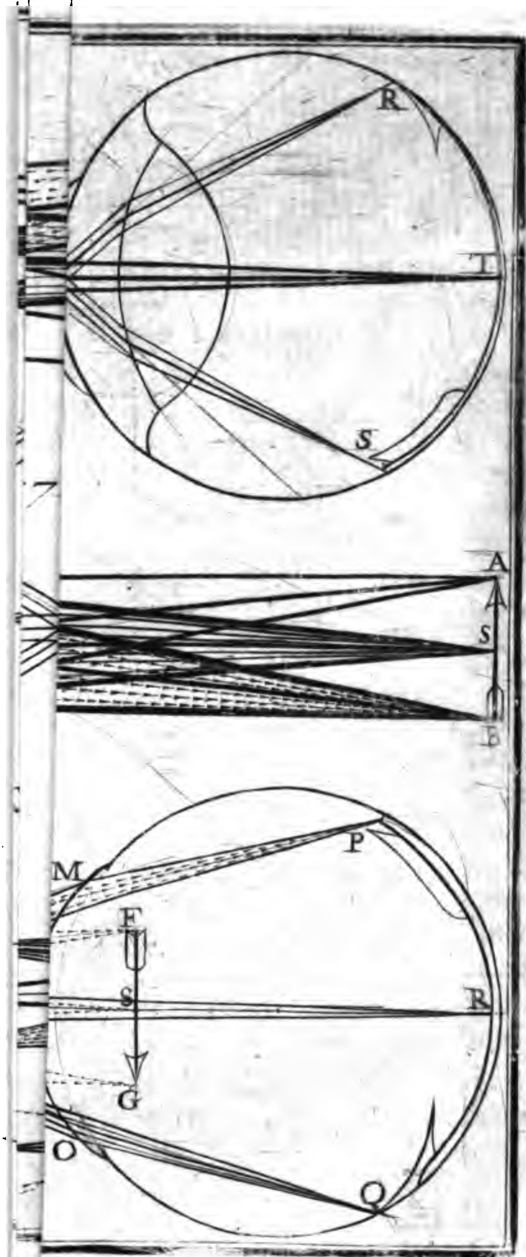
Now, let us conſider the Paſſage of a Ray of Light through another Drop at a certain Diſtance below this, *viz.* the Drop D, on

[i] See the Manner in which Light is reflected, Chap. VIII.
[k] See Chap. XI.

which let SD be the incident Ray. This Ray, after having been refracted and reflected as the former was in the other Drop, will emerge ſeparated into the Rays HR, HO, HY, *&c.* of which, if HR exhibits the *Red*, HO will paint the *Orange*, HY the *Yellow*, *&c.* and HV the *Violet* Colour; and the Ray incident upon this Drop being parallel to that which was incident upon the former, the Rays, exhibiting the ſeveral Colours ſeparated by this Drop, will be reſpectively parallel to the Rays exhibiting the correſpondent Colours ſeparated by means of the other Drop; that is to ſay, the Ray HR, which exhibits *Red* in this Drop, will be parallel to the Ray GR, which exhibits the ſame Colour in the other Drop; and ſo of the other correſponding Colours. Conſequently the Rays HO, HY, *&c.* which exhibit *Orange*, *Yellow*, *&c.* in this Drop, will all converge towards GR, which exhibits *Red* in the other: and therefore each of theſe would croſs that, if produced far enough. Let then the Ray HV in the Figure before us, which exhibits *Violet* in this Drop, be produced till it croſſes that which exhibits *Red* in the other produced alſo, ſuppoſe at the Point A where the Eye of the Spectator is placed. To this Eye therefore, upon this Suppoſition, will be repreſented in the Cloud at the ſame Time two of the *priſmatic* Colours, *viz.* *Red* and *Violet*, the

Red

Red above at G, and the *Violet* below at H. But if we ſuppoſe the Eye placed farther back, where a Ray, that exhibits another Colour in the Drop D, would croſs the Ray GR, or which comes to the ſame Thing, if we ſuppoſe the Drop D ſo much nearer to the Drop C, that that Ray may enter the Eye along with the other at A; then would the Colour of that Ray be exhibited along with the *Red*. For Inſtance, if the Drop D be placed ſo much higher, that the Ray HO which exhibits *Orange*, may croſs the Ray GR at A, then to the Eye will be exhibited the Colours of *Red* and *Orange*; and if there be a third Drop below this, ſo placed that the *Yellow* proceeding from it ſhall enter the Eye alſo at the ſame Time, then will three of the *priſmatic* Colours appear to that Eye, and ſo on for the other Colours, till the Situation of the Drop be ſuppoſed as low as where the Drop D is, and then the *Violet* and moſt refrangible Light is tranſmitted to the Eye; but from Drops below this, no Colour is tranſmitted thither, all the Rays which iſſue from a lower Drop, as E, paſſing below the Eye. And as thoſe Rays, which paſs through the lower Drops, are too low for the Eye at A, ſo thoſe which come from the higher ones, as B, are too high, as appears by Inſpection of the Figure; ſo that there is nothing but total Darkneſs both above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above and below. Now since the Colours of the Rainbow are the same with those of the *Prism*, it is evident, from what has been said, that between the Points G and H, all the Colours of that beautiful *Phænomenon* will appear, provided a sufficient Number of Drops be supplied from the Cloud, to prevent any *Hiatus* or Deficiency in the Light reflected by them.

BUT we have hitherto *tacitly* supposed, that the Rays SB, SC, SD, &c. were all incident with the same Degree of Obliquity upon the Surface of each Drop (that is, that they entered at the same Distance from that Point in the Surface of each Drop which is nearest the Sun) and that that Obliquity was certain and determinate: For Rays which enter the Drops with other Obliquities conduce nothing towards exhibiting the Colours of the Rainbow, and are therefore to be looked upon as *ineffectual* and *insignificant*. The Truth of which we shall now proceed to shew. After this, we shall be enabled to explain the remaining Particulars relating to the *Bow*.

LET then SA, SB, SC, &c. (Fig. 83.) represent the Sun's Rays falling upon the Drop XY, the first perpendicularly to the Surface of it, the other with different Degrees of Obliquity, according to their different Distances from the first; and let the Point at which the two first, *viz.* SA and SB, would meet by means of that Refraction which the oblique one SB

ſuffers in paſſing through the firſt Surface of the Drop, be H. Now it was remarked (Chap. III. Obſervat. 2. in the Notes) that when parallel Rays fall upon a convex Surface, the nearer any one of the oblique ones is to that which enters the Surface perpendicularly, the greater ſhall be the Diſtance at which it will meet the perpendicular one after Refraction at that Surface; that is, in the preſent Caſe, that the oblique incident Ray SB ſhall after Refraction at B, (ſuppoſing it to paſs through the ſecond Surface of the Drop without Refraction) meet the perpendicular Ray SA produced at a greater Diſtance than SC ſhall; and SC ſhall meet it at a greater than SD ſhall; SD at a greater than SE, *&c.* H then being ſuppoſed to be the Point where SB meets SA, let I be that where SC, K that where SD, L that where SE, M that where SF would meet it, *&c.* From whence we may obſerve,

THAT the farther we take the Rays from SA, the *nearer* are the Points which the refracted Rays fall upon the ſecond Surface of the Drop at, ſituated to X, till we come to the Ray SD; after which, the farther we take them from SA, the *farther* the Points they fall upon are from X. For Inſtance, the Ray SB falls upon N; SC upon O; SD upon P; but SE does not fall beyond P, but upon O, and SF upon N, *&c.* So that upon every Point below P there are two Rays incident; and the one of them

them is ſuch as enters the Drop on one Side of the Ray SD, and the other on the other; and the farther from it the one is on the one Side, the farther the other is on the other; and alſo the farther they are from it, the farther the Point they meet at is from P. Thus SC and SE meet at O, the Rays SB and SF at N, *&c.* The Uſe of which Obſervation we ſhall ſee by and by. But let it be remembered, that I would be underſtood here and in what follows of the Rays of one particular Colour only.

Now it is remarkable, that when two Rays fall upon a Drop, and at their Entrance are ſo refracted, as to meet in a Point at the other Surface, and are from thence reflected to ſome other Part of the Surface, and there paſs out of it; they ſhall after ſuch Emergency have the ſame Inclination towards each other, that they had before they entered the Drop. To explain this, let AB, CD, (Fig. 84.) repreſent two Rays incident upon the Drop BG, and let them, after Refraction at B and D, meet at the Point E, from whence being reflected, let them paſs out at F and G, and be refracted into the Lines FH and GI; then whatever Inclination the incident Rays AB, CD, have to each other, the emerging Rays FH and GI will have the ſame. Becauſe the Angles of Incidence and Reflection at E being equal, the Rays EF and EG will have the ſame Inclination to each other and to the Surface at F and G, that the Rays EB and

ED have to each other, and to the Surface at B and D. For if we conceive all these Rays to flow from the Point E as a Radiant, and BA, DC, to be the refracted ones of the incident ones EB, ED, as FH and GI are of EF and EG, it is evident, that under these Circumstances, the Rays BA and CA will have the same Inclination to each other, that FG and HI have; but the Degree of Refraction is the same, whether EB and ED, or AB and CD, be the incident Rays; because the refractive Power of the Drop is the same, whether the Rays pass one Way, or the other. The Proposition therefore is true.

From hence it follows, that the parallel Rays, SB, SF, (Fig. 83.) which after Refraction meet at the same Point N, will, if they are from thence reflected, suppose in the Lines NQ, NR, become parallel to each other, after their Emergency, suppose in the Lines QT, RV: But their intermediate ones SC, SD and SE, which fall upon quite different Points at the second Surface of the Drop, and are from thence reflected, will not do so, but will go out in Directions oblique to one another and to them; and will therefore pass on, not only a different Way from them, but from one another: So that the Rays QT and RV will be left to themselves, being deprived of their *intermediate* ones, by which Means they are rendered, as to all Intents and Purposes of Vision, entirely useless and *insignificant*.

Again,

Again, the Rays SC and SE, which meet at the Point O, will also be parallel among themselves after their Emergency from the Drop, but their *intermediate* ones will pass off another Way, though not so obliquely, with Respect to them, and to one another, as those in the foregoing Case; because the several Points they fall upon at the second Surface of the Drop, being situated between O and P, are nearer to each other, than the Points the *intermediate* Rays fell upon, in the former Case, were to N.

But such as are incident very near SD on each Side of it, will with their intermediate ones all fall upon, or at an insensible Distance from the Point P; so that these, after their Emergency, will all pass on parallel, or very nearly so, to each other; and therefore when they enter the Eye of a Spectator, though he be at a considerable Distance, will affect him sensibly enough to excite the *Idea* of their own Colour (for as was observed above, I speak now only of Rays of one Colour) which the other Rays considered in the forementioned Cases, for Want of their *intermediate* ones, were too weak to do, however near the Situation of the Eye might be. These therefore are the only Rays that exhibit the Colours of the *Bow*, and are hence called in Contradistinction to such as enter at other Points of the Drops, that is, with other Obliquities, *effectual* or *significant*.

It

It was proper therefore, in the Explication above, to ſuppoſe none to enter the Drop, but theſe. As to the Degree of Obliquity with which Rays muſt fall upon the Drops to become effectual, the Method of finding that will be ſhewn in the next Note.

SINCE then the effectual Rays enter each Drop with the ſame Degree of Obliquity (I ſtill mean ſuch Drops as exhibit the ſame Colour) the emerging Rays muſt neceſſarily make the ſame Angle with the incident ones in every Drop. The Magnitude of which ſhall be determined in the Note below [a]. That is, the Angle which the Ray SC (Fig. 82) makes with the emerging Ray GR which exhibits *Red*, will be the ſame in all the Drops that exhibit that Colour; and ſo of the reſt. Let then the Line

[a] We are here to determine the Angle, which an incident *efficacious* Ray of any Colour makes with the emerging Ray of the ſame Colour. In order to this, let AB, CD, (Fig. 84.) be two Rays incident upon the Drop BG, and let them be refracted to E, and after Reflection there, and a ſecond Refraction at F and G, let them emerge in the Lines FH and GI, making with the incident ones the Angle AKI, which is the Angle to be determined.

Let us call the *Ratio*, which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, I to R. Then from the Center L to the Lines BK, BE, and DE, draw the Lines LM, LN, and LO, reſpectively perpendicular, and with the *Radius* LO deſcribe the Arch OP, and draw the Line LB, and produce it to Q. Then will ABQ, or its Equal LBM, be the Angle of Incidence of the Ray AB, and LM its Sine: LBN will be the Angle of Refraction, and LN its Sine: Likewiſe LR will be the Sine of the Angle of Incidence of the other Ray CD, and LO

Line AI be supposed to be drawn from the Sun through the Eye of the Spectator, and produced towards the Point I. This Line, because of the Sun's immense Distance, will be parallel to the Sun's Rays SB, SC, SD; and therefore the Angle GAI which this Line makes with any one of the emerging Rays, for Instance GR, will be the same that the incident one makes with it. Let us then imagine the Line AI fixed, and the Line AG to revolve round it, always making the same Angle with it; then will the Line AG describe the Surface of a Cone whose *Apex* will be at A, and its *Axis* AI, and the Surface of this Cone will in all Parts of it make the same Angle with the Sun's Rays, because they are parallel to one

LO the Sine of its Angle of Refraction. We shall therefore have for the first Step of the following Process

this Proportion, *viz.*	1	LM : LN :: I : R
And for the second	2	LR : LO :: I : R
But by Construction	3	LO=LP
Therefore from the 2d and 3d Steps	4	LR : LP :: I : R
Now if we substract the two first Members of the fourth Step from the two first of the first Step respectively, by which Means the Proportion between the Terms will not be destroyed, we shall have	5	LM—LR : LN—LP :: I : R
But by the Figure	6	LM—LR=MR
And ——— ——— ———	7	LN—LP=NP
Therefore from the fifth, sixth, and seventh Steps, we have	8	MR : NP :: I : R

Parallel

one another, and to the Axis of the Cone; therefore Drops of Rain, whatever Part of this Surface they pass through in falling, will, in the Instant of Time that they pass through it, send a *red* Ray towards the Eye of the Spectator; for it is not necessary that the Drops should be all at the same Distance from the Eye. In like manner Drops of Water, passing through the Surface of a lesser Cone made by the Revolution of the Line HV about the Line AI, will exhibit *Violet*; and so for the intermediate Colours. So that the Rainbow, were we to see it entire, would be a compleat Circle having its Center in the Line AI, and consequently directly opposite to the Sun with respect to the Spectator's Eye.

Parallel to BE draw the Line DX, and on the Lines DR and DX let fall the Perpendiculars BT and BX, from the Point B. Then, because we suppose the Rays AB and CD *efficacious* ones, and therefore infinitely near one another, the little Triangles BTD, BXD, and NOP may be considered as right-lined ones, and the latter, *viz.* NOP may also be looked upon as right-angled at P. Upon this Supposition, the Triangles BTD and BLM will be similar, *for they are right-angled at T and M; and the Angles DBT and MBL want each the same Angle TBL to make either of them right, they are therefore also equal.* For the like Reason the Triangles DBX and BNL are similar, *being right-angled at X and N, and wanting equally the Angle XBL to make their Angles at B right ones.* But to the Triangle BDX the Triangle NPO is also similar, *for they are right-angled at P and X, they have their Sides NP and BX parallel, as being each perpendicular to the same Line BE; and their Sides NO and BD are also parallel, because the Points N and O, where the Perpendiculars LN and LO fall, must be in the Middle of the Lines BE and DE.* Now, since the Lines BE and DE are coincident at E, and the Points N and O are in the Middle of each, BD is double of NO; and consequently,

Eye. The Reason that it does not appear such, is because the Sun when the Rainbow is seen, is *in* or *above* the Horizon, and therefore the Center of the Bow being opposite to it, is *in* or *below* it, on the other Side the Spectator.

SINCE the Angle made by the Line SC with GR, or which is the same thing, GR with AI is 42 Degrees and two Minutes, as determined in the Note, it's evident that when the Sun is in the Horizon, the highest Point of the *Bow* is 42 Degrees and two Minutes above the Horizon,

quently, the Triangles NOP and BDX being similar, BX is double of NP. From whence we have the following Steps,

viz.	9	$BL : BM :: BD : BT$
And	10	$BL : BN :: BD : BX$
By comparing these two together	11	$BM : BN :: BT : BX$
But by Construction	12	$BT = MR$
And, as was just now shewn,	13	$BX = 2\,NP$
Consequently from the three last Steps	14	$BM : BN :: MR : 2\,NP$
But by the eighth Step	15	$I : R :: MR : NP$
Therefore from the last	16	$I : 2\,R :: MR : 2\,NP$
Consequently from the fourteenth and sixteenth Step	17	$BM : BN :: I : 2\,R$
But from the first Step	18	$LM : 2\,LN :: I : 2\,R$
Therefore from the two last	19	$BM : BN :: LM : 2\,LN$
And by changing the Places of the mean Terms in the last Step	20	$BM : LM :: BN : 2\,LN$
And by squaring each Term	21	$BMq : LMq :: BNq : 4\,LNq$
From whence by comparing the Antecedents and Consequents with the Antecedents we have	22	$BMq + LMq : BMq :: BNq + 4\,LNq : BNq$

U But

rizon, for then the Line AI is parallel to it; and as the Sun rises the Height of the *Bow* diminishes, and with it the Portion that is visible, till it is 42 Degrees and two Minutes high; after which the *Bow* appears no more, because then the Point I is above 42 Degrees below it.

THE *Phænomenon*, we have been explaining, constitutes what is called the *primary* or *interior Bow*; there is also another exterior to this, whose Colours are much more dilute and faint, which

But because the Triangle BML is right angled at M	23	BMq + LMq = BLq
And for the like Reason BNq + LNq is equal to BLq, therefore	24	BNq + 4 LNq = BLq + 3 LNq
Therefore from the three last Steps	25	BLq : BMq :: BLq + 3 LNq : BNq
But BLN being a right-angled Triangle	26	BNq = BLq — LNq
Therefore from the two last Steps	27	BLq : BMq :: BLq + 3 LNq : BLq—LNq
Now because BML is a right-angled Triangle	28	BLq = BMq + LM
Therefore from the two last Steps	29	BLq : BMq :: BL + 3 LNq : BMq + LMq—LNq
And substracting the two first Terms, *viz.* BLq and BMq out of the two last Terms respectively, we have	30	BLq : BMq :: 3 LNq : LMq — LNq
But by the first Step	31	LM : LN :: I : R
Therefore substituting I and R in the Room of LM and LN in the 30th Step, we have	32	BLq : BMq :: 3 Rq : Iq—Rq

The

which for that Reaſon is called the *ſecondary* Bow. The Progreſs of the Rays of Light through the Drops of Water, in forming this, is as follows.

LET A (*Fig.* 85.) repreſent the Eye of a Spectator, SB and SC two of the Sun's Rays entering the Drops, as expreſſed in the Figure, and after being twice reflected in each Drop, *viz.* at D and D, let them paſs out, the one at E, the other at F, by which Means they will be ſeparated into their homogeneous Colours, the *Violet* and moſt refrangible Light being

The Proportion therefore which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, when Rays of any particular Colour paſs out of Air into Water, being known, the Proportion, which the *Radius* BL bears to BM, will be thereby determined; and therefore the Angle BLM, of which BM is the Sine, will alſo be known, and therefore alſo the Angle LBM, which is equal to ABQ, the Degree of Obliquity wherewith the *efficacious* Rays enter the Drop. But the Line BM being known, the Line BN may be alſo had by the 17th Step, and therefore alſo the Angle BLN of which it is the Sine, and therefore the Angle LBN too, or its Equal LEN; and therefore alſo the Complement of this laſt to two right ones, *viz.* KEB. If now we ſubſtract the Angle LBN out of LBM, we gain the Angle EBK, and conſequently the third Angle in the Triangle EBK may be from hence known, *viz.* the Angle BKE. Now if we double this, we have the Angle AKI, which was the Angle ſought.

If a Computation be made after this Manner with the *Ratio* of 108 to 81 (or which is the ſame thing) that of 4 to 3, for the *Red* Rays, the Angle AKI (that is, the Angle GAI in Fig. 82.) will be found 42 Degrees and 2 Minutes; and if we uſe the Proportion of 109 to 81, which is the Proportion of Refraction in the *Violet* coloured Light, the Angle AKI (or HAI in Fig. 82.) will be 40 Degrees 17 Minutes. And the Difference between theſe two Angles (that is, the Angle GAH in Fig. 82.) will be the Breadth of the Bow.

conveyed from the uppermost Drop to the Eye at A in the Line EV; while the *Red*, and least refrangible, is carried from the lower one in the Line FR, and the intermediate Colours from the intermediate Drops: So that in this Bow the Colours will be in an inverted Order, with respect to those of the other, the *Red* being the innermost in this, which was the outermost in that. The Colours in this will be more dilute than in that, because the Rays in this suffer two Reflections, in that but one; and the Angles, which the incident Rays in this make with the emerging ones, are larger than the like Angles in the other, as shall be determined in the next Note, *viz.* 50 Degrees 57 Minutes for the *Red*, and 54 Degrees 7 Minutes for the *Violet*; this Bow therefore is exterior to, and encompasses the former.

As to the Means by which Rays of Light become *efficacious* in the Formation of this Bow, it is exhibited in Figure 86. where AB, CD represent two parallel Rays incident with such Obliquity upon the Drop, that they shall cross each other before they reach the other Side; which that it is possible appears from what was said, with regard to the Progress of the Rays through the Drop XY in Figure 83. Let them then cross in the Point E such, that after Reflection at F and G they may become parallel, suppose in the Lines FH and GI, then from the Nature of the Circle it is plain, that after

Reflection

Reflection at H and I they will cross again, suppose at K, and, after Refraction at V and W, will become parallel as at first. And such of these as also enter so very near one another, that their intermediate ones may suffer the like Refractions and Reflections with themselves, will be the *efficacious* ones, and exhibit the *Idea* of their own Colour, at a considerable Distance from the Drop. What the Obliquity is with which these must enter the Drops, and the Angle the emerging ones of each particular Colour will make with their incident ones, shall be determined in the Note below [b].

Now

[b] The Progress of the *efficacious* Rays through the Drop BH (Fig. 86.) being as explained above, it is evident, because the Angles of Incidence are every where equal to the Angles of Reflection, that the Lines BG, GI, and IW are all equal, and therefore the Arches BG, GI, and IW are so too; and likewise, that the Lines DF, FH, and HV are equal, and therefore also the Arches DF, FH, and HV: It is also apparent, that the Arch FG is equal to HI, therefore FG is half the Difference between the Arches FH and GI, and consequently it is half the Difference also between the Arches FD and GB which are respectively equal to these. Now, the whole Difference between these Arches is what remains when FG is taken from BD, therefore the Remainder when FG is taken from BD is double of FG, consequently FG itself is but one third Part of BD; for, if when one Quantity is taken from another, the Remainder be double to the Quantity taken away, it is plain that other must contain the Quantity taken away three times.

Now the Rays AB and CD being supposed infinitely near one another, the curvilineal Spaces BED, and FEG may be consider'd as similar Triangles, and therefore EG is equal to a third Part of EB, consequently N being the middle Point of the Line BG, EN is equal to EG, and therefore also a third Part of EB.

If now as in the 84th Figure, the Triangles BTD and BXD be formed; as also the Triangle NOP, NO will be a third Part of BD,

Now if we imagine the Lines EA and FA (Fig. 85.) to revolve about the Line AI which passes through the Eye of the Spectator and the Center of the Sun as before, and always to make the same Angles with it at A, they will describe the Surface of two Cones, in the larger of which, will be situated the Drops that exhibit *Violet*, and in the lesser those which exhibit *Red*. So that this Bow also, were it to appear entire, would be a compleat Circle, and

BD, and NP a third Part of BX. Therefore resuming the former Process at the 11th Step, we may proceed as follows,

Viz.	11	BM : BN :: BT : BX
By Construction	12	BT = MR
And by what was just observ'd	13	BX = 3 NP
Therefore from the three last Steps	14	BM : BN :: MR : 3 NP
But by the eighth Step	15	I : R :: MR : NP
Therefore from the last	16	I : 3 R :: MR : 3 NP
Consequently from the 14th and 16th Steps	17	BM : BN :: I : 3 R
But by the first Step	18	LM : LN :: I : R
Therefore from the last	19	LM : 3 LN :: I : 3 R
Therefore from the 17th and 19th	20	BM : BN :: LM : 3 LN
Or by changing the Places of the mean Terms in the last Step	21	BM : LM :: BN : 3 LN
And squaring the Terms	22	BMq : LMq :: BNq : 9 LNq
Therefore by comparing the Antecedents and Consequents with the Antecedents, we have	23	BMq + LMq : BMq :: BNq + 9 LNq : BNq
But by the Figure	24	BMq + LMq = BLq
And	25	BNq + 9 LNq = BLq + 8 LNq

Therefore

and the ſeveral Cones, through whoſe Surfaces

Therefore from the three laſt	26	$BLq : BMq :: BLq + 8\,LNq : BNq$
But by the Figure	27	$BNq = BLq - LNq$
Therefore from the two laſt	28	$BLq : BMq :: BLq + 8\,LNq : BLq - LNq$
But by the Figure	29	$BLq = BMq + LMq$
Therefore from the two laſt	30	$BLq : BMq :: BLq + 8\,LNq : BMq + LMq - LNq$
And ſubſtracting the two firſt Terms out of the two laſt Terms, we have	31	$BLq : BM :: 8\,LNq : LMq - LNq$
But by the firſt Step	32	$LM : LN : I : R.$
Therefore	33	$BLq : BMq :: 8\,Rq : Iq - Rq$

Now the Proportion of I to R being known, the Proportion which the *Radius* BL bears to BM is had by the laſt Step. But to avoid the Confuſion which a Multiplicity of Lines may occaſion, let the 86th Figure be transferred to the 87th with as many Lines as ſhall be neceſſary, in which let AB be the incident Ray, BG the refracted one as before. Then, becauſe the Proportion between BL and BM is known, the Angle LBM may be had, which is equal to ABQ, the Meaſure of the Obliquity with which the efficacious Rays enter the Drop; and therefore alſo its Complement to two right ones SBL. And the Line BM being known, the Line BN may be had, becauſe by the 17th Step BM is to BN as I to 3R, and therefore alſo the Angle LBN, or its Equal LGB, and conſequently BLG the remaining Angle of the Triangle BGL; but to this is equal the Angle GLH or HLV, and if theſe three be added together, and their Sum taken from four right ones, it will give the remaining Angle about the Center, *viz.* VLB, which being halved, gives the Angle SLB; but the Method of determining the Angle SBL was ſhewn before, and therefore LSB the remaining Angle of the Triangle LBS, may be had, which Angle doubled gives the Angle VSB or its Equal ASY, which is the Angle ſought.

If a Computation be made after this manner with the *Ratio* of 108 to 81 for the *Red* Rays, this Angle will be found to be 50 Degrees 57 Minutes; if with 109 to 81, for the *Violet*, it will be 54 Degrees 7 Minutes; and the Difference, *viz.* 3 Degrees 10 Minutes, will be the Breadth of the *Bow*.

the

the Drops as they form the Colours of it paſs, having one common *Axis* AI with thoſe in whoſe Surfaces the Drops forming the Colours of the other *Bow* were placed, this will be exterior to, and concentric with it, and will therefore ſurround it, as obſerved above.

As Rays of Light when they arrive at the Surface of a Drop of Water never all paſs out, but are in part reflected and in part refracted, it is evident that ſome Rays will paſs out of each Drop after having ſuffered three Reflections, ſome after four, &c. theſe alſo will conſtitute *Rainbows*; but becauſe the greateſt Part Part of the Rays will be loſt in ſuffering ſo many Reflections, that *Rainbow*, which is made by three Reflections, is ſcarce ever ſeen, much leſs ſuch as are made by more, &c.

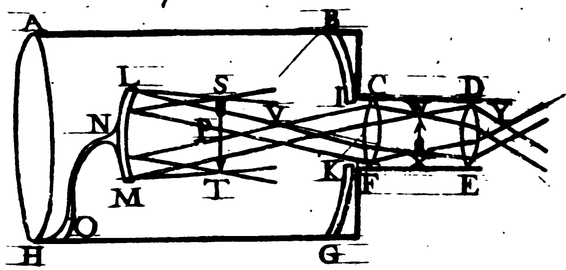
It is evident, that ſince the Line AI, *viz.* the common *Axis* of the Cones, on whoſe Surfaces the Colours of the *Bow* are formed, paſſes through the Eye of the Spectator, no two Perſons can ſee the ſame *Bow* at the ſame Time, or rather, that the Rainbows, ſeen by two Perſons at the ſame Time, are formed in different Drops of Rain, and in different Parts of the Heaven.

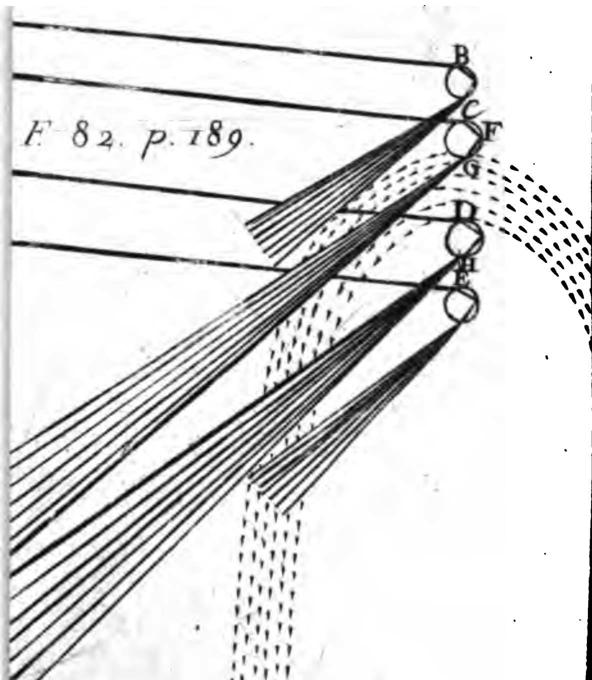
Accordingly, if a Perſon obſerves the Dew as it hangs upon the Graſs when the Sun ſhines, he ſhall ſee the Colours of the *Bow* in the Drops of Dew; but as he walks along, the Colours ſhall remove from Drop to Drop.

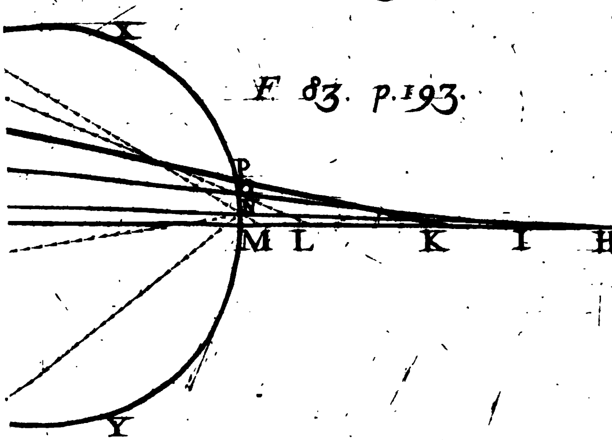
SOME-

Pl. 3. P. 208.

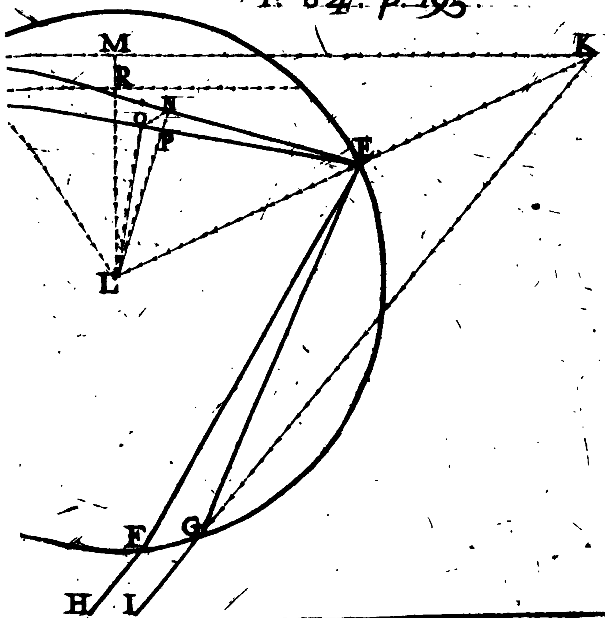
F. 81. p. 186.







F. 83. p. 193.



F. 84. p. 195.

Sometimes the lower Part of the *Bow* shall appear upon the Ground, and the upper Part of it not at all, and then it looks like a Rainbow lying along the Ground with the Extremities of its Legs turned upward into the Air: This is when the Sky is clear towards the Sun, but foggy on the opposite Parts, and only to a small Height from the Ground.

The Moon sometimes occasions the Appearance of a *Rainbow* after the same Manner that the Sun does, but the Colours are much more faint and dilute.

And lastly, If Water be continually thrown up into the Air opposite to the Sun, as from a Fountain, and there breaks into small Drops, the Appearance of the Rainbow will be exhibited in them.

See more on this Subject in *Antonius de Dominis de Radiis Visus & Lucis*; and the Authors referred to by Mr. *Johnson*, in his *Philosoph. Quæst.* Chap. VII. Q. 45 & 46.

Of the Obſcura Camera *and the* Magic Lanthorn.

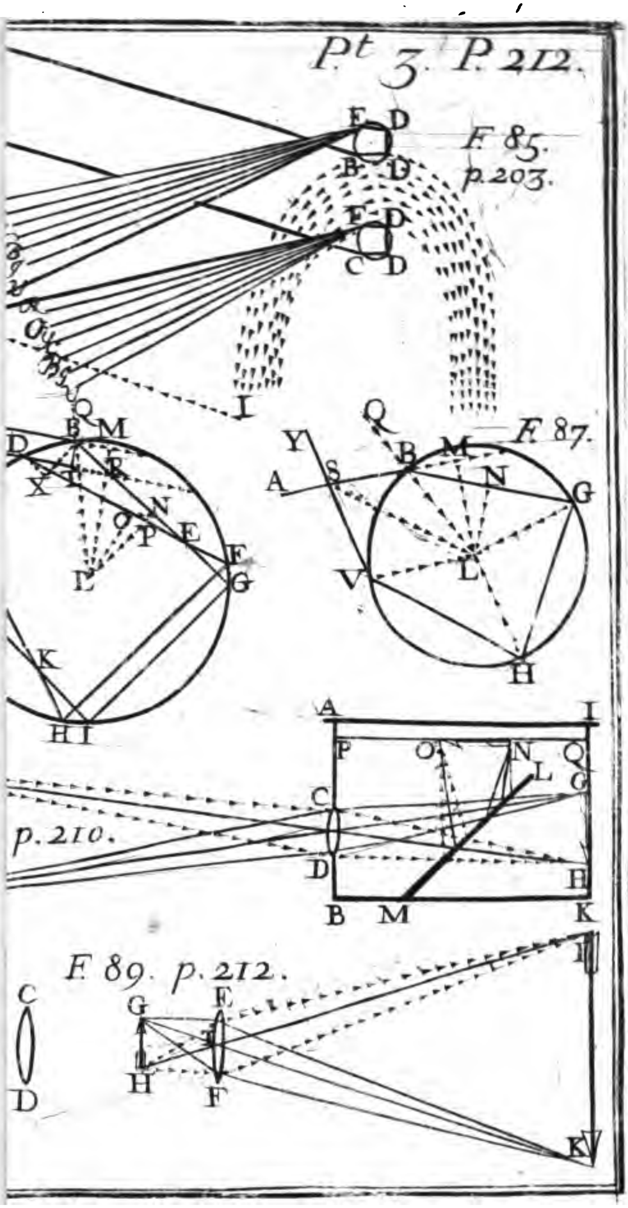
THE *Obſcura Camera* is of two Sorts; the one is no other than a convex *Lens*, fixed in an Hole in a Window-ſhutter, which *Lens*, when no other Light is permitted to enter the Room except what paſſes through it, will repreſent all the external Objects that are viſible through that Hole upon a white Paper held at the focal Diſtance of it, painted in their proper Colours. To illuſtrate this, let AB (Fig. 88.) repreſent a Window-ſhutter, CD a convex *Lens* fixed in an Hole therein, and let EF be an external Object; then will this Object emit Rays of Light of its own Colour from each Part, which paſſing through the *Lens*, as the Figure repreſents them, will be collected into Points at GH, and being there received upon a white Paper or other Surface, will repreſent the Object painted in its proper Colours, which Colours will be the ſtrongeſt of all when the Sun ſhines upon that Side of the Object that is next the Glaſs. But the Repreſentation will be *inverted*, becauſe the Pencils of Rays that flow from the Object croſs in the Middle of the Glaſs.

The

The other Sort of *Obſcura Camera* is that which is called the *Portable one*, and is of Uſe in drawing Pictures, taking Landſkips, &c. It is contrived after the following Manner, AIKB is a Box, in an Hole in whoſe Side the *Lens* CD is fixed (or rather at the Extremity of a ſhort Tube fixed in that Hole) and in the Situation LM is fixed a Piece of Looking-glaſs making an Angle with the Side of the Box of 45 Degrees; this, receiving the Rays in their Paſſage to GH, throws them upwards, and cauſes the Repreſentation to be made in NO, which is there received upon the under Side of ſome thin Subſtance PQ, that is in a ſmall Degree tranſparent, as thin Paper or Glaſs about half poliſhed, and ſo upon opening the Box appears ready to be drawn or copied out. But that the Colours may appear ſtrong, the Light muſt be kept from falling upon the Paper or Glaſs, as much as may be.

The *Magic Lanthorn* is an Inſtrument invented by *Kircher*, in order to repreſent Objects much larger and more luminous than they are. It is no other than a dark Lanthorn, in the Side of which there is fixed a ſhort Tube, and in the Tube two convex *Lenſes*, and between them a tranſparent painted Image of the Object we would repreſent. The Paſſage of the Rays through the *Lenſes*

Lenſes and the Image is thus. Let A (Fig. 89.) repreſent a burning Lamp placed as in a common Lanthorn, and let CD, EF, be the two *Lenſes* placed in the Tube abovementioned, and the Picture at GH. And let the Situation of the *Lens* CD be ſuch, that the Light which falls upon it from the Lamp may be all thrown upon the Picture GH, by which Means it will be ſtrongly illuminated, and being tranſparent, will throw out Rays in Plenty the other Way: Which Rays, in paſſing through the other *Lens* EF, let us ſuppoſe to be collected into their reſpective *Foci* on an oppoſite Wall at IK, and to form an Image there. Which Image will be larger than the Picture, in Proportion as the Diſtance IL is greater than LH; becauſe the Angles ILK and GLH are equal; and the Room being dark every where elſe, it will appear very bright, if the Picture be ſtrongly illuminated by the Lamp. And beſides the abovementioned *Apparatus*, there is ſometimes a concave Reflecter placed within the Lanthorn behind the Lamp, as at MN, to give a ſtronger Illumination to the painted Image at GH.


Pl 3. P. 212.
F. 85. p. 203.
F. 87.
p. 210.
F. 89. p. 212.

A
COMPENDIOUS SYSTEM
OF
Natural Philosophy.
With NOTES
Containing the
MATHEMATICAL DEMONSTRATIONS,
AND
Some Occasional REMARKS.

PART IV.
ASTRONOMY.
Containing the *real* and *apparent* Motions of the *Heavenly* *BODIES*.
With an Attempt to account for the *Phænomena* of the *COMETS TAILS*, After a new Manner.

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A COMPENDIOUS SYSTEM OF Natural Philoſophy.

PART IV.

ASTRONOMY.

The INTRODUCTION.

THE Science of *Aſtronomy* may be diſtinguiſhed into two Parts, the one relating to the Motion of the heavenly Bodies, as they really are in themſelves; the other as they appear to a Spectator upon the Surface of the Earth: and theſe two are frequently the very Reverſe of each other; that Body appearing to move *Weſterly*, which in reality at the ſame time moves *Eaſterly*; and that ſeeming to tend towards the *North*, which is ſtanding ſtill, or haſtening towards the *South*: and the contrary. And becauſe

the apparent Motions of theſe Bodies depend upon their real ones, the Order I ſhall proceed in, ſhall be firſt to lay down their real Motions; and then to ſhow how the apparent ones ariſe therefrom. But ſince the generality of People are apt to form their Judgment by their Senſes only, and are loath to imagine Things different from what they appear to be, to prepare ſuch Readers for a more eaſy Reception of what follows, it may not be amiſs to premiſe the following Particulars by way of Introduction.

I. Let us imagine only one great Body in the Univerſe, plac'd in the midſt of infinite Space, like a Ball in the Air; and then enquire whether of the two following Suppoſitions has the greater Probability on its Side; *viz.* That this Body will continue in its Place; or that it will not.

In perſuance of this Inquiry, perhaps it may be argued thus: Whereas no Bodies on which we can make tryal are obſerved to continue in their Places without a Support, neither will this Body remain in the Place it is put, but will immediately deſcend, as it were, towards the lower Regions of Space.

This Argument is founded upon the Obſervation of Bodies falling to the Ground for want of ſupport, which is not a parallel Caſe. For to ſay the leaſt that may be, we cannot

be

be ſure but that the Earth itſelf may be the Cauſe why Bodies deſcend towards it; as the Loadſtone is the Cauſe why Steel approaches it when placed within a certain Diſtance from it. And if ſo, then for any thing that appears from hence to the contrary, this Body, being the only one in the Univerſe, may remain in its Place without a Support. In ſhort, the Argument is juſt as if one ſhould reaſon thus: It has never yet been obſerved, but that Steel when placed at a ſmall Diſtance below a Loadſtone will aſcend upwards; therefore by Parity of Reaſon, if all the Bodies in the Univerſe were transformed into a Maſs of Steel, and placed alone in the midſt of infinite Space, that Maſs would aſcend. The Fallacy of which way of arguing is ſufficiently apparent.

There is a much greater Probability of Truth in the oppoſite Way, and it is more rational to argue thus: By all the Experiments and Obſervations made upon Bodies, it does not appear, that any ever moves out of its Place, unleſs impelled by ſome other; or when it moves towards another, as in the Caſe of Bodies falling towards the Earth, or of Steel tending towards the Loadſtone. Now the Loadſtone is certainly ſome way or other the cauſe why the Steel tends towards it, and therefore (by Parity of Reaſon) in all Probability the Earth is ſome way or other the Cauſe, why Bodies are diſpoſed to move towards it: and

if ſo, then ſuppoſing all the Bodies in the Univerſe to be collected into one, it would moſt likely continue where it was put, without requiring any thing to ſupport it, as having nothing which might impell it any way, or towards which it might move. This, I ſay, being the moſt natural and eaſy Deduction we can make in this Caſe, ſeems to have the greater Probability on its Side. And farther,

II. Since there are no Bodies, we can make Tryal upon, but what are diſpoſed to move towards ſome other Body, as Bodies towards the Earth, or Steel toward the Loadſtone, it is moſt probable that the ſeveral Bodies whereof the Univerſe conſiſts, would all move towards one another, if not obſtructed.

III. Let us ſuppoſe all the Bodies in the Univerſe to be united in one, and that Body to be put into Motion. Now whereas upon that Suppoſition there is nothing which may ſtop or deſtroy its Motion, it does not appear that it would ever ſtand ſtill, or loſe the leaſt Part of it. For by all the Experiments we can make upon the Bodies about us, there are none which, when put into Motion, can be ſtopped without ſome Force applied to them for that Purpoſe. They will *ſeem* indeed to ſtop of themſelves, which is always owing to ſome Impediment. Accordingly it is found, that the leſs Reſiſtance or Rubs Bodies meet with in their Way, the longer they continue

their Motion*(a)*. Whence we may infer, that this Body once put into Motion, would ever continue in that State*(b)*.

IV. Let us now ſuppoſe that all the Bodies in the Univerſe are compriſed in two, the one of which let us imagine to be immenſely greater than the other. Theſe Bodies, if not obſtructed, will (agreeably to the Concluſion made in Section II.) approach towards each other; but becauſe a little one requires proportionably leſs Force to put it into Motion than a larger one, the leſs will be the only one in which the Motion will be ſenſible, becauſe we have ſuppoſed the other to be immenſely greater, which will therefore be ſcarcely moved out of its Place by the leſs; juſt as it is with a Loadſtone and a Piece of Steel hung up by two Strings near one another; if the Steel be much leſs than the Loadſtone, it ſhall move almoſt all the Way towards the Loadſtone, while that ſhall ſcarce ſtir out of its Place; but if the Steel much exceeds the Loadſtone in Bulk and Weight, the Loadſtone ſhall then move to the Steel, and not the Steel to the Loadſtone, as in the other Caſe. This being granted,

Let us conceive the leſs Body to have a ſwift Motion given it in a Direction parallel to the Surface of the larger Body. This

(a) See Part I. Chap. IV.
(b) See more in Confirmation of this, Part I. Chap. IV.

Motion

Motion will in some Measure prevent its approaching to that Body, and the greater the Velocity is that is thus given to it, the longer will the Time of its Approach be protracted thereby, and the greater Space it will move over before it comes to the Body. The Velocity therefore which is thus given to it, may be so adjusted to the Degree of Tendency it has towards the other Body, that it shall move quite round it without ever touching it, returning to the same Place where it was when the Motion was at first given it. In this Case if it meets with no Opposition, it will move round it in the same Path over and over again; not unlike to a Stone whirl'd round the Hand in a Sling*(c)*. And as a smaller Body may continually move round a larger in this Manner, so may several less ones move round the same larger one at the same Time, provided the Distances of the less ones from each other be so great, that their mutual Tendencies towards each other do not disturb their Motions*(d)*.

This is the Case of the Planets and Comets with respect to the Sun, which is a Body immensely greater than any of them, placed as it were in the midst of an infinite Space, and

(c) See this explained more at large and illustrated, Part I. Chap. VIII.

(d) For the same Reason that the several smaller ones tend towards the larger, they will also tend one towards another, but with less Force in Proportion to their Smallness.

round

round which they perform their respective Revolutions *(e)*.

As to what we call the *Sky*, in which the Heavenly Bodies seem as it were fixed; it is no real Substance, but mere empty Space. The Reason why it appears to be a Substance, is as follows. Was there no Atmosphere surrounding the Earth *(f)*, whose Particles might reflect other Rays of Light to our Eyes, than those which come directly from the Sun, all Parts of the Heavens, even in spite of Sunshine, would be quite dark; and the Stars would be visible at Noon-day *(g)*. But since

(e) This Doctrine being allowed, we may see the Possibility of the Earth being habitable quite round: for if Bodies are not disposed to move any otherwise than as they tend towards some other, there can be no more Danger of our *Antipodes* falling from the Earth, than there is of our rising up into the Air. And as to their walking with their Head downwards with respect to us, there is neither Inconvenience, nor seeming Odness to them in that; for first, their Feet are turned towards the Surface of the Earth on which they walk, as well as ours are; and secondly, since both the Heavens and the Earth, which are the only Things they can judge of their Situation from, have the same Position with respect to their Bodies, when they stand or walk, that they have to us in the like Case, their Position cannot seem otherwise to them than ours does to us; for a Man cannot tell which way his Feet are turned, any otherwise than as his Judgment is directed by the Things about him; just as one cannot distinguish North from South in a strange Place, without a Compass, or seeing either the Sun or Stars.

(f) See the Nature and Constitution of the Atmosphere explained, Part II. Chap. III.

(g) See Chap. XIII.

the Atmoſphere of the Earth abounds with Particles capable of reflecting Light every way, ſome of it will fall upon our Eyes, to whatever Part of the Heavens they are directed, This Light, as all other, gives us the *Idea* of ſome Colour: having an Impreſſion of Colour upon our Minds, our Imagination preſents us with a Subſtance for it to inhere in. Juſt as when a Man views an Object in a Glaſs, the Light regularly reflected, gives him an Idea of the Colours of that Object duly ranged; but it is his Imagination that forms the Image he thinks he ſees.

Thus much by way of Introduction; which muſt not be underſtood as containing all the Reaſons for theſe Things; but only as tending to ſhew the Poſſibility, and in ſome Degree the Probability of them.

That which will be delivered afterwards, when we treat of the Phyſical Cauſes of the Motions of the Heavenly Bodies will ſet this Doctrine in a clearer Light, and abundantly evince the Reaſonableneſs of it. Proceed we in the mean time to conſider the Order theſe Bodies move in, and their reſpective Diſtances from the Sun, which makes the Subject of the firſt Chapter.

CHAP. I

Of the Bodies which compose the Solar System, *and their* real *Motions.*

THE Sun is a Body of prodigious Magnitude, situated in the midst of an unbounded Space, and the Fountain of Light and Heat to a certain Number of Planets and Comets, which continually move round it *(a)*: all these taken together make up what is called the *Solar System.*

The Planets are in number sixteen, six of which are called *Primary* ones; the other ten are distinguished by the Names of *Secondary* ones, *Moons*, or *Satellites.*

The *Primary* Planets are *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter* and *Saturn*. They revolve about the Sun at different Distances from it, moving from West to East *(b)*, in Orbits *(c)* nearly

(a) The Sun itself is not absolutely at Rest, but is subject to a very small Degree of Motion, which shall be consider'd when we treat of the Physical Causes of the Motions of the Heavenly Bodies.

(b) When any of the Heavenly Bodies are said by Astronomical Writers to move from West to East, or from East to West, it is always to be understood by way of the South. So that by the latter Expression is meant the same Way that the Sun appears to move in its daily Course, by the former is intended the contrary.

(c) By the Orbit of a Planet is meant the Way or Path it describes in moving round the Sun.

nearly circular, having the Center of the Sun directly within each of them, and nearly in the Middle; and they are not coincident one with another, but every one has about one Half a little above, and the other Half a little below each of the reſt. Or, to expreſs it in the common Way, the Planes *(d)* of their Orbits paſs through the Center of the Sun, and are ſo ſituated, that they make but ſmall Angles with each other.

Figure the firſt repreſents the *Solar Syſtem*, wherein the Point S is deſigned to denote the Center of the Sun; the Circle A B the Orbit or Path, which the neareſt Planet *Mercury* deſcribes in moving round it; CD, that in which *Venus* moves; FG, the Orbit of the Earth; H K, that of *Mars*; LN, that of *Jupiter*; and OP, that of *Saturn (e)*. Theſe Bodies, as they move round the Sun at very different

(d) The Plane of an Orbit is the Space included therein. Thus, if the Curve Line ADME, in Fig. II. repreſents an Orbit, the Surface of the Paper within that Curve repreſents the Plane of that Orbit. Or in other Words, the Plane of the Orbit of a Planet is a flat broad Space, out of which the Planet never ſtrays.

(*e*) That the Sun is at Reſt, and that the Planets move round it as deſcribed above, is an Opinion received of old by *Philolaus*, *Ariſtarchus* of *Samos*, and the whole Sect of the *Pythagoreans*. The *Egyptians* were early Obſervers of the Heavens, and from them probably this Notion was received in *Greece*; tho' the Notion of the Earth ſtanding ſtill in the Center, and the whole Heavens revolving round it, was generally received and defended till the Time in which *Copernicus* flouriſhed, *viz.* about the Year 1500, who reſtored the ancient Aſtronomy, and ſhewed in ſo clear a Manner how the Appearances of the Heavens might be

ferent Distances, the true Proportion of which is nearly represented in the Scheme, so they perform their respective Periods in very different Times; *viz. Mercury* in about three Months; *Venus* in about seven and a Half; the Earth in a Year; *Mars* in about two Years; *Jupiter* in twelve; and *Saturn* in not much less than thirty. And as they differ in their Periods, so they do surprisingly in their Mag-

be accounted for by it, that it became generally received, and from him was called the *Copernican* System.

There are two other *Hypotheses* of Note, *viz.* the *Ptolemaic* and the *Tichonic*. The *Ptolemaic* is so called from *Ptolemæus* a Mathematician of *Pelusium* in *Egypt*, a great Defender of this *Hypothesis*. He supposes the fix'd Stars to be stuck in the Firmament as they appear to the Eye; the Earth in the Center, round which he supposes the Moon to revolve; and at greater Distance, *Mercury*; at a greater still, *Venus*; at a greater than this, the Sun; at a still greater, *Mars*; then *Jupiter*; then *Saturn*; and beyond all, the Firmament with the fixed Stars; all in the same Time they appear to move round us. This *Hypothesis* was excellently well adapted to amuse the Vulgar, the Motions of the Sun and Planets being represented by it much after the same Manner as they appear to Sense. But since by the Help of Telescopes it has been discover'd, that *Venus* puts on different *Phases*, like the Moon, the Reason of which will be shewn in Chap. VI. this *Hypothesis* is entirely laid aside, as inconsistent therewith.

The *Tychonic Hypothesis* has its Name from *Tycho Brahe*, a *Danish* Nobleman. This Philosopher supposes that the Earth stands still, and that as the Sun moves round the Earth, the Planets move round that, just as the Secondaries move round their Primaries in the *Solar System* laid down above. There is another *Hypothesis* called the *Semitychonic*, and differs from the former only in this; that whereas that makes the Earth stand still, and the Sun with the Planets to move round it every Day; this gives the Earth a diurnal Motion round its *Axis*, and only supposes the Sun with the Planets revolving about it, to move round the Earth once a Year.

nitudes. For ſuppoſing the Circumference of the Sun to be repreſented by a Circle of ten Inches in Diameter, the ſeveral Circles Q, V, W, X, Y, Z, would nearly expreſs thoſe of the Planets *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn* reſpectively *(f)*.

Theſe Planets have no Light of their own; for when viewed through a Teleſcope, the Side only next the Sun is obſerved to be enlighten'd. They are alſo of the Form of a Globe; for one Edge of the ſhining Part ſometimes appears hollow, ſometimes ſtrait, and ſometimes convex, according as the Planet is ſituated with regard to the Spectator and the Sun; which it could not do, unleſs they were of that Form *(g)*.

Farther, the Planets do not only move about the Sun, as we have obſerved, but turn alſo about their own *Axes* *(h)* at the ſame Time, and

(f) Their Magnitudes, Diſtances, Times of Revolution, with their Rotation about their *Axes*, are more accurately expreſs'd in the following Table.

	Diameters in Engliſh Miles.	Diſt. from the Sun in Engliſh Miles.	Periodical Times.				Rotation about their *Axes*.			
			Days	H.	M.	S.	D.	H.	M.	S.
The Sun	763,460						25	6	0	0
Mercury	4,240	32,000,000	87	23	15	53	not diſcovered			
Venus	79,06	59,000,000	224	16	49	24	00	23	16	0
The Earth	7,970	81,000,000	365	6	9	14	00	23	56	4
Mars	4,444	123,000,000	686	23	27	30	00	24	40	0
Jupiter	81,155	424,000,000	4,332	12	20	25	00	9	55	54
Saturn	67,870	777,000,000	10,759	6	36	26	not known.			

(g) See the Reaſon why one Edge of the ſhining Part of a round Body appears ſometimes ſtrait, ſometimes hollow, ſometimes convex, largely explained Chap. VI.

(h) By the turning of a Planet about its *Axis* is meant, its turning about any two Points in its Surface that are oppoſite to one another;

and the ſame Way they do about the Sun, *viz.* from Weſt to Eaſt. As the former of theſe is generally compared to the revolving of a Stone in a Sling about the Hand, this latter may be compared to the ſpinning of a Top or the turning of a Wheel upon its Axle-tree. The Sun itſelf is not exempt from a Motion of this latter Kind.

This Motion of the Sun and ſome of the Planets about their *Axes* is diſcoverable by certain Spots, or diſtinguiſhable Parts on their Surfaces; which appearing firſt on one Extremity of their Diſks*(i)*, do by Degrees come forwards towards the Middle, and ſo paſs on till they reach the oppoſite Edge thereof, where they diſappear; and after they have lain hid about the ſame ſpace of Time that they continued viſible, they appear again as at firſt. By the Motion of theſe Spots paſſing in a right Line over the Diſk of a Planet, or riſing upwards one Half of the Way, and deſcending the other

another; and an imaginary Line ſuppoſed to be drawn through the Planet from one of thoſe Points to the other, is, while the Planet moves about thoſe two Points, called its *Axis*. Thus if the Reader holds a Ball between his Finger and Thumb, and turns it round, a Line ſuppoſed to paſs through it from Finger to Thumb, will in Aſtronomical Language be the *Axis* of that Ball. And if he takes it up by two other Points, and turns it, then will a Line paſſing through thoſe other Points become its *Axis*.

(i) This is a Term uſed by Aſtronomers for the Face of the Sun, Moon, or other Heavenly Body, when it is conſidered as a flat round Surface, as it appears to be: and the Breadth of it is conceived to be divided into twelve equal Parts, which they call *Digits*.

ther Half, the Situation of the Planet's *Axis*, about which it turns with reſpect to us, is evidently diſcovered. Theſe Spots are very viſible on the Surface of the Sun, *Venus*, *Mars* and *Jupiter*; but by reaſon of the Nearneſs of *Mercury* to the Sun, and the great Diſtance of *Saturn* from us, no Obſervations have been made that could diſcover any Spots in them, ſo that it is uncertain whether theſe Bodies revolve about their *Axes* or not. As to the reſt, the Times in which they thus revolve is expreſſed above in Note *(f)* in page 14. and the Inclination of their *Axes* in the Note below *(k)*.

In *Jupiter*, beſides his Spots, there are ſeveral broad Spaces running parallel to each other, called his *Belts*. Theſe, as alſo his Spots, are obſerved to undergo ſeveral Changes, neither keeping the ſame Magnitude nor Diſtance from one another. And ſome of his Spots have appeared only for a Time.

As to the Spots of the Sun, they are ſubject to much greater Variety of Changes. New ones appear, and old ones vaniſh, others ſucceeding in their Room; ſeveral ſmaller ones run together, and form one larger; and larger

(k) The Inclination of the Axis of the Sun to the Plane in which the Earth moves, is ſeven Degrees, and a half: that of *Venus* to the ſame Plane fifteen Degrees, according to an Obſervation of Signior *Blanchini*'s at *Rome* in the Year 1726. the Earth's 23 Degrees and 29 Minutes: and *Mars* and *Jupiter*'s are nearly at right Angles with their own Orbits.

er ones are ſometimes divided into ſmaller *(l)*. However, ſeveral have remained on the ſame Part of its Body long enough to determine the Time of its Motion about its Axis. See Note *(f)* in page 14.

In the Beginning of this Chapter it was obſerved, that the Planets move round the Sun in Orbits nearly circular, and in Planes croſſing each other in ſuch Manner, as to make very ſmall Angles with each other. As to the Form of their Orbits, they are *Ellipſes* (*m*), having one of their *Foci (n)* in the Center of the Sun. They differ very little from Circles, or to ſpeak properly, their Excentricities (*o*) bear a very

(l) We learn from Hiſtory, that the Sun has wanted its uſual Brightneſs, ſhining with a dim and obſcure Light for a Year together. This was probably owing to its being in a great Meaſure covered with Spots; for now there are ſometimes Spots ſeen upon it, that are larger than the Surface of the whole Earth.

(m) An *Ellipſe* is a Figure like an Oval; but ſtrictly ſpeaking, a Geometrical Curve, made by the tranverſe Section of a Cone or a Cylinder. But the Nature of it may be underſtood from the following Deſcription. Stick up two Pins at F and G (Fig. 2.) over which put a Thread, both Ends being tied together; then with a third Pin at P, within the Thread, keeping it upon the Stretch, deſcribe the Curve ADBE, which is an Ellipſe. And if we make Uſe of the ſame Thread, but increaſe or diminiſh the Diſtance between the Pins, the Figure deſcribed will ſtill be an Ellipſe, though of a different Kind. And when the Pins are brought cloſe together, the Figure deſcribed degenerates into a Circle.

(n) The Points F and G where the Pins were fixed, are called the *Foci* of the Ellipſe, and the middle Point between them the Center.

(o) The Diſtance between the Center and either of the *Foci* is the *Excentricity*.

a very small Proportion to their longer *Axes (p)*. And as to the situation in which their Planes lie in, it is in all nearly the same with that of the Earth's Orbit. See the Excentricities of their Orbits, and their Inclinations to that of the Earth's, in the Notes *(q)* and *(r)*.

The Primary Planets are again distinguished into two Kinds, *viz. Inferior* and *Superior*; those which are nearer the Sun than the Earth is, as *Mercury* and *Venus*, are called the *Inferior* Planets; the others the *Superior* ones.

The Secondaries are constant Attendants of the respective Primary ones to which they belong, revolving round them, while they themselves move round the Sun. Of the Primary Planets there are only the three largest, as far as the Observations of Astronomers have been able to discover, that have their Secondaries, *viz.* the *Earth*, *Jupiter*, and *Saturn*.

The

(p) The Line AB, which passes through the *Foci*, and is terminated at each End in the Ellipse, is called the *longer Axis*. The Line DE, which passes through the Center of the Ellipse in a Direction perpendicular to the former, and is terminated by the Sides of the Ellipse, is the *shorter Axis*.

(q) The Excentricities of the several Orbits of the Planets are as follow: Supposing the Distance of the Earth from the Sun 1000 equal Parts, the Excentricity of *Mercury*'s Orbit is about 80 such; of that of *Venus* 5; of that of the *Earth* 17; of *Mars* 141; of *Jupiter*'s 250; and the Excentricity of *Saturn*'s Orbit 247.

(r) Their Inclinations are as follows: *viz.* The Orbit of the Planet *Mercury* is inclined to that of the Earth 6 Degrees, 54 Minutes; that of *Venus* is inclined to it 3 Degrees, 24 Minutes; that of *Mars*, 1 Degree, 51 Minutes; that of *Jupiter*, but 1 Degree, 19 Minutes; and that of *Saturn*, 2 Degrees and 33 Minutes.

The *Earth* is attended but by one, which is the Moon. This Secondary revolves round the Earth from Weſt to Eaſt in 27 Days, 7 Hours and 43 Minutes, at the Diſtance of about 60 Semidiameters and an half of the Earth from its Center *(s)*; and performs its Rotation about its Axis, which is nearly perpendicular to the Plane of the Earth's Motion, in the ſame Time and the ſame Way that it revolves about the Earth.

The Planet *Jupiter* has four Moons, which revolve about it at different Diſtances and in different Times. The innermoſt performs its Revolution in about one Day and eighteen Hours, nearly at the Diſtance of ſix Semidiameters of *Jupiter* from its Center; the ſecond revolves about it in about three Days and an Half, at the Diſtance of about nine ſuch Semidiameters; the third in about ſeven Days, at the Diſtance of 14 Semidiameters; the fourth and outermoſt Satellite requires almoſt 17 Days to perform its Courſe in, and is diſtant from its Center 25 Semidiameters nearly *(t)*.

D *Saturn*

(s) The Excentricity of the Moon's Orbit is about 3,3 of the Semidiameter of the Earth; and its Inclination to that of the Earth is 5 Degrees and 18 Minutes. The Reaſon that the Moon appears ſo large, and to outſhine all the other Planets, both Primary and Secondary, is becauſe of the Smallneſs of its Diſtance from us in reſpect of theirs; for had it been as far removed from us as ſome of them, it had never been ſeen at all by us.

(t) More accurately as in the following Table.

Satell.

Saturn has no leſs than five Satellites: The firſt and innermoſt revolves about it in one Day and 21 Hours, at the diſtance of about 5 Semidiameters and an half of *Saturn* from its Center; the Second performs its Courſe in two Days aud 17 Hours, at the Diſtance of 6 Semidiameters and a Quarter; the Third in about 4 Days and an half, and is diſtant from it about 8 Semidiameters and 3 Quarters; the fourth compleats its Period in about 16 Days, at the Diſtance of 20 Semidiameters; the fifth and outermoſt requires above 79 Days to perform a Revolution in, and is 59 Semidiameters of *Saturn* diſtant from its Center *(u)*.

These Satellites, as alſo thoſe of *Jupiter*, perform their Revolutions about their Primaries from Weſt to Eaſt, as their Primaries do about the Sun.

The Orbits of the ſecondary Planets are expreſſed in Fig. I. by the pricked Circles; *viz.*

Satell.	Periodical Times. D.	H.	M.	S.	Diſt. in Semid. of *Jupiter*.
1	1	18	27	34	5,667
2	3	13	13	42	9,017
3	7	3	42	36	14,384
4	16	16	32	09	25,299

(*u*) More accurately as in the following Table.

Satell.	Periodical Times. D.	H.	M.	S.	Diſt. in Sem. of *Saturn*'s Ring.
1	1	21	18	27	2,10
2	2	17	41	22	2,69
3	4	12	25	12	3,75
4	15	22	41	14	8,70
5	79	07	48	0	25,35

viz. that of the Moon at E, thoſe of the Satellites of *Jupiter* at I; and thoſe of *Saturn* at T; the Centers of the *Earth*, *Jupiter* and *Saturn* being ſuppoſed to be in the Points E, I, and T reſpectively.

As to the Orbits of the Satellites of *Jupiter* and *Saturn*, Aſtronomers have not obſerved that they differ from Circles; and whether theſe Secondaries revolve round their *Axes* or not, has alſo not been diſcovered (*x*).

The Secondaries are alſo Bodies having no Light of their own, being obſerved to be eclipſed as oft as they enter the Shadows of their Primaries.

Beſides its Satellites, *Saturn* is alſo encompaſſed with a flat broad Ring, not unlike the Verge of an Hat, but it no where touches the Body of *Saturn*, the Space between it and *Saturn* being equal to the Breadth of the Rim. The Diameter of the Ring, meaſuring acroſs from one Outſide to the other, is to the Diameter of *Saturn*'s Body as 7 to 3 (*y*).

The Form of the Ring encompaſſing the Body of *Saturn*, is expreſs'd in Fig. I. at RR. The

(*x*) The Orbits of *Jupiter*'s Satellites lie almoſt in the ſame Plane in which *Jupiter* himſelf moves round the Sun; but thoſe of *Saturn* deſcribe their Orbits nearly in the Plane of the Ring produced, except the fifth, the Situation of whoſe Orbit deviates a little therefrom. See the Inclination of the Ring in the next Note.

(*y*) The Ring, according to Meſſrs. *Roëmer*, *Picard* and *Hugens*, is inclined to the Orbit of the Earth 31 Degrees: But Dr. *Halley* affirms it to be nearly parallel to the Plane of the Earth's Equator.

The Comets revolve alſo round the Sun in Orbits, whoſe Planes paſs through its Center; but they are not confined to any particular Direction, as the Planets are, ſome moving one Way, and ſome another: their Orbits are of a very elliptical or oval Form, ſo that their Lengths vaſtly exceed their Breadths; and they lie not nearly in the ſame Plane, like thoſe of the Planets, but admit of any Inclination to one another. But more of theſe in another Place(z). A Portion of the Orbit of a Comet is expreſſed in Fig. I. at *abcd* (*a*).

Thoſe

(z) Chap. XI.

(*a*) The Claim which the Syſtem here laid down has to a Preference to all others, will appear from the following Conſiderations.

I. The Planet *Venus* moves round the Sun, and not round the Earth. For through a Teleſcope it ſometimes appears with a bright and round Face like the full Moon, and ſometimes horned like the new Moon; (ſee Chap. VI.) and when it appears full, it is ſeen near the Body of the Sun, as well as when it appears horned: which things could not be, did it move either round the Earth alone, or round neither the Sun nor the Earth. For did it move round the Earth alone, or round both the Sun and the Earth, it muſt be ſeen ſometimes in that Part of the Heavens which is oppoſite to the Sun. Did it move round neither the Sun nor the Earth, it could not appear both horned and full when it is ſeen near the Body of the Sun: it remains therefore, that it revolves round the Sun, and not round the Earth. Its Orbit therefore, agreeably to the abovementioned Syſtem, includes the Sun, and is within the Orbit of the Earth.

II. In like Manner we are aſſured, that *Mercury* turns round the Sun in a leſſer Orbit than that of *Venus*; becauſe it puts on the ſame Phaſes with *Venus*, and is never ſeen ſo far from the Sun as that Planet is.

III.

Those bright and shining Bodies, which besides the Planets and Comets are dispersed every

III. The Orbit of *Mars* includes both the Earth and the Sun; and the Earth is not in the Center thereof. For this Planet is capable of appearing opposite to, or in any other Situation in respect of the Sun, which it could not be, unless it moved round the Earth; and it always appears full, or nearly so; which it could not do if it ever came between the Sun and the Earth: it moves therefore also round the Sun. And farther, when *Mars* is in the opposite Part of the Heavens to the Sun, it appears about five times larger than when it is near the Sun; which shews that it is so many times nearer the Earth in one Situation than in the other: the Earth therefore is not in the Center of its Motion. All which is agreeable to the System above laid down

IV. Since the like is observable both of *Jupiter* and *Saturn*, (though, by Reason of their greater Distance from the Sun and us, the Diversity in their apparent Magnitudes in different Parts of their Orbits is not so great as in *Mars*) it is reasonable to conclude, that these Planets also have both the Earth and the Sun within their Orbits, and that rather the Sun than the Earth is in the Center of the same.

V. Since then the Earth is placed within the Orbits of the Planets *Mars*, *Jupiter*, and *Saturn*, these Planets cannot appear to us to stand still or to go backward, as Observation shews they sometimes do appear, unless the Earth moves: and since, as we have shewn, the Planets *Venus* and *Mercury* revolve about the Sun, and not about the Earth; since likewise the Earth is placed between the Orbits of *Mars* and *Venus*, and the Periodical Time of the Earth, if it does move, is, in point of Magnitude, between the Periodical Times of those two, it has been thought reasonable to suppose that the Earth revolves in like Manner about the Sun, as the Planets do, and is therefore reckoned in the Number of them.

VI. But what wonderfully confirms this Conclusion, is, that Harmony which upon this Supposition runs through the whole *Solar System*; *viz.* that the Motions of all the Planets, both Primary and Secondary, are governed and regulated by one and the same Law; which is, that *the Squares of the Periodical Times of the Primary Planets are* to each other, *as the Cubes of their Distances* from the Sun; and likewise *the Squares of the*

every where throughout the Heavens, are the *fixed Stars.* They are termed *fixed*, becaufe they are obferved to keep the fame Diftances from each other in all Ages, having no Motion like that of the Planets or Comets. They are far removed out of the *Solar Syftem*, and fhall therefore be confidered in a Chapter by themfelves.

the periodical Times of the Secondaries of any Primary are to each other, *as the Cubes of their Diftances* from that Primary. Now the Moon, which in the Syftem above laid down is a Secondary of the Earth, in the other Hypothefes is a primary One; and fo the Rule cannot take Place; becaufe her Periodical Time, confidered as that of a Primary one, does not agree therewith. Whereas according to the other Suppofition, the Motions of the Heavenly Bodies, as we fhall fee in its proper Place, are to be accounted for with the greateft Simplicity; and a regular and beautiful Fabrick is thereby exhibited, worthy its divine Architect.

By the *Square* of the periodical Time of a Planet is meant the Time it takes up in revolving about the Sun, or about its Primary, multiplied by itfelf *Once*; and by the *Cube* of its Diftance, is meant its Diftance multiplied by itfelf *Twice*. Thus if the Time a Planet moves once round its Orbit in, be 30 Years, then 30 times 30 is the *Square* of its periodical Time: and if the Diftance of a primary Planet from the Sun, or of a Secondary from its Primary, be 81 Millions of Miles, then 81 Million multiplied by 81 Million, and that again by 81 Million, will exprefs the *Cube* of the Diftance of that Planet.

CHAP.

CHAP. II.

Of the fixed Stars.

THE *fixed Stars* are those bright shining Bodies, which, besides the Planets and Comets, are dispersed throughout the Heavens: they are termed fixed, as appearing always at the same Distance from each other.

It is observable of the *fixed Stars*, that, when viewed through a Telescope, they appear only as mere Points destitute of all sensible Magnitude. This is supposed to be owing to their very great Distance from us. That they appear larger to the naked Eye than with a Telescope, is ascribed to that Irradiation or Brightness which is observed to surround shining Bodies when seen in the Dark, and which the Telescope takes off *(a)*.

From their shining with so strong and sparkling a Light, when at the same Time they are so far off that through a Telescope they appear as mere Points, is inferr'd, that they shine with

(a) That the Stars would appear of no sensible Magnitude to the naked Eye, were it not for that Irradiation or Brightness mentioned above, is confirmed from their being observed to *twinkle*; which the Planets and other Bodies of sensible Magnitude do not. The Reason of their *twinkling* is, that the least Particle of Dust or Vapour passing between the Eye and them is capable of intercepting their Appearance for a Time, which in larger Bodies it is not.

with their own proper and unborrowed Light: for if they borrowed their Light, they would be invisible to the naked Eye, since the Satellites of *Jupiter* and *Saturn*, which appear of very distinguishable Magnitudes through a Telescope, are so.

There is another remarkable Circumstance relating to the starry Heavens; that the better Telescopes we make use of, the more Stars appear: from whence it is thought, that all these *fixed Stars* are not placed at equal Distances from us, but that they are every where interspersed at great Distances beyond one another throughout the Universe; and that probably the different Appearances which they make in point of Splendor and Magnitude may be rather owing to their various Distances from us, than to any Difference in their Magnitudes.

The Distance of the nearest *fixed Star* is computed to be at least 500 thousand Millions of Miles from us; and their Magnitude is judged to be such, that if they were as near us as the Sun is, they would not appear unlike it (*b*).

Their appearing under various Magnitudes has given Occasion to a Distribution of them into several Orders or Classes: those which appear largest, are called *Stars of the first Magnitude*,

(*b*) The Method of computing the Distance of the *fixed Stars* is shewn in Chap. V.

Magnitude; thoſe which are neareſt to them in Luſtre are reckoned of the *ſecond Magnitude*; and ſo on, till we come to the leaſt Stars that are viſible to the naked Eye, which conſtitute the laſt Claſs, and are ſaid to be of the *ſixth Magnitude*. For ſuch as are to be ſeen only by the help of a Teleſcope, are not included in thoſe ſix Claſſes, but are diſtinguiſhed by the Name of *Teleſcopic* Stars; the Diſtribution of the Stars into Claſſes being made long before the *Teleſcopic* ones were diſcovered. But we are not to underſtand from hence, that all the Stars which are viſible to the naked Eye, are reducible to ſome one of theſe Claſſes preciſely; for there are ſcarcely two Stars to be found exactly of the ſame Size: they are only to be ranked in that Claſs to which their Magnitude and Luſtre gives them the juſteſt Claim.

There is alſo another Diſtribution of the Stars into *Conſtellations*. The Ancients, that they might the better diſtinguiſh the Stars with regard to their Situation in the Heavens, divided them into ſeveral *Aſteriſms* or *Conſtellations*, that is, Syſtems of Stars, each Syſtem conſiſting of ſuch as are near each other. And to diſtinguiſh theſe Syſtems from one another, they gave them the Names of ſuch Men or Things as they fancied the Space they took up in the Heavens repreſented, or of thoſe whoſe Memories, in Conſideration of ſome no-

table Exploit, they had a Mind to tranſmit to future Ages.

Beſides the Stars contained within theſe Conſtellations, there are ſome others, which for that Reaſon are called *Informes.* Of theſe the later Aſtronomers have made ſome new Conſtellations.

There is alſo a Diviſion of the Heavens into three Parts; *viz.* 1. The *Zodiac*, or that Portion of the Heavens in which the Planets would appear to move to an Eye placed in the Sun. The Breadth of this Space depends on the Inclination of the Orbits in which the Planets move to one another, and includes twelve Conſtellations, commonly called the Signs of the *Zodiac*; *viz. Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, and *Piſces*: and becauſe moſt of theſe are Animals, that Space is called the *Zodiac*. 2. All that Region of the Heavens which lies on the North Side the *Zodiac*; which contains twenty-one Conſtellations: and 3. That on the Southern Side, which contains fifteen;

There is a remarkable Tract that goes quite round the Heavens, called the *Milky Way*. This Space has a peculiar Whiteneſs, occaſion'd by an infinite Number of ſmall Stars therein, none of which are to be ſeen diſtinctly without a Teleſcope, yet conſpire to render that Part of the Heavens, where they are, much more

more luminous than the reſt. This Tract is in ſome Parts ſingle, in others double *(a)*.

Notwithſtanding that ſeeming Impoſſibility of *numbring* the Stars, their relative Situations one to another have been ſo carefully obſerved by Aſtronomers, that they have not only been able to number them, but even to diſtinguiſh the Places of each in the Heavens, and that with greater Accuracy, than any Geographer could ever point out the Situations of the ſeveral Cities or Towns upon the Surface of the Earth. And not only the Places of thoſe few, if I may call them ſo, which are to be ſeen

(a) Hugenius, in the Year 1656, looking by chance through a large Teleſcope at three ſmall Stars very cloſe to one another in the Middle of *Orion*'s Sword, ſaw ſeveral more as uſual: But three little Stars very near one another, together with four larger ones, ſhone out as it were through a whitiſh Cloud much brighter than the ambient Sky; which being very black and ſerene, cauſed that lucid Part to appear like an Aperture that gave a Proſpect into a brighter Region. He viewed it many Times, and found it continued in the very ſame Place and of the ſame Shape. But in the *Philoſophical Tranſactions*, No. 347. there is an Account of a later Diſcovery of five more ſuch lucid Spots, though leſs conſiderable than this of *Hugens*; the Middle of which, we are there told, is at preſent in II. nineteen Degrees, with South Latitude twenty-eight Degrees, forty-five Minutes; and that it ſends forth a radiant Beam into the South-Eaſt; as another in the Girdle of *Andromeda* ſeems to do in the North-Eaſt. It is alſo there remarked, that tho' theſe Spots are in Appearance but ſmall, and moſt of them but a few Minutes in Diameter, yet ſince they are among the *fixed Stars*, as it appears they are by their having no Parallax, they cannot fail to occupy Spaces immenſely great, and perhaps not leſs than our whole *Solar Syſtem*: in all which Places, it ſhould ſeem, there is a perpetual uninterrupted Day.

with the naked Eye have been pointed out and regiſtered by them, but even of ſuch as are diſcoverable only by the Teleſcope. The firſt among the *Greeks*, who numbered thoſe which are viſible to the naked Eye, and regiſter'd their Places in a Catalogue, was *Hipparchus*. He flouriſh'd about 120 Years before *Chriſt*, and number'd 1022 Stars. To this Catalogue the Places of ſeveral more have been added by ſucceeding Aſtronomers, as *Ptolemy*, *Copernicus*, *Tycho Brahe*, *Ricciolus*, and Dr. *Halley*; which laſt went to the Iſland of St. *Helena*, where he reckoned up and ſet down the Places of ſuch Stars in the Southern Hemiſphere as are not viſible to us in this.

The laſt Catalogue is that of *Flamſtead*, who reckons 3000, of which a great Part are *Teleſcopical*. The Number of Stars which may be ſeen at one Time with the naked Eye in one Hemiſphere ſeldom exceed a Thouſand; which perhaps may appear ſtrange, ſince at firſt Sight they ſeem to be innumerable. But this Appearance is only a Deception of Sight, ariſing from a confuſed and tranſient View. Let a Perſon ſingle out a ſmall Portion of the Heavens, and after ſome Attention to the Situation of the more remarkable Stars therein, begin to count, and he will ſoon be ſurpriſed to find how few there are therein.

Yet neverthleſs, if we conſider the Teleſcopic Stars along with thoſe which are viſible

to the naked Eye, we may venture to pronounce them infinite: ſo prodigious are the Numbers that may be ſeen in a good Teleſcope.

Thoſe in Mr. *Flamſtead's* Catalogue mention'd above, that are Teleſcopical, are only the more remarkable ones, ſuch whoſe Longitudes and Latitudes (that is, their true Situation in the Heavens) it was thought worth while to regiſter and put down. Dr. *Hook*, with a Teleſcope of 12 Feet, ſaw 78 Stars among the *Pleiades*, that is, thoſe which are commonly called the ſeven Stars; and with a longer Teleſcope ſaw more *(b)*. And in the ſingle Conſtellation of *Orion*, which in *Flamſtead's* Catalogue has but 80 Stars, there have been ſeen 2000 *(c)*.

There is one Thing more which is very remarkable in the fixed Stars, *viz.* that ſome of thoſe which were taken Notice of, and had their Places regiſter'd, have ſince diſappeared. Some have diſappeared for a Time: Some new ones have appeared, and afterwards diſappeared; particularly a remarkable one in the Conſtellation *Caſſiopeia*, in the Year 1572, which ſome time after its firſt Appearance outſhone the biggeſt of the fixed Stars, and in ſixteen Months Time by degrees vaniſhed quite away, and was never ſeen ſince. In the *Pleiades*, which uſed to be reckoned 7, there are but 6 to be ſeen with the naked Eye.

From

(b) Hook's Micrograph. pag. 241

(c) Antonius Maria de Rheita, Rad. Sydereomiſt. pag. 197.

From the Similitude there appears to be between the *fixed Stars* and the Sun, it is generally ſuppoſed by Philoſophers, that they are not placed in the Heavens by way of Ornament only, or to ſupply us with a faint Light in the Abſence of the Moon; but that each of them is placed in the midſt of a Syſtem of planetary Worlds, and that it directs their Motions, and ſupplies them with Light and Heat, in the ſame Manner that the Sun does the ſeveral Bodies of which the Solar Syſtem is compoſed *(d)*.

As to thoſe Alterations in the Appearances of ſome of the fixed Stars mentioned above, Sir *Iſaac Newton* conjectures, that, as it is poſſible the Sun may ſometimes receive an Addition of Fewel by the falling of a Comet into it (the only Uſe *(e)* they ſeem to be of,) ſo the ſudden Appearance of ſome of the Stars, which formerly were not viſible to us, may be owing to the falling of a Comet upon them, and occaſioning an uncommon Blaze and Splendor for ſome Time. But that ſuch as appear and diſappear periodically, and increaſe by very ſlow

(d) How the Sun directs the Motions of the Planets will be ſhewn afterwards.

(e) Sir *Iſaac Newton* computes, that the Comet which appeared in 1680 approached towards the Sun within leſs than a ſixth Part of the Sun's Diameter; and from thence concludes, that it muſt have been retarded by the Reſiſtance of the Sun's Atmoſphere, which is very large; the Conſequence of which is, that it muſt come nearer and nearer the Sun every Revolution, till at laſt it falls into its Body.

slow Degrees, seldom exceeding those of the third Magnitude, may be such as having large Portions of their Surfaces obscured by Spots, may by revolving about their *Axes*, like the Sun, expose their lighter and darker Parts to us successively.

CHAP. III.

Of such of the apparent *Motions of the Heavenly Bodies, as arise from the Motion of the* Earth *about its* Axis.

HAving now laid down the *true* and proper Motion of the Planets, and shewn, as far as the Observations of Astronomers have been able to inform us, the Nature of the Sun, the Planets, and the fixed Stars; we are in the next Place, to enquire particularly into their *apparent* Motions, and shew how it comes to pass that these seem to us so different from what they really are. And these are principally of two Sorts, *viz.* those which arise from the Motion of the Earth round its *Axis*, and those which are owing to the Motion of the Earth and Planets about the Sun. The former of these shall be consider'd in this Chapter; the latter in those which follow.

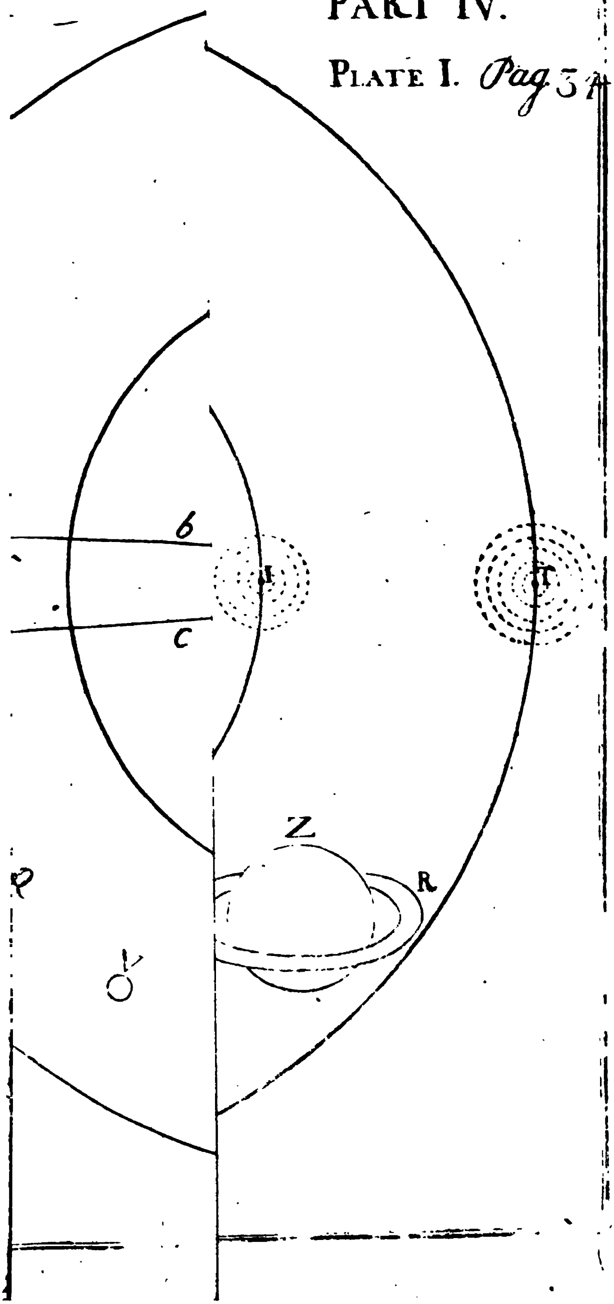
Since then, as we observed above, the Earth turns round its own *Axis* from West to East, every Spectator upon its Surface must necessarily be carried round it the same Way; and consequently those Parts of the Heavens which lie hid towards the East, will by and by come into his Sight, and those which are visible to him will depart out of it towards the West. From hence it is that the Spectator, not being sensible of his own Motion (the Reason of which is, because all things about him move along with him) imagines the whole Heavens to turn round the contrary Way, *viz.* from East to West, every 24 Hours, which is nearly *(f)* the Time the Earth performs one Revolution about its *Axis* in. This may be illustrated in the following Manner.

Let the Circle STV (Fig. 3.) represent the Earth, S the Place of a Spectator, ABC so much of the Heavens as is visible to him in that Situation, and let A be the Place of the Sun or any other of the heavenly Bodies: When S, the Place of the Spectator, is carried by the Rotation of the Earth about its *Axis* to T, the visible Part of the Heavens will become B A D; and the Point A, which before was just at one Edge of the visible Portion of the Heavens, is now in the midst of it, or directly over the Spectator's Head. Again, when

(f) The Reason why this is not the *exact* Time, is, because of the Motion of the Earth in its Orbit performed in the mean while; as shall be explained in Chapter the XV.

PART IV.

PLATE I. Pag 37



when the Spectator is got to V, the visible Part of the Heavens is become ADX, and the Point A is got to the other Extremity of it, and just ready to disappear; after which it is seen no more till the Spectator arrives at the Point S again.

From this Motion of the Earth arises that apparent Revolution of the Planets and fixed Stars once in 24 Hours; as also that of the Sun, and therewith the Succession of Day and Night.

That the Days vary from each other both in Point of Length and Heat, is owing to the Motion of the Earth round the Sun, and will be accounted for in the next Chapter.

CHAP. IV.

Of the apparent *Motion of the* Sun, *arising from the* Earth's *revolving about it.*

AS when a Person sails along the Sea-Coast, the Shore, the Villages, and other remarkable Places upon Land appear to change their Situation, and to pass by him; and this apparent Change of Place is of two Sorts, (the one, is that of Bodies at rest, the

Change of whose Places depend solely on that of the Spectator; the other is, that of Bodies in Motion, whose apparent Change of Place depends as well on their own Motion as on that of the Spectator;) so it is in the Heavens: To a Spectator upon the Earth, as it moves along its Orbit, the Sun, the Planets, and the fixed Stars appear to change their Places; which Change in those which are at rest is owing wholly to the Motion of the Earth; in those which move, it is to be ascribed to their own conjointly with that of the Earth: The former Kind shall be considered in this Chapter and the next, the latter in those which follow. To begin with that of the Sun.

As a Spectator in the Sun would see the Earth describing a Circle in the Heavens, which is that where the Plane of the Earth's Orbit (were we to suppose it extended far enough) would cut them; so a Spectator on the Earth, being carried round the Sun therewith, would imagine *that* to move through the same Circle, and in the same Direction, passing from Star to Star till it has appeared to move quite round the Heavens. This Circle is called the *Ecliptic*, and from what was observed above of the Zodiac, necessarily passes through twelve Signs thereof. From hence it is, that the Sun is said to pass through the Signs of the Zodiac once a Year.

To illustrate this, let S (Fig. 4) represent the Sun, ABC the Orbit of the Earth, and DEF the starry Firmament. Then if we suppose the Spectator to be in the Sun, as the Earth moves from A to B, it will seem to move from D to E, describing the Arch DE in the Heavens, and so on; but if the Spectator be upon the Earth, then whilst it moves from A to B, the Sun will seem to describe the Arch GH in the opposite Part of the Heavens: and whilst the Earth is passing from B to C, the apparent Place of the Sun will pass from H to I; and so on through the whole Circle. So that the Sun appears to describe the same Circle among the fixed Stars in a Year, that the Earth would seem to describe in the same Time to a Spectator in the Sun.

Having thus shewn, that the annual Motion of the Earth gives the Sun an apparent one in the *Ecliptic*, we shall now proceed to enquire how the Variety of Days and Nights, as to their Lengths and the different Seasons of the Year, are to be accounted for. In order to which we will premise the three following Observations.

I. As the Earth turns round its *Axis*, there are two Points in its Surface which have no Motion from thence, *viz.* the two Extremities of its *Axis*. These Points are called the *Poles* of the Earth; that which respects the

 North,

North, is called the *North Pole*, the other the *South Pole*. But every other Point of the Earth's Surface describes a Circle, which will be greater or less, as the Point that describes it is farther from or nearer to either Pole; and consequently that Circle, whose describing Point is equally removed from either, will be the largest of all. This Circle is called the *Equator*, by Mariners the *Line*; and its corresponding Circle in the Heavens, or that which the Plane of this Circle, were we to suppose it extended thither, would mark out, the *Equinoctial* Circle; and those lesser Circles described by other Points of the Earth's Surface are called *Parallels* of this.

II. The *Axis* of the Earth, as was observed above, making an Angle with the Plane of its Orbit of 23 Degrees and an half, does the same with that of the *Ecliptic*, those two Planes being coincident one with another. Thus supposing the Line AB (Fig. 5.) to represent the Plane of the *Ecliptic* seen edgeways, and the Circle DFE the Earth; the *Axis* of it is not coincident with the Perpendicular DE, but stands inclined in the Situation FG, supposing the Arch DF to contain 23 Degrees and an half. And,

III. In whatever part of its Orbit the Earth is, its *Axis* has the same Inclination to the Plane

Plane thereof, and is every where parallel to itself: that is, if a Line be conceived as drawn parallel to the *Axis* while the Earth is in any one Point of its Orbit, the *Axis* will in every other Situation of the Earth be parallel to the said Line. Thus supposing the Line FG to represent the Situation of the *Axis* of the Earth when at DFG, and to be parallel to the Line HI; then when the Earth is at *dfg*, or any other Part of its Orbit, its *Axis* *fg* will still be parallel to the same Line HI *(a)*.

These things being observed, let us imagine the Plane of the *Ecliptic* to be represented by the Line AB (Fig. 6.) seen edgeways as before. Let the Point S represent the Place of the Sun, and PEpQ the Earth, whose *Axis* let be Pp, and its Poles P and p, the former the North, the latter the South; and let EQ be the Equator and TC, RN, &c. Parallels thereof. And let us in the first Place consider the Earth in the Situation KL, or directly beyond the Sun with respect to the Eye. It is evident from the Figure, that in this Situation of the Earth, the Sun's Place S is in the Plane of the Equator EQ produced; and consequently, while the Earth

(a) N. B. The Axis of the Earth deviates a little from its Parallelism every Revolution of the Earth about the Sun, but in so small a Degree that the Consideration of it is better omitted in this Place; it shall be taken Notice of in the next Chapter.

Earth revolves upon its *Axis* Pp, the Sun will appear to describe that Circle, and will enlighten one Side of the Earth from Pole to Pole. Whence half of the Equator, and half of each of its Parallels RN, TC, &c. will be enlightened thereby, while the other half being on the opposite Side the Earth, will be in the Dark. And consequently every Place on the Surface of the Earth, since it must describe some one of those Parallels as it is carried round by the diurnal Motion, will be as long in the Light as in the Dark; that is, the Days and Nights will then be of equal Length all over the Earth *(b)*.

But because the *Axis* of the Earth is every where parallel to itself, as was just now observed; when the Earth is carried by its annual Motion into the Situation MO (which we will suppose to be a quarter of a Circle distant from its former Situation, the Poles of the Earth being still turned the same Way as before) it is manifest that the Sun's place S is not now in the Plane of the Equator EQ produced, (for that would pass below the Sun towards L) but directly over C, a Point in the Parallel TC; and therefore as the Earth revolves about its *Axis*, all places of the Earth that

(b) When the Sun appears to enter the first Degree of the Sign *Aries*, the Earth is in that Part of its Orbit where these Phænomena happen.

that lie in this Parallel will come directly under the Sun once in 24 Hours, as those that lay under the Equator did in the former Situation of the Earth: and since the Rays of the Sun always enlighten one half of the Globe of the Earth at a Time, they will reach beyond the North Pole as far as F; and on the other Side no farther than I. From whence it follows, that the Tract of the Earth which lies within the Circle FG enjoys continual Day-light, while the Earth is in this Part of its Orbit; and on the contrary, that it is continual Night to that Part of the World that lies within the opposite Circle HI: that is, the Sun will not appear to set in the former, nor rise in the latter. And farther, greater Portions of those Parallels which lie between the Equator and the Circle FG will be in the illuminated Hemisphere than in the dark one, as appears by Inspection of the Figure, in which the latter Hemisphere is distinguished from the other by being shaded; and on the contrary, greater Portions of such as lie between the Equator and the Circle HI will be in the Dark, than in the Light. And hence it is that the Earth being in this Part of the Orbit, the Days are longer than the Nights on the Northern Side the Equator; whereas on the Southern Side, the Nights are longer than the Days; and the Difference between the Lengths of the Days and Nights is so much

the

the greater, as the Place is more remote from the Equator on either side, because the Difference between the Portions of the Parallels which are in the illuminated Hemisphere, and those which are in the contrary one, are greater the farther they are from the Equator: But to such as live under the Equator, Day and Night are in this Situation of the Earth also equal to one another, one half of the Equator EQ being here also in the illuminated Hemisphere (c).

Let us now conceive the Earth to have moved through another quarter Part of its Orbit, and to be in the Situation KL again, but with this Difference, that it is now on the contrary side the Sun to what we supposed it at first. In this Situation also it is evident, that the Plane of the Equator EQ being produced, will pass through the Sun, and that the illuminated Hemisphere will pass from Pole to Pole as before; for we are still to conceive the *Axis* of the Earth parallel to its first Situation; so that the Sun will seem to describe the same Circle in the Heavens, and the Days and Nights will be equal every where, as

(c) In this Situation of the Earth the Sun appears to be in the first Degree of *Cancer*, and to the Inhabitants of the Northern Hemisphere of the Earth introduces the Summer; to those of the other, the Winter Quarter.

as when the Earth was in the opposite Point of its Orbit *(d)*.

But when the Earth has passed through another quarter of its Orbit (which we will suppose it to have done, when it is in the Situation X Y) its *Axis* Pp still remaining parallel to its former Direction, the Place of the Sun will now be vertical to R, a Point under the Parallel RN; so that to all the Inhabitants that live under this Parallel, the Sun will be vertical once in 24 Hours, as it was to those who live under the Parallel TC, when the Earth was at MO; and that Portion of the Earth's Surface which is included within the Circle FG, will be wholly in the Dark, and that within the opposite one HI, will be wholly in the Light; and lesser Portions of the Northern Parallels will be in the Light than in the Dark, and greater Portions of the Southern. So that the Inhabitants of the Northern Hemisphere will then have their Days at the shortest, and those of the Southern at the longest. Those who live within the Circle FG will have no Day, and those within HI no Night. Tho' to those who live under the Equator, the Days and Nights are in this, as in all other Situations of the Earth, of an equal Length, the Equator

G being

(d) In this Situation of the Earth, the Sun enters the first Degree of *Libra*, and makes *Autumn* to us, and *Spring* to those on the other side the Equator.

being every where cut into two equal Parts by the Boundary of Light and Darkneſs *(e)*. From

(e) When the Earth is in that Part of its Orbit where theſe Phænomena happen, the Sun is then entering the firſt Degree of *Capricorn*, and brings on Winter to us, and Summer to the Inhabitants of the other Hemiſphere.

There is another Way in which theſe *Phænomena* may be illuſtrated; and that is as follows. Let S (Fig 7.) repreſent the Sun, and the Circle ABDD the Earth's annual Courſe round it; EFGH the Earth, P the North Pole, RT the Arctic Circle, IKLE the Tropic of *Cancer*, FNH the Northern half of the Equator; the ſhaded Part the obſcure Hemiſphere, the other the illuminated one: And let X be the Place of the Sign *Aries* in the Heavens, Y that of *Cancer*, M that of *Libra*, and Q that of *Capricorn*.

When the Earth is in that Part of its Orbit that reſpects M or *Libra*, the Sun appears in the oppoſite one, *viz.* at X, or in *Aries*; and the Circle GPE, which ſeparates the light Hemiſphere from the dark one, paſſes through the Poles of the Earth, and therefore cuts each Parallel of the Equator into two equal Parts, and thereby renders the Days and Nights every where equal.

But while the Earth in its annual Courſe moves from M towards Q, which we will ſuppoſe to be a quarter of a Circle diſtant from it, its *Axis* being directed as before, the Pole P will gradually advance into the illuminated Hemiſphere, and that Portion of the Parallels that are therein will continually encreaſe until the Earth is arrived there: in which Poſition the Boundary of Light and Darkneſs paſſes through FRH, the Pole P, and all the Space within the Arctic Circle falling wholly within the illuminated Hemiſphere; and thoſe Portions of the ſeveral Parallels that are in the Light, will be longer than thoſe which are in the Dark, all which is evident from the Inſpection of the Figure. So that in the Northern Hemiſphere of the Earth, the Days will be at longeſt, and the Nights at the ſhorteſt.

Again, as the Earth is moving from Q towards X, the Pole P begins to incline towards the Line that diſtinguiſhes Light from Darkneſs, for the like Reaſon that before it receded from it; and therefore thoſe Portions of the Parallels that are in

From this apparent Shifting of the Sun from one Side to the other, those Parallels of the Equator, mentioned in the foregoing Explication, have been distinguished from the rest by particular Names, as the two *Tropics*, and the two *Polar* Circles. The *Tropics* are TC and RN; the former is called the *Tropic of Cancer*; because when the Sun seems to describe that Circle, it then appears to be in the first Degree of that Part of the Ecliptic which is distinguished by the Sign *Cancer*: the latter is called the *Tropic of Capricorn*; because at that time of the Year when it appears to describe that Circle, it is entering the first Degree of the Sign *Capricorn* The two *Polar* Circles are FG and HI; the former the *Arctic*, the latter the *Antarctic*. These are just as far distant from their respective Poles, as the Tropics are from the Equator; *viz.* 23 Degrees and an half; this being a necessary Consequence of the Sun's illuminating but one half of the Globe of the Earth at a Time.

G 2 The

in the Light gradually lessen till the Earth arrives there; at which Time that Line will pass through it again, and so make the Days and Nights every where equal.

But afterwards the Pole will begin to fall into the obscure Hemisphere, and so recede gradually from the Light, until the Earth comes to Y; in which Situation not only the Pole, but the whole Arctic Circle will be involved in Darkness; and those Portions of the several Parallels that are in the Light, will be shorter than those which are in the Dark; so that the Days will be at the shortest, and the Nights at the longest.

The Days and Nights being of an equal Length, when the Sun appears to be in those Points where the Equator cuts the *Ecliptic*, (*viz.* the first Degree of *Aries* and *Libra*) they are from thence called the *Equinoctial Points*, and the Times of the Year answering thereto the *Equinoxes*; one the *Vernal*, the other the *Autumnal*; and of the *Equinoctial Points*, that where the *Ecliptic* crosses the Equator in its Ascent towards the North Pole, is called the *vernal Equinoctial Point*; that where it crosses it in its Descent towards the South Pole, the *Autumnal*; and a Circle imagined to pass through them and the Poles of the Earth, is called the *Equinoctial Colure*.

And because when the Sun seems to describe either of the *Tropic* Circles (that is, when it enters the first Degree of *Cancer* or *Capricorn)* it being about to revert to the Equator, does as it were seem to stand still, neither receding from, nor approaching towards it, these Points are called the *Solstitial Points*; and the Times of the Year answering thereto are termed the two *Solstices*; the one, the *Summer*, the other the *Winter*. And a Circle conceived to pass through the Poles of the Earth and these Points, is called the *Solstitial Colure.* And lastly, these four Points of the Heavens being those in which the Sun appears to be when the four Seasons of the Year (*viz.* the *Spring*, *Summer*, *Autumn*, and *Winter)*

ter) begin, are diſtinguiſhed by the Names of *Cardinal Points*.

If a right Line perpendicular to the Plane of the *Ecliptic* be ſuppoſed to paſs through the Center of the Sun, and to be extended both ways to the Heavens, this Line is called the *Axis of the Ecliptic*; and the two Extremities thereof, the *Poles of the Ecliptic*. But this Line is oft conceived to paſs through the Center of the Earth inſtead of the Sun; for they are ſo near to each other with reſpect to the Diſtance of the Heavens, that the Difference is not material.

The Sun being not placed in the Center of thoſe *Ellipſes* which the Planets deſcribe in moving round it, but in one of their *Foci*, as was obſerved above, it follows, that they muſt be farther from the Sun in one Part of their Orbits than in another. And not only ſo, but they muſt alſo move faſter in one Part of their Orbits than in another; becauſe, as has been already demonſtrated in Part I. Chap. VIII. page 46. whatever Orbit a Body deſcribes, if it be retained therein by any centripetal Force whatever, it will deſcribe equal *Areas* in equal Times; that is, it will move faſter in Proportion as it approaches nearer to the Body it revolves about, and ſlower the farther it is from it. And accordingly it is obſerved by Aſtronomers, that the nearer a Planet

Planet comes to the Sun in its Orbit, the swifter it moves.

The Consequence of this is, that the Sun does not appear to move uniformly along the *Ecliptic*, but in some Days to describe a greater Portion of it than in others. In Winter it moves the fastest, in Summer the slowest; insomuch that it takes up eight Days more in passing from the *Vernal* to the *Autumnal Equinox*, than in its passage from the *Autumnal* to the *Vernal*; altho' in either of these Intervals of Time it moves through just half of the *Ecliptic (f)*.

When an *Ellipse* is considered as the Orbit of a Planet, the longer *Axis* thereof is called the *Linea Apsidum*; and if the *Focus* F be the Place of the Sun *(g)*, the Point A is called the *Summa Apsis (h)*, or the *Aphelion*; and the opposite

(f) Hence it is, that in one Part of the Year the Sun is farther towards the East, and in another farther towards the West, than it would seem to be, was its apparent Motion in the *Ecliptic* uniform; so that it passes the Meridian sometimes sooner, sometimes later, than the Time at which a Clock that moves uniformly would indicate the Hour of *Twelve*. This is one Reason that Sun-dials are observed to go sometimes slower, and sometimes faster. When the Sun appears too far toward the East, they are too slow; when it appears too much to the Westward, they are too quick. But see this farther explained, Chap. XIV.

(g) See Fig. 2.

(h) This is the general Name to distinguish that Point of an Orbit, in which a revolving Planet is at the greatest Distance from the Body it revolves about. With respect to a Body revolving about the Sun, it is called the *Aphelion*; but with respect to the Moon considered as revolving about the Earth, it is

opposite one B, the *Ima Apsis*, or the *Perihelion*. The Distance of a Planet from its *Aphelion* is called its *Anomaly*; and this is of two Kinds, *mean* and *real*; the latter is its real Distance from its *Aphelion*; the former is that Distance at which it would be from it, supposing its Motion in its Orbit to be uniform; and the Difference between these two, Astronomers call the *Equation*, or *Prosthaphæresis* of the Planet.

The *Ellipses* the Planets describe do themselves turn round the Sun, though with a very slow Pace, their *Aphelia* respecting sometimes one Part of the Heavens, and sometimes another. In this Age the *Aphelion* of the Earth's Orbit points towards the former Part of the Sign *Capricorn*. This is the reason that the Sun is farther from us in Summer than in Winter; for the Sun appearing in *Cancer* in the former Season, the Earth is then in *Capricorn*, and therefore in its *Aphelion*, that is, its greatest Distance from the Sun.

The Sun being farther removed from us in Summer than in Winter, it may be asked how it comes to pass, that since the Sun is the Fountain of Heat, as well as Light, our Winters are so much colder than our Summers? In Answer to this it is to be considered, that the Rays

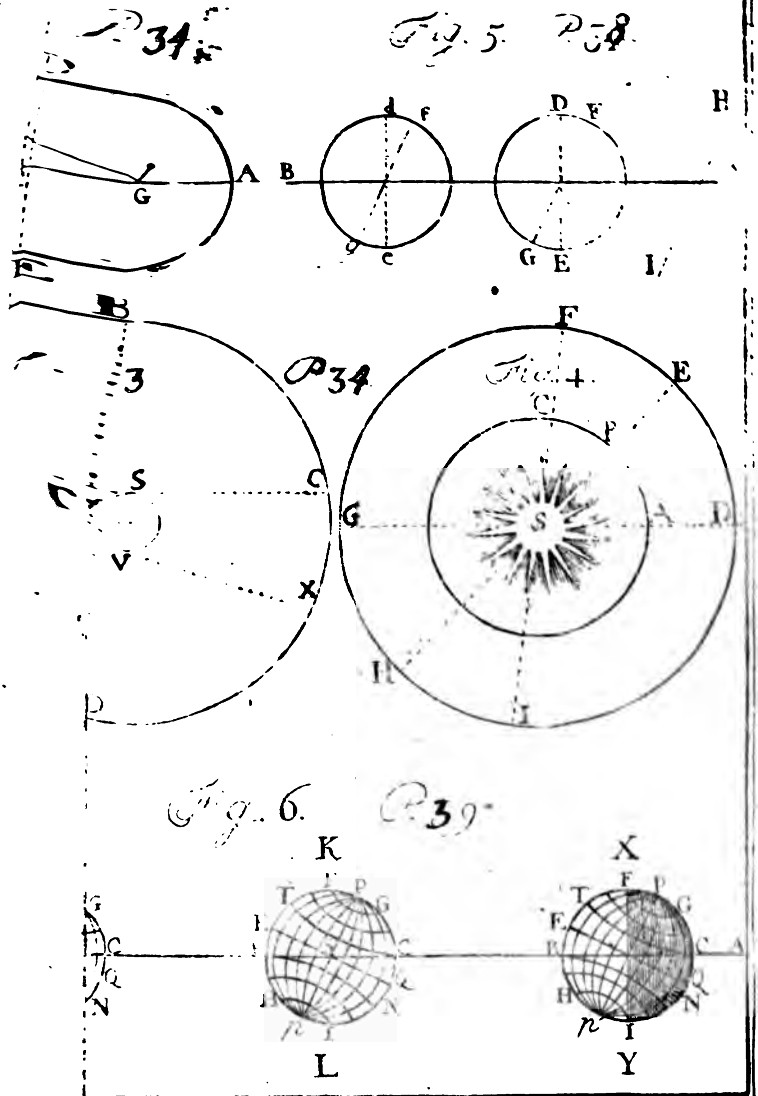
is called the *Apogeon*. And the *Ima Apsis* is the nearest Point, which with respect to a Body moving about the Sun is called the *Perihelion*, but with respect to the Moon the *Perigeon*.

Rays of the Sun fall with much less Obliquity upon the Surface of the Earth on our side the Equator in the Summer, than in the Winter; and therefore they not only act more forcibly upon it, but a greater Quantity of them fall upon a given Place. Again; in the Winter the Sun being much lower in the Heavens when at its Meridian Height, than in Summer, its Rays pass through a longer Portion of the Earth's Atmosphere, by which great Part are intercepted, and some by various Refractions and Reflections turned another Way *(i)*. And lastly, in Summer the Sun continues with us sixteen Hours, and is absent but eight; whereas in Winter it is with us but eight Hours, and is absent sixteen. All which things conspire to make a considerable Alteration with respect to Heat and Cold.

If this be so, why is not the Weather hotter when the Sun is in the Tropic of *Cancer*, its Rays then falling with the least Obliquity, and passing the shortest Way through the Atmosphere, and the Days being then at the longest, than it is about a Month afterwards, when the Sun is in the beginning of the next Sign? In answer to this it is to be remembred, that Bodies are not always the hottest at that very Instant the greatest Degree of Heat is applied to them: they require Time to

(i) See the State of the Atmosphere, Part II. Chap. III.

PART IV. PLATE II. Pag. 50.



to heat as well as to cool; it is the Length of Time therefore that the Heat is applied to them, as well as the Degree of it, that determines the Quantity of Heat communicated to them.

For the like Reaſon we commonly find it warmer about one or two of the Clock in the Afternoon, than at twelve, when the Sun is in its Meridian Altitude, and its Rays fall thickeſt and moſt forcibly upon the Earth.

The Sun appearing by means of the annual Motion of the Earth (as was illuſtrated in the beginning of this Chapter) to move from Weſt to Eaſt in the Heavens, it happens, that if any Star riſes or ſets along with the Sun at any time, that Star will after a few Days riſe or ſet before it; becauſe the Sun's apparent Place in the Heavens will be removed to the Eaſtward of the Star. So that thoſe Stars which at one time of the Year ſet with the Sun, and therefore never appear at all, ſhall at another time of the Year riſe when the Sun ſets, and ſhine all Night. And as any one Star ſhifts its Place with reſpect to the Sun, and in Conſequence of that with reſpect to the Hour of the Night, ſo all the reſt do; and from hence it is, that all thoſe Stars which at one time of the Year appear on any one ſide of the Pole-Star in the Evening, ſhall half a Year after, at the ſame Hour, appear on the contrary Side thereof.

CHAP. V.

Of the Earth's annual Parallax, *the* Nutation *of the* Poles, *and the* Precession *of the* Equinoctial Points.

SInce the *Axis* of the Earth is always parallel to itself, it is in different Parts of the Year directed towards different Parts of the Heavens; and therefore to a Spectator on the Surface of the Earth, who cannot be sensible of his Change of Place, the apparent Places of the fixed Stars are different, when viewed from opposite Points of the Earth's Orbit. Thus let S (Fig. 8.) represent the Place of the Sun, AB the Orbit of the Earth, and when the Earth is at A, let its *Axis* be directed towards the Point C in the Heavens; then when the Earth is arrived at B, the opposite Point of its Orbit, its *Axis* being parallel to its former Position, will be directed to a different Point in the Heavens, *viz.* to D, as far distant from the former as the Point A is from B, that is, a Space equal to the Diameter of the Earth's Orbit: And therefore to a Spectator on the Surface of the Earth, the Places of the fixed Stars appear removed as far as from C to D, *viz.* by the Quantity of the Angle, CBD; or, because the Lines AC and BD are parallel, by the Angle BCA, which is that under which the Orbit of the Earth

Earth would appear from a fixed Star at C. This Angle is by Aſtronomers called the *Parallax of the Orbis magnus*, or the *Earth's annual Parallax.*

If this Angle could be accurately taken, the Diſtance of a fixed Star at C, with reſpect to that of the Sun at S, might be found by the Rules of *Trigonometry (a).* But by reaſon of the great Diſtance of the fixed Stars, that Angle is ſo exceeding ſmall that it cannot be taken with any tolerable Accuracy, even with the beſt Inſtruments. It has appeared to ſome to contain about 47 Seconds of a Minute, according to which the Diſtance of the Stars muſt be five hundred thouſand Millions of Miles from us; but to others, particularly to *Flamſtead*, who has taken great Pains to obſerve it accurately, it has appeared to contain but 42 Seconds, according to which the Diſtance of the Stars muſt be much greater.

Altho' we hitherto have looked upon the *Axis* of the Earth as being always parallel to itſelf, yet it is not ſo, but is ſubject to a twofold Motion; to the one is owing the Nu-

(a) For the Angle DBC being taken by Obſervation, its Equal ACB is had; and the Angle BAC, or SAC, ſubtends an Arch of the Heavens that meaſures the Diſtance between the Place of the Sun and the ſaid Point, and may therefore eaſily be obtained: and conſequently, by the Rules of *Trigonometry*, the Proportion which the Line AC or the Diſtance of the Star from the Earth, bears to the Line AB, which is double the Sun's Diſtance, may be found.

tation of the Poles, to the other the Preceſſion of the Equinoctial Points, which are now to be explained. The firſt is a Deviation of the Earth's *Axis* from its Paralleliſm with itſelf twice in the Year, and a Returning to it as oft: For inſtance, the Inclination of its *Axis* to the Plane of the *Ecliptic* increaſes while the Earth is moving from the Solſtitial to the Equinoctial, and diminiſhes as much in its Paſſage from the Equinoctial to the Solſtitial Points. So that the Inclination of the *Axis* of the Earth to the Ecliptic is, notwithſtanding this, at oppoſite Points of its Orbit always the ſame. This Nutation therefore does not in the leaſt interfere with what was ſaid with regard to the Earth's annual Parallax, in which though we ſuppoſed that *Axis* always parallel to itſelf, yet we conſider'd it only in oppoſite Points of its Orbit. This is called the Nutation of the Poles *(b)*. The other indeed claſhes a little with our former Suppoſition; but taking a ſingle Revolution of the Earth by itſelf, as we have hitherto done, it is ſo ſmall as to be quite inſenſible, tho' not ſo when we take a many Revolutions together. Accordingly, by comparing ſeveral Years together, Aſtronomers have obſerved that the *Axis* of the Earth deviates

(b) As the Inclination of the *Axis* of the Earth is thus twice augmented and twice diminiſhed, while the Earth revolves about the Sun, in like manner it is twice augmented, and twice diminiſhed in each Revolution of the Moon about that.

viates a little from the Direction it formerly had; notwithſtanding which Deviation, its Inclination to the *Ecliptic* remains the ſame. To illuſtrate the manner in which this Alteration in its Direction is made, let S (Fig. 9.) repreſent the Sun, the Circle XEY the Earth, AB the Plane of the *Ecliptic*, EL the *Axis* of it, which we will now ſuppoſe to paſs thro' the Center of the Earth, and let P Q denote the *Axis* of the Earth. On the Surface of the Earth, and round the Points E and L, let us conceive the two ſmall Circles PTX and QVY to be deſcribed. Then if at any Time the Poſition of the Earth's Axis is in the Line PQ, as we have ſuppoſed, after ſeveral Revolutions of the Earth about the Sun it ſhall be found in the Situation CD; and after ſeveral more Revolutions it ſhall be removed into the Situation TV, *&c.* the one Pole of the Earth deſcribing the Circle PCTX, and the other QDVY: ſo that notwithſtanding this Change in the Direction of the *Axis* of the Earth, its Inclination to the Plane of the *Ecliptic* remains the ſame, the Poles of the Earth being every where at the ſame Diſtance from E and L, the Poles of the *Ecliptic*. But this Motion of the Poles is ſo very ſlow, that they do not perform their Revolution in leſs than 25,920 Years, which is at the Rate of one Degree in 72 Years. It is performed in a Direction contrary to that of

of the Order of the Signs, or from Eaſt to Weſt. Which is termed by Aſtronomers, a Moving in *Antecedentia*; whereas when a Body moves according to the Order of the Signs, its Motion is ſaid to be in *Conſequentia*.

From hence it follows, that the Solſtitial Colure, which paſſes both through the Poles of the *Ecliptic* and of the Earth, muſt neceſſarily turn round the *Axis* of the *Ecliptic* in the ſame Direction, or contrary to the Order of the Signs; and with that the Equinoctial Colure alſo, becauſe theſe croſs one another at right Angles in the Poles of the Earth. Whatever Star therefore the Equinoctial Points are directed towards at any particular Time, after 72 Years they will not be directed to the ſame, but to ſome other Star or Point of the Heavens, ſituated one Degree towards the Weſt. This Motion of the Equinoctial Points is called their *Preceſſion*. And from hence the Stars ſeem to move towards the Eaſt, and thereby to have their Longitude (which is always reckon'd upon the *Ecliptic* from the vernal Equinoctial Point) increaſed. The Conſtellations therefore ſeem all to have deſerted the Places allotted to them by the firſt Aſtronomers. For Inſtance, the beginning of the Sign *Aries*, which was near the vernal Equinoctial Point, and gave Name to that Point of the *Ecliptic*, is now advanced about an whole Degree forwards: So that *Aries* is now where

Taurus

Taurus used to be; *Taurus* where *Gemini*, &c. But to avoid Confusion, Astronomers have thought fit to let the several Portions of the *Ecliptic*, where these Constellations were at first observed to be, retain their old Names *(c)*; so that the vernal Equinoctial Point is still reckoned the first *Degree of Aries* *(d)*.

(c) Those Portions of the Ecliptic, where the Constellations were at first, are called *Anastra*, those where they now are, are distinguished by the Titles of *Stellata*.

(d) The ancient Astronomers observing this Motion of the fixed Stars, and not knowing how to account for it, concluded that they really had such a Motion, and called the Time of an entire Revolution of them, *Annus Magnus*, or the great Year. And imagined that when that great Year was ended, and the Stars reinstated in their proper Places, all Things would begin to be done over again in the World, not only in the same Order and Manner, but by the same Persons they were done before.

The Causes of this Precession, as also of the Nutations above mentioned, will be accounted for when we treat of the Causes of the Motions of the heavenly Bodies.

CHAP.

CHAP. VI.

Of the Phænomena *which arise from the* Motion *of the* Earth *and of the Inferior Planets,* Mercury *and* Venus *conjointly.*

SInce the Orbits of *Mercury* and *Venus* are included within that of the Earth, it is evident neither of them can come to an Opposition to the Sun; that is, appear in the opposite Part of the Heavens with respect to it, but must always accompany it in its apparent Motion through the Ecliptic, just as the Satellites of the primary Planets do in reality accompany them. But whereas they perform their respective Revolutions about the Sun in Periods different from that of the Earth, it is obvious they must be sometimes on one side of the Sun, and sometimes on the other with respect to us; sometimes be between the Sun and us, and sometimes directly beyond it. When they are between the Sun and us, they are said to be in their *inferior Conjunction*; when beyond it, in their *superior Conjunction*: and when they appear to be removed the farthest from the Sun, on either Side, they are then said to be in their greatest *Elongation*.

To

To illustrate this, let S (Fig. 10.) represent the Sun, ABCD the Orbit of one of the inferior Planets, suppose *Venus*; E the Place of the Earth in its Orbit; FG and HI a Portion of the Heavens. Then will A be the Place of its *inferior*, C that of its *superior Conjunction*; in both which Cases it will appear in the Heavens to be at K, which is the apparent Place of the Sun. When it is at B or D, where a Line joining its Center and that of the Earth touches its Orbit, it then appears at I or H, and is said to be in its greatest Elongation, which is about 48 Degrees (*a*). But this is liable to some Alteration; because as its Orbit is an *Ellipse*, it is sometimes nearer the Sun, and sometimes farther from it, in its greatest Elongation.

From hence we see the Reason why the Planet *Venus* is sometimes said to be our Morning Star, and sometimes our Evening Star: For after it has passed its *inferior Conjunction* with the Sun, and is ascending towards its *superior* one, it is all that Time on the *Western* side of the Sun, and therefore necessarily rises

I every

(*a*) Hence the Distance of *Venus* from the Sun, with respect to that of the Earth from the same, is found. For if in the Triangle EBS, we take the Angle at E, when that at B is a right one, the Proportion between the opposite Sides is had from the known Rules of *Trigonometry*. The Reason why the Angle at B is not always a right one, is, because the Orbit of the Planet is not a Circle, but an *Ellipse*.

every Morning before it, and is then our *Morning Star*. On the other hand, while it deſcends from its *ſuperior* to its *inferior Conjunction*, it is to the *Eaſtward* of the Sun, and therefore riſes and ſets after it; and appears to us only in the Evening after the Sun is down.

Farther; altho' the Motion of the inferior Planets in their Orbits is direct, or according to the Order of the Signs, yet they frequently appear to be *retrograde*, or to move the contrary Way; and ſometimes to ſtand ſtill. When they are near their *inferior Conjunction* with the Sun, they ſeem to move one Way, and when near their *ſuperior Conjunction*, they appear to go another. And at that Time when their direct or progreſſive Motion is changing into a regreſſive one, or the contrary, they ſeem to ſtand ſtill. All which may be illuſtrated in the following Manner.

Let ABCD (Fig. 10.) repreſent the Orbit of the Planet *Venus*, S the Place of the Sun, E the Earth, FG a Portion of its Orbit, and HI the Sphere of the fixed Stars, as before; and becauſe *Venus* moves faſter than the Earth, let us ſuppoſe the latter to ſtand ſtill, and the former to move with the Differences of their Velocities; Then while *Venus* deſcribes that Portion of its Orbit that is repreſented by DAB, it will ſeem from the Earth to move from H to I in the Heavens, which is contrary to its real Motion, and to the Order of the Signs;

and

and in moving over the remaining Part of its Orbit, it will ſeem to paſs from I to H. And was the Earth really to ſtand ſtill, as we have ſuppoſed, it would be ſtationary, or ſeem to have no Motion either Way, at the Points B and D.

But as the Earth does not ſtand ſtill, but continues to move on in its Orbit, *Venus* will not ſeem ſtationary exactly at the Points B and D; but during its Motion through a certain Part of its Orbit, as LM, where ſuppoſing the Lines EL and OM parallel to each other, their Orbits are ſo inclined to each other, that the Space LM ſhall be to the Space EO as the Velocity with which *Venus* moves is to that with which the Earth moves; for it is evident, that in that caſe the Earth will paſs through EO while *Venus* paſſes through LM, and therefore it will appear all the Time in the ſame Part of the Heavens, becauſe the Diſtance between thoſe parallel Lines is nothing, when compared to that of the Heavens.

There is alſo another Particular to be taken Notice of with regard to the apparent Motion of the Inferior Planets, *viz.* that they do not ſeem to deſcribe the *Ecliptic* in the Heavens as the Sun does, but are obſerved to be ſometimes above and ſometimes below it. The Reaſon of this is, that their Orbits are inclined to the Plane of the Earth's, having one Half above it and the other below it, on which

account they interſect the Plane of the *Ecliptic* in a Line that paſſes through the Center of the Sun (the Center of the Sun, as was obſerved above, being in the Planes of each Orbit.) This Line is called the *Line of the Nodes*, and the two Points of the *Ecliptic*, through which it paſſes when produced to the Heavens, are called the *Nodes*. Theſe Planets therefore never appear in the *Ecliptic* except when they are in their *Nodes*, and in all other Parts of their Orbits ſeem to be more or leſs diſtant from it, according as they are ſituated with reſpect to them and the Earth. But if we ſuppoſe them to be ſeen from the Sun, their greateſt Diſtance from the *Ecliptic* will be when they are fartheſt removed from their *Nodes*.

Their Diſtances from the *Ecliptic*, as it would appear if ſeen from the Center of the Earth, is called their *Geocentric Latitude*; as it would appear from the Center of the Sun, is called their *Heliocentric Latitude*.

There is a remarkable *Phænomenon* relating to the inferior Planets, which was not known to Aſtronomers, till diſcovered by the Teleſcope; and that is, that in their different Situations with reſpect to the Earth, they aſſume different *Phaſes*, or Appearances, like thoſe of the Moon. Which may be explained in the following Manner.

Let

Let A, B, C, D, (Fig. 11.) represent the Situation of one of the inferior Planets (*Venus* suppose) in several Parts of its Orbit, and let S be the Place of the Sun, and I that of the Earth in its Orbit KL: and if from the Center of the Sun to that of *Venus* in its several Stations we draw the Lines SA, SB, *&c.* respectively; and at right Angles to these the Lines MN, MN, *&c.* these last Lines will represent Circles upon the Surface of the Planet, separating the enlightened from the obscure *Hemisphere.* And if from the Earth to the Planet we conceive the Lines IA, IB, *&c.* to be drawn, and at right Angles to these the Lines OP, OP, *&c.* respectively, these latter will represent Circles separating that Part of the Planet which is turned towards us from that which is not. These we may call *Circles of Vision.* Now it is evident, that when *Venus* is in its inferior Conjunction at A, the Circles MN and OP coincide, and no Part of the Planet besides its unilluminated Hemisphere is turned towards us; in which Case it is invisible, unless it happens at that Time to be in or near the Plane of the *Ecliptic*, which if it be, it appears as a Spot upon the Surface of the Sun. When it has advanced in its Orbit as far as B, there is a small Part of the illuminated Hemisphere included between the Lines OB and BM, on that side the Circle of Vision which is next the

the Earth; in which caſe, ſince by reaſon of its great Diſtance from us we cannot diſtinguiſh the *Convexity* of its Surface, it appears as a flat round Surface, a ſmall Portion of which is illuminated, the reſt being dark; and ſo appears *horned*, like the new Moon *(b)*. When it has arrived at C, then is half its illuminated Hemiſphere turned towards us, and it appears as a round Surface, half of which is enlightened, and half not. When it has got to D, then is the greateſt Part of its illuminated Hemiſphere turned towards us, and it appears as a round plain Surface almoſt wholly enlighten'd, or what the Aſtronomers call *gibbous*. When it is in its ſuperior Conjunction

(b) The Reaſon why *Venus* or any other Planet (the Moon ſuppoſe) appears *horned*, when a ſmall Portion only of her illuminated Hemiſphere is turned towards the Eye of the Spectator, will perhaps in ſome Meaſure appear from the Conſideration of the 12th Figure. In which let ABCD repreſent the Planet *Venus*, or any other Body of a globular Form; which that it might the better do, I have drawn ſome of thoſe Circles which are uſually found on Globes deſigned to repreſent the Earth. Now if half this Globe be illuminated, and half be in the dark, and if only ſo much of the illuminated Part as is included within the Space ABCE be turned towards the Eye, the remaining Part AECD being obſcure, it is plain, that the illuminated Part ABCE, when the whole Surface ABCD, by reaſon of the great Diſtance of the Globe, appears *flat*, muſt ſeem to be *horned*. And again, if the Space ABCE was a Portion of the dark Hemiſphere, and AECD of the illuminated, 'tis certain that when both are conſidered as making up one flat Surface, the illuminated Part muſt appear convex, or *gibbous*, on the Side AEC, which is contiguous to the unilluminated Hemiſphere ABCE.

junction at E, the Circles MN and OP coincide again, and the whole enlightened Hemisphere is turned towards the Earth; in which Situation the whole illuminated Face of the Planet is turned towards the Earth, and, if not intercepted by the Body of the Sun, appears like the Moon at the *Full*. After which it puts on the same Appearances again, but in an inverted Order, till it arrives at A, where it is *invisible*, as before *(c)*.

What has been here illustrated with regard to *Venus*, is to be understood in Reference to the like *Phænomena* of *Mercury*; only with this Difference, that the Orbit of *Mercury* being less than that of *Venus*, its greatest *Elongation* is not so great as that of *Venus*, not exceeding about 33 Degrees. Upon which Account this Planet is rarely to be seen with the naked Eye. And as *Mercury* revolves round the Sun in less Time than *Venus* does, its *direct Motions*, *Stations* and *Retrogradations*, as also its Conjunctions with the Sun, succeed one another quicker than those of that other Planet.

(c) Before the Invention of the Telescope, when *Copernicus* revived the old *Pythagorean Hypothesis*, it was objected to him, that, according to that Scheme, *Venus* ought to assume the like Phases with the Moon. To which he replied, that possibly some Means would be found out, whereby succeeding Astronomers would discover that it did so. The first that observed it was *Gallilæo*, who thereby greatly confirmed the Truth of the *Copernican* System.

CHAP.

CHAP. VII.

Of the Phænomena *which are owing to the* Motion *of the* Earth, *and that of the superior Planets* Mars, Jupiter, *and* Saturn *conjointly*

THE ſuperior Planets do not always ſeem to accompany the Sun, as we have ſhewn in the foregoing Chapter that the inferior ones do; but are ſometimes in Conjunction with it, ſometimes in Oppoſition: and are liable, as was obſerved before of the Planet *Mars*, to be ſeen at any Diſtance from it.

To illuſtrate this, let S (Fig. 13.) repreſent the Place of the Sun, ACE the Orbit of the Earth, NRM that of one of the ſuperior Planets (ſuppoſe *Mars*) and let OP be a Portion of the *Zodiac*. Now ſince theſe Planets perform their Revolutions about the Sun in different Times from that wherein the Earth goes round its Orbit, it is poſſible, that when the Earth is at E, *Mars* may be at M, or it may be at N; in the former Situation, it is in Conjunction with the Sun, in the latter it is in Oppoſition: or it may be at R, or at T, or in

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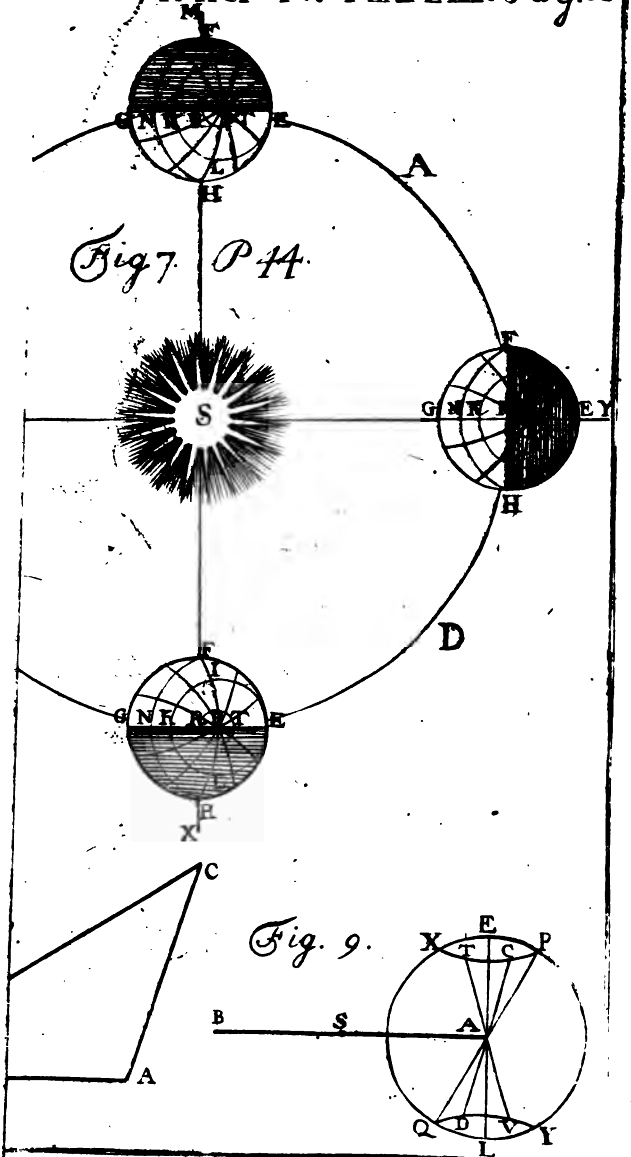


in any Situation with reſpect to the Sun and the Earth whatever.

The ſuperior Planets have this in common with the inferior ones, *viz.* that according as the Earth is ſituated with reſpect to them, they ſometimes ſeem to move forwards according to the Order of the Signs, ſometimes contrary to that Order; and between the Times of their being thus *direct* and *retrograde*, they ſeem to ſtand ſtill, as ſhall be made evident by the following Illuſtration.

Things remaining as before in the thirteenth Figure, ſince the Motion of the Earth is quicker than that of *Mars*, let it be ſuppoſed that *Mars* ſtands ſtill at N, and that the Earth is moving with the Difference of their Motions from B to C: It is obvious, that in this Caſe *Mars* will ſeem to paſs from O to P, which is contrary to the Order of the Signs; but while the Earth is paſſing over the other Part of its Orbit, *viz.* CAB, the Planet *Mars* would ſeem to move back again from P to O, even though it had no Motion of its own. Since then it is moving that Way all the Time, it neceſſarily ſeems to move according to the Order of the Signs; tho' with this Difference, that it ſeems to move faſter than it would do were it not for the Motion of the Earth in the mean time. Analogous to what was obſerved of the inferior Planets (ſince *Mars* does not ſtand ſtill, as we

have ſuppoſed) it will not appear to be *Stationary* when the Earth is at B and C, but during the Motion of the Earth through a certain Part of its Orbit, as LH, which, ſuppoſing the Lines LN and HI parallel, is to the Portion of *Mars*'s Orbit NI (through which that Planet is moving at the ſame Time) as the Celerity with which the Earth moves is to that of *Mars*.

What was ſaid in reſpect of the inferior Planets, that they do not always appear in the *Ecliptic*, holds alſo with regard to the ſuperior ones. For their Orbits are alſo inclined to that of the Earth, one Side of them being above, and the other below it; ſo that thoſe have alſo their *Heliocentric* and *Geocentric* Latitudes as well as the other, and are never to be ſeen in the *Ecliptic*, but when they are in their *Nodes*.

What has been illuſtrated with regard to *Mars*, is applicable to the other two ſuperior Planets, only with this Difference; that the Motion of *Saturn* is more frequently changed from progreſſive to regreſſive, than that of *Jupiter*; as alſo its Oppoſitions and Conjunctions with the Sun are more frequent. The Reaſon of this is, becauſe as *Saturn* moves much ſlower than *Jupiter*, the former is more frequently overtaken and paſs'd-by by the Earth than the latter. And as theſe *Phænomena* happen ofter in *Saturn* than in *Jupiter*, ſo they

do

do more frequently in *Jupiter* than in *Mars* for the ſame Reaſon.

So ſmall is the Diſtance of the Earth, with reſpect to that of *Jupiter* and *Saturn*, from the Sun, that in whatever Part of their Orbits they be ſituated with reſpect to the Earth, that Side of them, as to Senſe, which is turned towards the Sun, is always towards the Earth; on which Account they always appear to us as ſhining with a full Orb. But it is not ſo with reſpect to *Mars*; for ſince that Planet is not much farther removed from the Sun than the Earth, it is obvious that that Side of it which is turned towards the Sun will not always be towards the Earth. For inſtance; though when *Mars* is at N, and the Earth at A or E, its illuminated Hemiſphere is directly towards the Earth, yet it is turned away from it conſiderably when the Earth is at B or C; at which time therefore it appears to want ſomething of its uſual Rotundity, or to be *Gibbous* like the Moon, a little before or ſoon after the *Full.*

As the inferior Planets *Mercury* and *Venus* have their reſpective Elongations from the Sun when ſeen from the Earth; ſo the Earth, if we imagine it to be viewed from the ſuperior Planets, hath alſo its Elongation from the Sun. But this being a Phænomenon, which has not happened to our Aſtronomers to ſee, they give it a different Name, and call it when

at the greateſt, from a Phænomenon in the ſuperior Planets depending upon it, *the Parallax of the Semidiameter of the Earth's annual Orbit in thoſe Planets:* by which is meant, the Difference between the Place in the Heavens the Planet would appear in if ſeen from the Center of the Sun, and that in which it would appear if ſeen from the Center of the Earth, when at its greateſt Elongation from the Sun with regard to that Planet. To explain this, let N be the Place of the Planet, and B that of the Earth in its greateſt Elongation from the Sun, as ſeen from that Planet. Then will the aparent Place of the Planet as ſeen from the Sun (that is, its true or *Heliocentric* Place) be Q, but its *Geocentric*, or apparent Place as ſeen from the Earth, will be O. Now becauſe the Arch QO ſubtends an Angle (*viz.* QNO) at the Planet which is equal to SNB, *viz.* that which SB, the Semidiameter of the Earth's annual Orbit ſubtends, the abovementioned Difference in the apparent Situation of the Planet is called *the Parallax of the Semidiameter of the Earth's annual Orbit in that Planet* (*a*).

This

(*a*) Hence, as was obſerved of the Inferior Planets, the Diſtances of the ſuperior ones from the Sun with reſpect to that of the Earth from thence, may be found. For in the Triangle SBN, all the Angles being known, the Relation between the two Side BS and SN is had by the known Rules of *Trigonometry*. The Angle SBN is had from immediate Obſervation,

This Angle in *Mars* is but 42 Degrees, in *Jupiter* not above 11: In *Saturn* it is but the fourth part of the greatest Elongation of *Mercury* seen from the Earth, *viz.* about 6 Degrees. So that since *Mercury* by reason of its Nearness to the Sun is rarely seen by us, in all Probability the Astronomers in *Saturn* (if there be any) do not know that there is such a Body in the Universe as our Earth.

servation, as being subtended by an Arch in the Heavens between the apparent Place of the Planet and that of the Sun. The Angle at N is equal to the Difference between the real and apparent Place of the Planet; the first of which is found by Computation, the latter by Calculation. In *Jupiter*, the best Way of finding that Angle is from his *Satellites*, the Method of doing which shall be shewn when we have considered the Phænomena of those Planets.

CHAP. VIII.

Of the Phænomena *of the Moon.*

HAving now finished what relates to the *Phænomena* of the *Primary* Planets, we proceed to consider those which are owing to the Motion of the *Secondary* ones; and first of the Moon, which is the constant and only Attendant of the Earth, and is always so near it, that if the Earth and the Moon were viewed together from the Sun, the Moon

Moon at ſuch Time as it would appear the fartheſt from the Earth would not ſeem to be at a greater Diſtance from it than what is equal to one third Part of the Breadth of the Sun's Body, as it appears to us.

Since the Moon is an opake ſpherical Body, and revolves about us in ſuch Manner as to paſs between the Sun and the Earth, it is abſolutely neceſſary it ſhould put on different *Appearances*, according to its various Poſition with reſpect to the Sun and the Earth; its illuminated Hemiſphere being conſtantly turned towards the Sun the Fountain of Light, and for that reaſon ſometimes a greater and ſometimes a leſs Portion of it turned towards us. To ſhew this, little need be ſaid, after ſo full an Explication of the ſeveral Phaſes of *Venus*, as was delivered in Chap. VI.

When the Moon is between the Earth and the Sun, that is, in Conjunction with the Sun, its whole illuminated Hemiſphere is turned from us, in which Station it is ſaid to be *New*; afterwards when by its Motion in its Orbit it emerges out of the Sun's Rays, a ſmall Portion of that Hemiſphere which is turned towards the Sun is alſo turned towards the Earth, and then (as was obſerved of *Venus*) the Moon, by reaſon of its great Diſtance from us, appearing as a flat Surface, one Edge of

of that Surface appears brighter than the rest of it, and so the Moon in this Situation appears *horned*; and because it moves from East to West, as was observed above, it at this Time appears to the Eastward of the Sun; upon which account it seems to follow the Sun in its daily Course, rising and setting after it. When the Moon is got to a *quartile Aspect*, as Astronomers call it, (that is, to the Distance of a quarter of a Circle in the Heavens from the Place of the Sun) then is half of its illuminated Hemisphere turned towards us, and half from us, in which Case it appears to us as a plain round Surface *half illuminated*, and half not. When it has advanced a little farther it its Orbit, and got farther from the Sun, more of its illuminated Hemisphere becomes visible to the Inhabitants of the Earth; whence it appears like a plain round Surface, somewhat defective of Light on that side which is from the Sun, and is said to be *Gibbous*. When it comes to an Opposition with respect to the Sun, it then turns the same Side directly towards the Earth that is towards the Sun, at which time it appears to us wholly enlightened, and is therefore called the *full Moon*. And as it performs the other Part of its Revolution, it assumes first a *gibbous*, then an *halved*, afterwards an *horned* Face; and when it arrives at its Conjunction with the Sun, it disappears

disappears for a while, the obscure part only being turned towards us, as before.

When the Moon is horned, the darker Hemisphere, or that which is turned from the Sun, has a faint Light diffused over it, sufficient to render it visible. This Light is owing to a Reflection of the Sun's Rays from the Surface of the Earth to that part of the Moon. For as the Moon when at the *Full*, by means of its enlightene Hemisphere which is then turned towards the dark Side of the Earth, shines upon it, and affords a certain Degree of Light; so the Earth when the Moon is at the *New*, has its enlightened Hemisphere, (or that side where it is *Day*) turned towards the obscure Hemisphere of the Moon, and enlightens that, but in a much greater Degree than the unilluminated Hemisphere of the Earth (or that side where it is Night) is enlightened by the *full* Moon; the Earth being considerably larger than the Moon. So that as that Body is a Moon to us, our Earth is a much larger and more luminous one to that.

The Time in which the Moon performs one entire Revolution about the Earth from any Point of the Zodiac to the same again, is called a *Periodical* Month. This Month consists of 27 Days, 7 Hours, and 43 Minutes. But the *Synodical* Month, which is the Time the Moon takes up in passing from one Con-

junction

l. II.

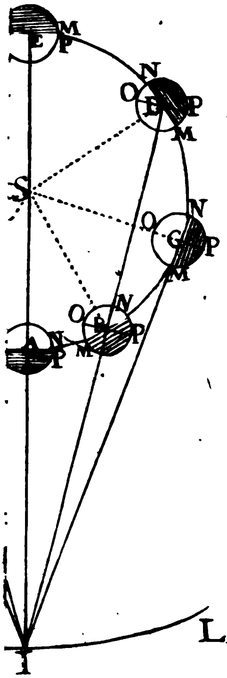


Fig. 10.

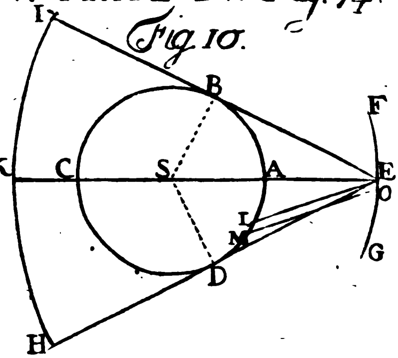


Fig. 12. P. 64.

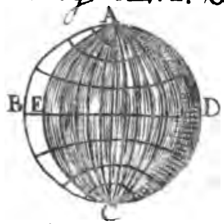
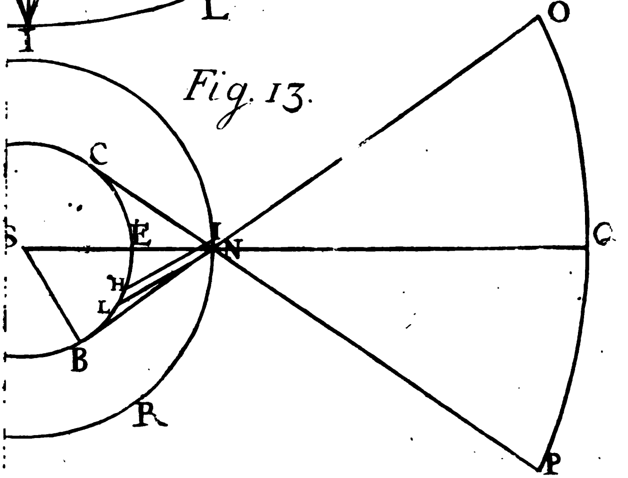


Fig. 13.



junction with the Sun to another, is larger than this by about two Days and 5 Hours. The Reason of this is, that while the Moon is passing from one Conjunction to another, the Sun by means of the Earth's Motion in its Orbit, is in appearance advanced towards the East; so that when the Moon comes round to that Part of the Heavens where the Sun was at the foregoing Conjunction, it is not then in Conjunction with it again, but has such a Portion of the Heavens to describe, as takes up so much more time, as, being added to the *Periodical* Month, makes it up 29 Days, 12 Hours, 44 Minutes, and 3 Seconds; which is the *Synodical*. But perhaps the 14th Figure may make this clearer: in which let S represent the Sun, E the Earth moving in its Orbit AB from A towards B, and let MNO denote the Orbit of the Moon, and M its Place therein when in Conjunction with the Sun. Now while the Moon describes its Orbit MNO, let it be supposed that the Earth advances in its Orbit from E to *e*, in which Case the Moon's Orbit will be found at *mno*, and the Point M will be at *m* in the Line *ed*, if we suppose it parallel to ES. And therefore when the Moon comes to *m*, it will have performed an entire Revolution about its Orbit; but will not be in Conjunction with the Sun, but has still the Arch *mp* to describe

before it reaches that Point of the Heavens in which the Sun will then appear to be.

The Plane of the Moon's Orbit is not coincident with that of the Earth, but makes with it an Angle of about 5 Degrees, cutting it in two Points oppoſite to each other, after the ſame Manner as the Equator and the Ecliptic were obſerved to cut each other; ſo that the Moon does not ſeem to deſcribe the Ecliptic in moving round its Orbit, but one half of the Way it declines towards the *North*, the other half towards the *South*. The two Points where the Moon's Orbit paſſes through the Plane of the Ecliptic, are called the *Nodes*; and a right Line drawn from one of theſe Points to the other, will paſs through the Center of the Earth (that being in the Plane of the Moon's Orbit as well as in that of its own) and is called the *Line of the Nodes.* That Node through which the Moon paſſes from the *South Side* the Ecliptic to the Northern is called the *Aſcending Node*; the other thro' which it paſſes from the *North* to the *South* Side, is called the *Deſcending Node (a).*

The Extremities of the Line of *Nodes* are not always directed towards the ſame Points of the Ecliptic, but continually ſhift their Places from Eaſt to Weſt, or contrary to the Order

(a) Aſtronomers often expreſs the former of theſe Nodes by this Mark ☊, which they call the *Dragon*'s *Head*; the latter by ☋, which they call the *Dragon*'s *Tail.*

Order of the Signs; performing an entire Revolution about the Earth in the Space of something less than 19 Years.

As was observed of the primary Planets, the Moon also moves not in a Circle, but in an *Ellipse*, one of whose *Foci* is in the Center of the Earth. And the *Linea Apsidum* of it is not carried along with the Earth in a Direction parallel to itself, but continually turns round the Center of the Earth thro' which it passes from West to East, or according to the Order of the Signs, in the Space of about 9 Years.

The Moon being subject to the Influence of the Sun as well as the Earth, it is not only liable to various Irregularities in its Motion, but its Orbit is wont to put on different Forms and Motions, all which depend on its Situation with respect to those two Bodies. They are these that follow.

I. The nearer the Moon is to its *Syzigies*, (that is, its Conjunction or Opposition with the Sun) the greater is its Velocity; and the nearer it is to the *Quadratures* (that is, the first and last Quarter) the slower it moves *(b)*.

II. It does not describe *Areas* proportionable to the Times, except in the Quadratures and Conjunctions.

(b) This Irregularity was observed by *Tycho Brahe*, who gave it the Name of the Moon's *Variation*.

III. Its Orbit is ſomewhat more curved in the Quadratures, than it would be if the Moon were only influenced by the Earth; and leſs curved in the Syzigies, the Moon running farther off from the Earth in the Quadratures than in the Syzigies.

IV. When the Earth is in its Perihelion, (that is, in that Part of its Orbit which is neareſt the Sun) the Periodical Time of the Moon is greater than when it is in its Aphelion, or fartheſt from the Sun: ſo that the Lunar Months of one Part of the Year exceed thoſe of another.

V. The *Linea Apſidum* of the Moon's Orbit goes forwards when the Moon is in the Syzigies, and backwards when in the Quadratures; but it goes farther forwards than backwards every Time, ſo that at length it revolves quite round, as obſerved above.

VI. The Excentricity of the Moon's Orbit is greater and leſs in every Revolution of the Moon; it is greateſt when the Moon is in the Syzigies, and leaſt when it is in the Quadratures: and conſequently while it paſſes from the Quadratures to the Syzigies, it is encreaſing; and while it paſſes from the Syzigies to the Quadratures it is diminiſhing. And if we compare ſeveral Revolutions of the Moon together, its Orbit is the moſt Excentrical when the Situation of it is ſuch, that the *Linea Apſidum* is in the Syzigies, and leaſt when

when in the Quadratures. So that when both the Moon and the *Linea Apſidum* of its Orbit are in the Syzigies, its Excentricity ſhall be half as great again, as when they both happen to be in the Quadratures.

VII. The Line of the Nodes does not move uniformly backwards, that is, contrary to the Order of the Signs; but when it is in the Syzigies it ſtands ſtill, and moves the faſteſt of all when in the Quadratures.

VIII. The Inclination of the Lunar Orbit to the Ecliptic (*c*) is alſo liable to Change; it is greateſt when the Nodes are in the Syzigies, and leaſt when they are in the Quadratures.

All theſe Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion; and alſo greater when the Moon is in Conjunction with the Sun than in Oppoſition.

IX. There is another Particular in the Moon's Motion, which is by ſome reckoned among its Irregularities; which is, that it moves faſter in its Perigeon, than in its Apogeon; but this is not owing to any Perturbation in its Motion ariſing from the Influence of

(*c*) Some Aſtronomers meaſure the Inclination of the Moon's Orbit to the Ecliptic, not from the Ecliptic, but from a Perpendicular to that, and ſo call its Inclination 85 Degrees inſtead of 5. And ſo of the Inclination of the Equator to the Ecliptic; reckoning that 66 Degrees and an half, and not 23 and an half, as is the moſt uſual Way.

of the Sun; but is common to all the Planets, and is occasion'd only by its describing equal Areas in equal Times in an elliptical Orbit, as the other Planets do.

All these Irregularities observable in the Motion of the Moon, naturally flow from the Supposition, that the Earth and the Moon are retained in their Orbits by Gravity, in the Manner laid down and explained in the Introduction to this Part, as will appear when we have considered the Physical Causes of the Motion of the heavenly Bodies.

The only uniform Motion the Moon has, is its Rotation about its *Axis*, which it performs the same Way, and in the same Time, that it does its Revolution about the Earth. The Consequence of this would be, that if its Revolution in its Orbit were performed with an uniform Celerity, and that Orbit were a Circle having the Earth in its Center, and also if the *Axis* of the Moon were perpendicular to the Plane of that Orbit, the Moon would in all Positions turn exactly the same Side towards the Earth. This is so obvious as to need no Illustration. But since its Motion in its Orbit is not uniform, nor its Orbit a Circle, neither its *Axis* perpendicular to the Plane thereof, the Moon does not keep the same Face exactly towards us, but turns it a little, first one Way, then another; appearing thereby subject to a two-

fold

fold *Libration*, *viz.* from East to West, and from North to South. To the one of which the two first Causes abovementioned concur; the other is owing to the third. And first to illustrate that *Libration* which is from East to West. Let the Elliptical Orbit BBB (Fig. 15.) represent that of the Moon, let E be the Place of the Earth, and let the Circle ABCD represent the Moon in its Apogeon at A; and for the present, let us suppose the *Axis* of the Moon perpendicular to its Orbit. Then because its Motion about its *Axis* is uniform, and its Motion from its Apogeon the slowest of all; it is evident, that at the Time when it has revolved a quarter of the Way round its *Axis* and obtained, suppose the Situation FBCD, it will not have advanced a quarter part of the Way through its Orbit; and therefore the Face BCD, which was turned towards the Earth at first, cannot now be turned to I the Center of the Ellipse, much less to E the Place of the Earth, but to some other Place, as K, on the other Side the Center: But when it has advanced to G, the same Face BCD will be turned towards E, that was when the Moon was at A; because as it has advanced half through its Orbit, it has advanced half round its own *Axis*. Again, as it moves quicker in that Part of its Orbit, that's near its Apogeon, it will have proceeded above a quarter part of the Way through

through its Orbit, and ſo will have got into the Situation HBCD ſuppoſe, when it has turned a quarter round its *Axis*; and that Part of the Moon which was viſible when the Moon was at A, will be turned towards K, as it was when the Moon was at F. And it is obſervable from the Figure, that the Face BCD, which was turned directly towards the Earth at E, when the Moon was either at A or G, is, when the Moon is at F, turned to the *Right*, or towards the Weſt, with reſpect to a Spectator viewing it from E; but to the *Left*, or towards the Eaſt, when the Moon is at H. And this is the firſt kind of *Libration* which was to be explained. As to that which is from North to South, it may be thus illuſtrated. Let HI (Fig. 16.) repreſent the Plane of the Moon's Orbit, being ſuppoſed to be ſeen edgeways; let E be the Place of the Earth, and let the Circle AMB*m* repreſent the Moon, the Line AB being its *Axis*, and M*m* one of the Parallels of its Equator; and when the Moon is at H, let the Point M be that which is turned directly towards the Earth. Now when the Moon has revolved half round its *Axis*, which will be when it has got to I the oppoſite Point of its Orbit, the Point M will be found at *m*; but becauſe of the Parallelſm of its *Axis*, not this, but ſome other Point, as N, is that which is turned directly towards

towards the Earth at E; so that to a Spectator at E, the visible Part of the Moon librates from North to South, and returns to its place once in each Revolution, one of the Poles as A being only visible, when the Moon is in one Part of its Orbit, suppose at I, and the Pole B only when it is at H; because the Circle represented by the pricked Line C D terminates the View of a Spectator at E, in either Situation of the Moon (*a*.)

As to the Surface of the Moon, whoever views it with a good Telescope, will perceive some Parts of it shining very bright, while others are as dark and obscure. The bright-

(*a*) It may be worth considering here, what kind of Motion the Earth seems to have to the Inhabitants of the Moon, if any such there be. And first, we will suppose that the Hemisphere, the Moon turns towards us, is always exactly the same. In this Case it is evident, that to such as live in the Middle of that Hemisphere, the Earth must always seem to be directly over their Heads; because a Line drawn from the Center of the Moon through the Place where they are, would pass through the Earth: and to such as live at the Extremity of that Hemisphere, the Earth must always appear in their Horizon. But since the Moon, as seen from the Earth, has these libratory Motions abovemention'd, it is impossible that the Earth should always appear directly Vertical to the one, or in the Horizon of the other; but must seem always nearly at the same Degree of Elevation; continually shifting its Situation from East towards the West, and from West towards the East, and at the same time from North towards the South, and from South towards the North, according as the Libration of the Moon, as seen from the Earth, is made in that Time.

er Parts are thought by ſome to be Land, the other Water; becauſe leſs Light is reflected from Water than from Land. But *Keil* is of Opinion, that ſome of thoſe darker Places may be occaſioned by the Shadows of higher Places falling upon them, and others may conſiſt of a Soil of a darker Colour, that reflects leſs Light than the reſt *(b)*. However this be, it is moſt certain, that there are Mountains in the Moon, and thoſe very high ones. For when the Moon is viewed through a Teleſcope, the Line which ſeparates the illuminated Hemiſphere from the dark one does not appear ſtrait when the Moon is *halved*, or uniformly bent when it is *horned* or *gibbous*, as when we view it with the naked Eye; but ſeems jagged, broken, and uneven; which evidently ſhews that its Surface is ſo too: For if its Surface were ſmooth and even, no Reaſon could be given why the Light of the Sun ſhould not reach as far in one Place as another. But the Boundary of Light and Darkneſs in the Moon is not only thus jagged and uneven, but ſome Parts even within the darkened Hemiſphere, that lie near the Edge of it, are enlighten'd by the Rays of the Sun long before ſuch as lie nearer the illuminated Hemiſphere, and continue ſo till after the Parts about them are

(b) See his Aſtronomy, Lect. X.

are in the *Dark*. These therefore can be no other than the Tops of high Mountains, on which the Sun shines as it does on those on Earth, both before and after it is visible in the Vallies below. The perpendicular Height of a very remarkable Mountain in the Moon, called St. *Katharine* (c), has been found by Mensuration to be nine Miles, which is three Times that of the highest Mountain we have upon the Earth (d).

(c) See the Names Astronomers have distinguished the Parts of the Moon by, in a Figure annexed to the Lecture referred to in the foregoing Note.

(d) The Method of Measuring the Height of a Mountain in the Moon is this: Let ABDE (Fig. 17.) represent the Moon, C its Center, SA a Ray of Light from the Sun, ABD the illuminated Hemisphere, AED the dark one, F the Top of a Mountain beginning to be illuminated. Then with a Telescope in which there is a Micrometer fixed, let the proportionable Lengths of the Lines FA and AD be taken. Then because the real Length of the latter, which is the Diameter of the Moon is known, the true Length of the former will also be had. Since then the Line SAF is a Tangent to the Moon, the Triangle FAC will be a right-angled one, and F C will be its Hypothenuse, whose Square being equal to the Squares of the other two Sides, it may from thence be also known. Now if from this we substract the Semidiameter of the Moon, the Remainder is the perpendicular Height of the Mountain,

CHAP. IX.

Of the Eclipses *of the* Sun *and the* Moon.

AN *Eclipse* is a Deficiency of Light in some of the heavenly Bodies, occasioned by the Interposition of some other between that and the Sun. Thus when the Moon passes through the Shadow of the Earth, it being by that means deprived of the Sun's Light, loses its Brightness, and is then said to suffer an Eclipse. So when the Moon is interposed between the Sun and the Earth, the Sun being in Appearance deprived of its Light, is said to be eclipsed; though it is the Earth which is properly so, that, and not the Sun, being the Body where the Light is wanting.

One primary Planet is never eclipsed by the Interposition of another; because they are so small with respect to the Magnitude of the Sun, that the Shadow of any one of them ends in a Point before it reaches the Body of another. Thus when the Earth is interposed between *Mars* and the Sun, its Shadow vanishes before it reaches to *Mars*. And even when the Moon is interposed between the Sun and the Earth, if the Earth be in its *Perihelion*, and the Moon in its *Apogeon* at

at that Time, the Shadow of the Moon will not reach the Earth, and ſo the Moon will not ſeem to cover the whole Body of the Sun, but appear as a large dark Spot, ſurrounded with a Circle of Light proceeding from the Limb or Edge of the Sun.

From what has been obſerved it appears, that there can be no Eclipſe of the Moon, but when the Earth is interpoſed between it and the Sun, that is, at the Time of its Oppoſition, or when it is *Full*; nor any of the Sun, except when the Moon is between the Earth and the Sun, that is, at the Time of its Conjunction with the Sun, or when it is *New*. And the Reaſon why theſe Eclipſes do not happen at every New and Full Moon, is, becauſe, as was obſerved above, the Plane of the Moon's Orbit is inclined to that of the Earth's, ſo that the Moon is ſometimes above the Ecliptic, and ſometimes below it, at the Time of its Conjunction with, or Oppoſition to the Sun, in which caſe the Shadow of the one paſſes above or below the other *(a)*.

The

(a) It is not requiſite to conſtitute an Oppoſition or Conjunction of a Planet with the Sun, that it be in a right Line that paſſes through the Sun and the Earth; it may be in any Degree of Latitude it is capable of, and is nevertheleſs ſaid by Aſtronomers to be in Conjunction or Oppoſition with the Sun, provided it be in the ſame Part of the Zodiac, that is, if it have the ſame Degree of Longitude therewith.

The only Time therefore in which Eclipses either of the Sun or Moon can happen, must be when the *Nodes* are in or near the Syzigies, that is, when the Line of the *Nodes* points towards the Sun. Now if the Line of the *Nodes* always pointed towards the same Points of the Heavens, or remained parallel to itself, then the Sun appearing by means of the Earth's Motion to run through the Ecliptic once a Year, it is obvious that one Extremity of this Line would point towards the Sun each half Year: but since this Line has a slow Motion contrary to that of the Sun, by which it revolves round in nineteen Years, as observed above, it meets the Sun with one or the other Extremity before it has got quite half round, and so points towards it twice in somewhat less than half a Year: So that it sometimes happens that it points towards the Sun three times within the Space of a Year, which is as oft as possible. And it frequently happens, that the Sun being in its Approach towards one of the Nodes at the Time of the New or Full Moon, and near enough to occasion an Eclipse then, the Moon shall get about to the other Node, before the Sun be too far removed from the former, and so there will be another Eclipse then. Upon which account it is possible there may be two Eclipses, while the Sun is passing by either Extremity of the Line of the Nodes; the one of which (if it be

be at the New Moon) will be an Eclipſe of the Sun, and the other will happen at the next Full Moon, and will therefore be an Eclipſe of the Moon; or if the firſt be at the Full, the other will be at the New. But if the Moon be within a Day or two of paſſing thro' one of its Nodes at the Time of its Conjunction, it will paſs by the other without ſuffering an Eclipſe there.

When it happens that the Moon is in one of its Nodes at the Time of its Oppoſition to the Sun, then is it ſaid to be *centrally* eclipſed, its Center being then in a right Line with thoſe of the Sun and Earth. In this Caſe it is alſo *totally* eclipſed, every part of it being hid from the Sun's Rays. But the Shadow where the Moon paſſes through, is about three times as broad as the Face of the Moon, ſo that it may ſuffer a total Eclipſe, though it be at ſome Diſtance from the Node at the Time of its Opppoſition; an Eclipſe therefore may be *total*, when it is not a *central* One; but an Eclipſe of this Kind will be of ſhorter Duration than the former. Farther, if it be ſo far removed from the Node that a Part of its Diſk only falls into the Shadow of the Earth, it is ſaid to be a *partial* Eclipſe, and is of ſhorter Duration than the former. If the Moon be above 13 Degrees from the Node, there is no Eclipſe.

The

The Duration of a *central* Eclipſe of the Moon, is about 4 Hours; the firſt of which is taken up in entring the Shadow, two more it continues quite immerſed therein, and the fourth is ſpent in getting out.

What is here ſaid, may be illuſtrated in the following Manner. Let AB (Fig. 18.) repreſent a ſmall Portion of the Plane of the Earth's Orbit ſeen edgeways, CD a part of the Moon's Orbit, and let the Point where theſe Lines croſs each other be one of the Nodes, and E,E,&c. the Shadow of the Earth paſſing along the Ecliptic. If the Moon falls into the Shadow at N, it is both a total and central Eclipſe; if at O, it is total, but not central; if at P, the Eclipſe is only partial. And if the Moon is at Q at the Time of its Oppoſition, it then paſſes by the Shadow without being eclipſed at all *(b)*.

As all opake Bodies when illuminated by the Rays of the Sun caſt a Shadow from them, ſo they have alſo a *Penumbra*, which is an imperfect kind of a Shadow every where ſurrounding the former, and growing larger and larger,

(b) We have hitherto ſuppoſed that the Shadow of the Earth reaches the Moon, as it would do, were it not for the Atmoſphere of the Earth, which by refracting the Sun's Rays as they paſs through it towards the Earth, throws them into its Shadow; which refracted Rays falling upon the Moon render it viſible to us, altho' placed within the Limits of the Shadow, and is the Occaſion of that reddiſh Colour, with which the Moon appears at that Time.

larger, as we recede from the Body. As the other is owing to a total Interception of the Rays of Light, this is occaſioned by a partial one. The 19th Figure will explain this better.

Let ABCL repreſent the Body of the Sun, KI the Earth, and MN the Moon; and let the Lines AM, BM, &c. be drawn as in the Figure; then will the Space MNFG, within which none of the Rays can come, be the Shadow thereof. But it is evident that beſides this there are other Spaces, *viz.* MDE and MEF, &c. within which but Part of the Sun's Rays can come; and the nearer thoſe Spaces are to the Shadow, the leſs Light they are capable of receiving. The Light which falls within theſe Spaces is called the *Penumbra.*

This being premiſed, we may proceed to conſider the different Kinds of Solar Eclipſes.

The Moon being ſmaller than the Earth, and having a conical Shadow, as being alſo leſs than the Sun, a very ſmall Part of the Surface of the Earth can be covered by the Shadow of the Moon at the ſame Time, though, as obſerved above, the whole Body of the Moon may be involved in that of the Earth; ſo that an Eclipſe of the Sun is viſible but to a few Inhabitants of the Earth, whereas one of the Moon may be ſeen by all that live on that Hemiſphere of the Earth that is turned towards it.

In order to conſtitute a *central* Eclipſe of the Sun, it is not neceſſary that the Moon ſhould be exactly in the Line of the Nodes at the Time of its Conjunction; for it is ſufficient to denominate an Eclipſe of the Sun *central*, that the Center of the Moon be directly betwixt the Center of the Sun and the Eye of the Spectator: for to him the Sun is then *centrally* eclipſed. But ſince the Shadow of the Moon can cover but a ſmall Portion of the Earth, it is obvious this may happen when the Moon is not in one of her Nodes. And not only this, but the Sun may be eclipſed *centrally*, *totally*, *partially*, and *not at all*, at the ſame time. For inſtance, when the Moon is interpoſed between the Sun and the Earth, to thoſe who live where the Center of the Shadow paſſes (as in the Middle between F and G in the laſt Figure) the Sun will be *centrally* eclipſed; to thoſe who live within the Shadow, but not in the Center (ſuppoſe nearer to F than G) *totally*; to thoſe who live in the *Penumbra*, as between F and D, or G and I, *partially*, and that more or leſs as they live nearer or farther from the Shadow; to thoſe who live without the *Penumbra*, as between D and K, *not at all*.

But as the Shadow paſſes along the Surface of the Earth, different Inhabitants, and thoſe

those very distant ones, will see the same kind and degree of Eclipse, though not at the same Hour *(d)*.

(d) For the Method of predicting Eclipses, see *Keil's* Astronomy, Lect. XIV. Or *Whiston's* Astronomical Lectures, Lect. XII.

CHAP. X.

Of the Phænomena *of the* Satellites *of* Jupiter *and* Saturn, *their* Eclipses *and* Occultations: *And also of* Saturn's Ring.

THE Satellites of *Jupiter* and *Saturn* are subject to the same kind of Motion with the Moon, except that their Orbits, as far as can be collected from the Observations of Astronomers, are circular, or very nearly so; and therefore, if we suppose them to be seen from their Primary ones, they will exhibit nearly the same *Phænomena*.

All the Satellites of *Saturn* revolve about it in almost the same Plane, *viz.* that of its Ring; excepting the fifth, the Plane of whose Orbit deviates a little therefrom. Those of *Jupiter* move also in a Plane that is proper to themselves, and nearly coincident with

that in which *Jupiter* itſelf moves about the Sun.

Theſe Planets, *viz.* the Secondaries both of *Jupiter* and *Saturn*, when viewed from the Earth, appear to have their Progreſſions, their Stations, and Retrogradations different from thoſe of their Primaries; that is, when a Primary is progreſſive, its Secondary may be regreſſive, or otherwiſe; and *vice verſa.* For inſtance, when a Secondary is beyond its Primary, that is, in its ſuperior Conjunction with reſpect to the Earth, it ſeems to move according to the Order of the Signs; but when it is in its inferior one, it ſeems to move the contrary Way; and when its progreſſive Motion is changing into a regreſſive one, the Planet appears ſtationary.

To illuſtrate this; let S (Fig. 20.) repreſent the Sun, I the Planet *Jupiter* or *Saturn*, BCG the Orbit of one of its Satellites, and let EPF be the Orbit of the Earth. While the Satellite paſſes through OHB, the ſuperior Part of its Orbit, it will appear from the Earth to move the ſame Way with its Primary; but while it moves through the remaining Portion BLO, it will ſeem to go the contrary Way; as is evident if we conſider that its Motion in its Orbit is quicker than that of its Primary.

Theſe Satellites and their Primary ones mutually eclipſe each other in the ſame manner

ner in which it has been ſhewn that the Earth and the Moon do. But there are three different Caſes in which the Secondaries diſappear to us. The One is, when a Secondary is directly behind the Body of its Primary with reſpect to the Earth; this is called an *Occultation* of the Planet: Another is, when it is directly behind its Primary with reſpect to the Sun, and ſo falls into its Shadow, and ſuffers an Eclipſe, as the Moon, when the Earth is interpoſed between that and the Sun. The laſt is, when it is interpoſed between the Earth and its Primary; for then it can-be diſtinguiſhed from the Primary itſelf. All which may be illuſtrated in the following Manner.

While the Planet deſcribes OHB, the ſuperior Part of its Orbit, it is capable of diſappearing twice; once when its Primary is interpoſed between it and the Earth, as when it is at G or K, or ſome intermediate Point, according as the Earth is ſituated at that Time; and alſo at H, where it is involved in the Shadow of its Primary. When the Earth is between F and Q, or F and P, that is, when the Sun is to the Eaſtward of *Jupiter*, the Eclipſe of the Secondary happens firſt, *viz.* when it paſſes through the Shadow at H; and the *Occultation* afterwards, when the Planet lies hid at K. When the Earth is between E and Q, or E and P, the Occultation

tation is first, and the Eclipse afterwards. While it passes through the inferior Part of its Orbit, it disappears also, according to the Situation of the Earth, either at D or C; or when it is between those Points, being directly between the Earth and its Primary, and therefore not distinguishable from the latter. When the Satellite is at L in Conjunction with the Sun, its Shadow falls upon its Primary, and eclipses a Part of its Disk, in like manner as the Moon, in the like Situation with respect to the Earth, eclipses a Part of that (*a*). As

(*a*) From the Occultations and Eclipses of the Satellites of *Jupiter*, three notable Problems are determined: *viz.* 1. The Parallax of the Orbit of the Earth in that Planet, and thereby (as we observed Chap. VII. in the Notes) its Distance from the Sun. 2. The successive Propagation of Light; and 3. The Longitude of Places upon the Earth.

1. The Method of finding the Parallax of the Orbit of the Earth, is this: Let the Interval of Time which lapses between the Middle of an Occultation of one of the Secondaries, suppose at G, and the Middle of its Eclipse afterwards at H, which is the Time the Planet describes the Arch GH in, be observed. The Time then of its whole Revolution being known, that Arch itself may be had, which is the Measure of the Angle GIH, or its equal EIS, which is the Parallax of the Orbit of the Earth in that Planet.

2. If Light were *instantaneous*, a Spectator at P would see an Eclipse of one of the Secondaries of *Jupiter*, at the same Time that he would do if on the contrary side the Sun at Q. But it appears from Observations, that when the Earth is at Q, those Eclipses happen sooner, and that when it is at P they happen later than they ought to do by Computation. Which shews that Light takes up some time in passing from Q to P. From Observations of this kind it has been computed by *Romer*, that Light is eleven Minutes in passing from the

As to the *Phænomena* of *Saturn*'s Ring, it is observable, 1. That its Inclination to the Plane of the Ecliptic is about 31 Degrees: 2. That the Thickness of it is insensible; and therefore

the Sun to us; which is upwards of 81 Millions of Miles: But it has since been found by the more accurate Observations of others, that it passes over that Space in about 7 Minutes.

3. The Longitude of a Place is its Distance East or West of some other Place; and as Places lie more or less East or West one of another, the Hour of the Day in one is more or less different from the Hour of the Day in the other. That is, as Places differ in point of Longitude, so they do proportionably in their Account of Time. For instance, if a Place lies fifteen Degrees to the Eastward of another, it is one Hour past Mid-day there, when it is but Mid-day at the other: The Reason is, because the Sun is an Hour in passing over each fifteen Degrees of its daily Course. In like Manner, if a Place lies so many Degrees to the Westward of another, the Sun comes an Hour later to that Place than to the other; so that it is but Eleven of the Clock there, when it is Twelve at the other. And so proportionably for any other Distance. If therefore at two Places it be observed at what Hour the beginning of an Eclipse of one of *Jupiter*'s Satellites happens at each of them, and if that Hour be not the same in both, the Difference will shew how far those Places lie East or West of one another. Or if instead of the Observation at one of the Places, an Almanack be made use of, that shews the Time when the Eclipse will happen at that Place, and that Time be compared with the Time of its happening observed at the other Place, the Differences between those Times, if there be any, will shew the same.

This Method very rarely succeeds at Sea, because the Tossing of the Ship is inconsistent with the Accuracy required in Observations of this kind.

The like may be done by observing the Time when an Eclipse of the Moon happens, or when it comes to a fixed Star: But these are *Phænomena* that are much less frequent than the other.

therefore when it is so situated, that the Plane of it, if produced, would pass through the Earth, it becomes invisible. Neither is it to be seen when the Plane of it passes between the Sun and the Earth; for then the Sun shines upon that Side of it which is turned from us; in which case, the dark Side being next the Earth, it is not perceptible to us. But in this case a black List or Circle appears to encompass the Planet, which in all Probability is no other than the Shadow of the Ring thereon. The Extremities of the Ring which appear on either Side the Planet are by some called its *Ansæ*.

CHAP. XI.

Of the Comets.

THE Comets are opake, spherical, and solid Bodies like the Planets; and like them perform their Revolutions about the Sun in elliptical Orbits, which have the Sun in one of their *Foci*. The Particulars in which they differ from the Planets, are, that they move in various Directions, some the same Way with the Planets, others the contrary; neither are their Motions confined within the Zodiac, their Orbits admitting of any Inclination

75.

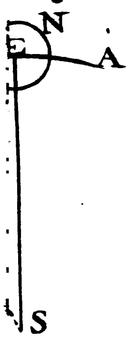


Fig. 15. P. 81.

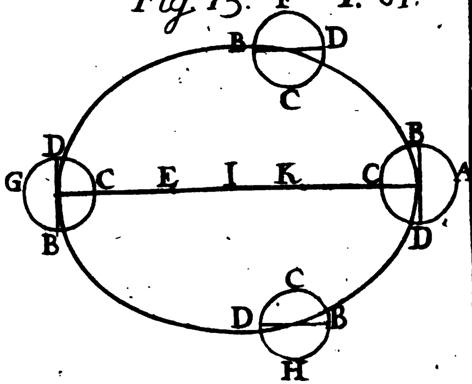
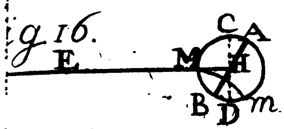


Fig. 16.



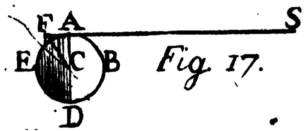


Fig. 17.

Fig. 18. P. 90.

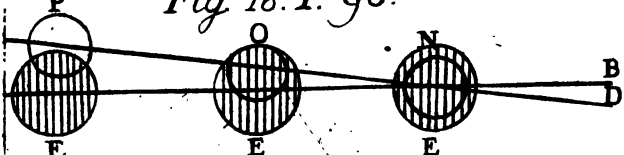
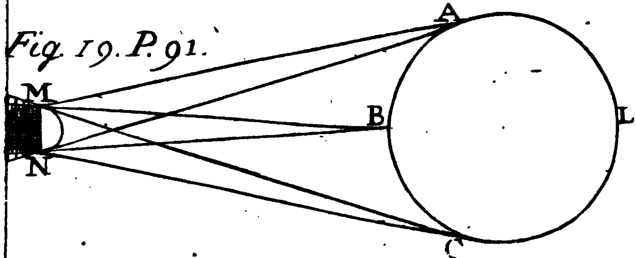


Fig. 19. P. 91.



tion to the Ecliptic whatever: And the Excentricity of their Orbits is so very great, that some of the Comets perform the greatest Part of their Motion almost in right Lines, tending in their Approach to the Sun almost directly towards it, after which they pass by it; and when they leave it, march off again nearly in a right Line, till they are out of sight, as if they were hastening back to the fixed Stars; and return not till after a Period of many Years (*a*).

As the Comets approach the Sun, their Motion grows proportionably swifter, for they describe equal *Areas* in equal Times about its Center, as the Planets do. Hence it is, that when they are in their *Perihelia*, that is, at their least Distance from the Sun, their Motion is immensely swifter than when they are in their *Aphelia*, or greatest Distance from it. This is the Cause that they are visible to us for so short a time; and when they disappear, are so long before they come near enough to be seen again (*b*).

O The

(*a*) See a Portion of the Orbit of a Comet described, Plate I. Fig. 1. at *abcd*.

(*b*) From the Observations of Astronomers, it seems probable, that the Comet which appeared in the Year 1682 was the same which was seen before in the Year 1607, and the Year 1531, and therefore may be expected again in the Year 1758, after a Period of about 151 Years. And that the great Comet which appeared in the Year 1680 was the same that was seen in the Time of K. *Henry* I. in the Year 1106, and in 531, and in

The Ancients were divided in their Opinions concerning them, some considering them as wandring Stars, others as Meteors kindled in the Atmosphere of the Earth, subsisting for a Time, and then dissipated: others looked upon them as ominous Prodigies. But it is put beyond all doubt by the more accurate Observations of the late Astronomers, that they are a kind of Planets. That they are not Meteors is obvious, for if they were, they could not bear that vast Heat, which some of them in their *Perihelia* receive from the Sun. The great Comet which appeared in the Year 1680 was within a sixth Part of the Sun's Diameter from its Surface, and therefore must acquire a Degree of Heat intense beyond all Imagination (*c*).

As

in the forty fourth Year before Christ, in which *Julius Cæsar* was murthered. If so, then the Period of this Comet is about 575 Years. There are between 20 and 30 that have appeared since the Year 1337, but no two Appearances seem to belong to the same Comet, except those abovementioned. See Dr. *Halley*'s Synopsis of the Astronomy of Comets, published at the End of *Gregory*'s Astronomy, in which there is a Table expressing the Motions of all the Comets that have been hitherto duly observed. The Comet of 1680 was in one Part of its Orbit within half the Sun's Breadth of the Way of the Earth.

(*c*) One Method by which Astronomers investigate the apparent Course of a Comet, is this: They observe what two Stars are directly one on one side of the Comet, and the other on the other; which is done by holding up a Thread between the Eye and the two Stars, and extending it in such manner, as that it shall seem to cross each Star: then they look out two other Stars in

such

As to those *Phænomena* of them which arise from the Motion of the Earth; they agree in a great Measure with those of the Planets. For instance, those Comets which move according to the Order of the Signs, a little before they disappear, become more than ordinarily *slow* or *retrograde*, if the Earth at that time be between them and the Sun; but more than ordinary swift, if the Earth be on the opposite Side: and the reverse of this happens to those which move contrary to the Order of the Signs. This is occasioned by the Motion of the Earth, as was observed of the superior Planets: For when the Earth goes the same Way with a Comet, but with a swifter Motion, the Comet seems *retrograde*; when with a slower Motion, the Comet's apparent Motion becomes slower; and when the Earth moves the contrary way, it becomes swifter.

Few of the Comets are to be seen in their Access to the Sun, but in their Recess appear with long fiery Tails, pointing directly, or nearly so, towards that Part of the Hea-

such Situation also, that the Comet shall appear in a Line that passes from one to the other; which are found as before: then they extend a Thread upon the celestial Globe, from one of the two first Stars to the other; and another Thread from one of the two last Stars to the other: and the Point on the Globe where the Threads cross is the apparent Place of the Comet at the Time the Observation was made. This they do daily, and so trace out its apparent Course in the Heavens.

vens which with respect to the Comet is opposite to the Sun. Some are visible before they reach the Sun, and begin to put forth their Tails, which at first are short and thin, seldom exceeding fifteen or twenty Degrees in Length, but grow longer and denser as the Comet comes nearer the Sun. If the Comet passes very near the Sun, it then sends forth fiery Beams of Light every Way. After this it puts forth a Tail forty, fifty or sixty Degrees long, which as the Comet recedes farther from the Sun, continually diminishes both in Length and Splendor; but is larger and longer at any Distance in its Recess from the Sun, than at an equal Distance in its Access to it.

This great Splendor and Length of the Tails, Sir *Isaac Newton* thinks arises from the Heat which the Sun communicates to the Comet as it passes near it; and accounts for it after the following Manner. As the Ascent of the Smoak in a Chimney is owing to the Impulse of the Air, with which it is entangled (for the Air about a Fire being rarefied by the Heat thereof, has its specifick Gravity thereby rendered less than that of the more distant Air: Upon this account it ascends, and carries along with it the Smoak with which it is engaged), in like manner he thinks the Tail of a Comet may rise from the Atmosphere thereof into those Parts which are opposite to the Sun, being carried up by the Æther about the

Comet, rarefied to a very great degree by the Heat thereof. And thinks his Opinion greatly confirmed by the Appearance of the Tails; for when accurately observed, they are found not to rise always in a Direction precisely opposite to the Sun, but to deviate or incline a little from thence towards those Parts which the Comet has lately left; and not only so, but to be bent into a certain Curvature, the Extremities of the Tails deviating from the true Opposition more in proportion than the other Parts; and to be more dense seemingly, and better defined on the convex, than on the concave Side: And farther, that the longer the Tail is, the more sensible is the Curvature, as being the greatest at the greatest Distance from the Body of the Comet. Upon these Accounts he thinks it evident, that the *Phænomena* of the Tails of Comets depend on the Motion of their Heads, and that the Heads furnish the Matter which forms the Tails. For as in the Atmosphere of the Earth, the Smoak of a Body ascends perpendicularly if the Body be at rest, and obliquely if the Body be moved any otherways than directly upwards, or downwards; so in the Heavens, where all the Bodies gravitate towards the Sun, Smoak and Vapour must ascend from the Sun, and rise perpendicularly, if the smoaking Body be at rest; and obliquely, if the Motion of the Body be oblique to the Di-

rection the Vapour ascends with. And because the Force by which the Vapour ascends, is strongest near the Body of the Comet, the Obliquity with which it ascends will be the least near the Body, and greater at greater Distances, and consequently the Column will be incurvated or bent towards those Parts the Comet leaves: And because the Vapour in the preceding Side is something more recent, that is, has ascended something more lately, it will be something more dense on that Side, and on that account must reflect more Light, as well as be better defined; the Vapour on the other Side languishing by degrees, and vanishing out of sight (*d*).

This is the Sentiment of Sir *Isaac Newton*; but I must beg Leave to differ from him in this Matter. For I think it somewhat improbable on account of the great Rapidity with which Comets move, that the Æther diffused throughout the heavenly Regions should gravitate

(*d*) Some have supposed, that the Heads of Comets are transparent, and that their Tails are no other than a Beam of the Sun transmitted through them: But were the Heads of Comets transparent, they themselves would scarce be visible. Others, that they arise from the Refraction of the Rays of Light in their Way from the Comets to us: But if so, then both the Planets and fixed Stars ought to have Tails also. *Kepler* ascribed the Ascent of the Tails to the Rays of the Sun carrying the Particles of the Comet's Atmosphere away with them, that is, impelling them into the Regions opposite to it. But we have no Instance of any thing in Nature like this; this is therefore an *Hypothesis* that cannot be supported.

gravitate sufficiently towards the Sun (especially at Distances equal perhaps to twice that of the Earth from it) to cause the Vapours, or the Atmosphere of the Comets, to ascend into those Parts, towards which the Comets are tending. For when a Body ascends in a Fluid, as being specifically lighter than it, its Motion at or near the beginning of it, is very small; so that if the Fluid have but a small Degree of Motion the contrary Way, the Resistance thereof will readily stop the Ascent of the Body, and will carry it the same Way it moves itself. This is the Reason the Air, which causes the Flame of a Candle to ascend, can scarcely blow downwards with so small a Degree of Motion, or (which is all one) the Candle itself can scarcely be lifted up with so gentle an Hand, but the Flame will turn downwards. How is it then likely, that when a Comet moves with an incredible Velocity (*viz.* at the Rate of above a thousand Miles in a Minute, as some of them do at the Time when their Tails rise almost directly before them :) How is it then likely, I say, that the Æther should by its Gravity alone raise the Vapour of the Comet with such Force, as to cause it to overcome its Resistance, when that Resistance arises from so great a Rapidity of the Comet? Would it not rather carry it with it the other Way? And if we augment the Density of the Æther, and therewith its Gravity,

Gravity, then the Resistance of it will be the greater; and so the Case will be more desperate still. If we suppose it to be rarer, it will be lighter, and so less able to buoy the Vapours of the Comet up. The Question then is, how are those Tails to be accounted for? My Conjecture is as follows: It is well known that when the Light of the Sun passes through the Atmosphere of any Body (as the Earth) that which passes on one Side is by the Refraction thereof made to converge towards that which passes on the opposite one; and this Convergency is not wholly effected, either at the Entrance of the Light into the Atmosphere, or at its going out; but that beginning at its Entrance, it increases in every Point of its Progress. It is also agreed on all Hands, that the Atmospheres of the Comets are very large and dense. I suppose therefore, that by such time as the Light of the Sun has passed through a considerable Part of the Atmosphere of a Comet, the Rays thereof are so far refracted towards each other, that they now begin sensibly to illuminate it (or rather the Vapours floating therein) and so render that Part which they have yet to pass through, *visible* to us; and that this Portion of the Atmosphere of a Comet thus illuminated, appears to us in the Form of a Beam of the Sun's Light, and passes under the Denomination of a Comet's Tail.

Thus,

Thus, when a Burning-Glaſs is expoſed to the Sun's Rays, if there be the leaſt Smoak or Duſt hovering about the Glaſs, the Rays of the Sun, as ſoon as they are brought near enough together by the refractive Power of the Glaſs, render ſo much of the Smoak or Duſt as they have yet to paſs through, very diſtinguiſhable from the reſt. Indeed, by the accurate Reflection of the Glaſs, the Light of the Sun is brought to a *Focus*, wherefore the illuminated Space ends in a Point: but it happens not ſo to Rays refracted by the Atmoſphere of a Body; becauſe the Extremities of an Atmoſphere do not refract near ſo ſtrongly as thoſe Parts which are nearer the Body. Let us now ſee how this Hypotheſis will anſwer the *Phænomena* of the Tails.

When a Comet has been in its *Aphelion*, by means of its long Abſence from the Sun, the Vapours with which its Atmoſphere was full, are now condenſed and fallen down, the Atmoſphere itſelf grown cold, ſettled into its natural State, and reduced to a ſmall Compaſs; therefore as the Comet approaches the Sun, the Tail of it is inviſible to us, or at leaſt appears very ſhort: When it comes into the hotter Regions near the Sun, the Atmoſphere begins to rareſy and dilate itſelf, and new Vapours riſing up into it, encreaſe the Quantity of Matter therein, on which account the Tail

grows longer. When the Comet is in its *Perihelion*, if it be then very near the Sun, by means of the exceſſive Heat thereof, not only the hinder Parts of its Atmoſphere are ſtrongly illuminated by refracted Light, but the other Parts of it by the direct Light of the Sun, ſo that the whole Comet, Atmoſphere and all, is as it were on Fire; which cauſes an infinite Number of Beams and Irradiations to be ſent out from every Part; in which State the Comet is ſaid to be *Hairy*. Afterwards when the Comet has got to ſome Diſtance from the Sun, and this great Illumination ceaſes, and the Atmoſphere is extended by Rarefaction to a monſtrous Size, and now more replete with Vapours than ever, which like the Steam of a boiling Cauldron have been continually riſing into it, the Rays of the Sun have a great Way to go within it, after they are ſo far refracted towards each other, as to render the Atmoſphere viſible. So that the Tail now appears at its greateſt Length; but from this Time grows continually ſhorter and ſhorter (as the Atmoſphere by the Comet's loſing its Heat contracts its Dimenſions, and the Vapour floating therein ſubſides on to its Body) till the Comet is out of Sight.

But the greateſt Difficulty is yet behind, and that is, to account for the Deviation of the Tail of the Comet from thoſe Parts which are oppoſite to the Sun, towards thoſe which

the Comet has left. For it will most certainly be thought, that, upon this Hypothesis, the *Axis* of the refracted Rays, which is also the *Axis* of the Tail, ought to be in a right Line, that passes through both the Center of the Sun and of the Comet. Now if we consider the exceeding great Rapidity with which a Comet moves, and that the Propagation of Light is not instantaneous, this Difficulty will also vanish: For since Light is propagated in Time, that which passes near the Body of a Comet will not, when it comes at the Extremity of its Atmosphere, enlighten a Portion of it that lies in a right Line drawn through the Center of the Sun and the Place of the Comet where it now is, but where it was at the Instant when that Light passed by it: And therefore the illuminated Point of the Extremity of the Comet's Atmosphere, that is, the End of its Tail, will not be opposite to the Sun, but always in a right Line, that passes through the Center of the Sun and a Point of the Orbit behind the Comet, or which it has lately left; as will readily appear to any one that shall duly attend to this matter. And because this is true of every other Portion of the Tail, in proportion to its Distance from the Head, the whole Tail will decline from those Parts which are opposite to the Sun, towards those which the Comet has lately left. And thus is the abovemen-

tioned Deviation to be accounted for. But it muſt not be concealed here, that that very ſucceſſive Propagation of Light, which occaſions this Deviation, is capable of augmenting or diminiſhing it in Appearance, or even of entirely taking it away, according to the Situation the Comet is viewed from. For, on account of the ſucceſſive Propagation of Light, all diſtant Bodies that are in Motion appear not ſo far advanced as they really are; and the faſter they move, and the farther they are from us, the greater is the Difference between their real and apparent Place. Let us then ſuppoſe, that the Extremity of a Comet's Tail is farther from the Eye of a Spectator than the Body of the Comet, by a Space equal to the Semidiameter of the Earth's Orbit, then will Light be ſeven Minutes longer in coming from the Extremity of the Tail than from the Head; and therefore the Tail will appear to be farther behind its true Place than the Comet does, by a Space equal to that through which the Extremity of the Tail moves in ſeven Minutes; and conſequently the apparent Deviation in this Caſe will conſpire with the true one, and render it in appearance proportionably larger. Again, when the Extremity of the Tail is nearer the Earth than the Comet, then Light comes quicker from the former than from the latter, and therefore the Comet appears farther behind its true Place than the Extre-

Extremity of its Tail does, and so the true Deviation appears the less, or not at all, according as the Case may happen.

The apparent Deviation of a Comet's Tail depends also on another Circumstance, *viz.* the Situation of the Spectator with respect to the Plane of the Comet's Orbit; for when a Spectator is in the Plane of that Orbit, the Deviation becomes insensible to him, as not being able to distinguish nicely the exact Distances of the several parts of the Tail from his Eye: the farther he is removed out of that Plane, the more sensible it becomes.

There is another Particular, *viz.* that the Extremities of the Comet's Tail deviates from the true Opposition more in proportion than the other Parts, so that they are bent into a certain Curvature: and farther, that they are more lucid and distinct on the convex than on the concave Side. All which may be accounted for, if we allow the heavenly Regions the least imaginable Resistance; for in that Case, the Atmosphere of a Comet will be somewhat denser on the Side towards which the Comet tends; and therefore the Light which passes on that Side will suffer a greater Degree of Refraction throughout its whole Progress along the Atmosphere, that is, to the end of the Tail, than such as passes on the other: so that, that Side of the Tail which moves first, will be more lucid and better defined than the

the other: And because this Resistance will make the greatest Alteration in the extreme Parts of the Comet's Atmosphere, the Extremity of the Comet's Tail will deviate from the true Opposition more in Proportion than those Parts which are nearer the Body; so that the Tail (at least that side of it which moves first) will become convex. And if the Refraction of those Rays which pass on that side towards which the Comet tends, and on that account are most refracted, be so great as to cause them to cross those which pass on the other side the Comet, before they get out of its Atmosphere, their Refraction being irregular, they will cross the other dispersedly and in several Places, which, as is obvious to conceive, will necessarily give that other side of the Tail a concave Form, and render it imperfectly defined.

Thus I have ventured to throw in a Conjecture of my own; but propose it rather as a *Quere*, than a Solution; leaving it to be farther examined into by those who may have had better Opportunities of acquainting themselves with *Phænomena* of this Kind. *Valeat quantum valere potest.*

CHAP.

CHAP. XII.

Of the Parallax *of the* Heavenly Bodies.

THE Difference between the apparent Places of the Heavenly Bodies, when viewed from the Center of the Earth, and when ſeen from the Surface thereof, is called their *Parallax*. To explain this,

Let AB (Fig. 21.) repreſent a Portion of the Earth's Surface whoſe Center is C, HV the Heavens, and let APN be the Horizon of a Spectator at A; and P the Situation of a Planet therein. The Place of this Planet among the fix'd Stars, as ſeen from the Center of the Earth, will be M, which is called its *true* Place: But when view'd by a Spectator upon the Surface of the Earth at A, it appears in the Horizon at N, which is called its *apparent* Place; and the Arch MN, which meaſures the Diſtance between the one and the other, is the *Parallax* of that Planet. If the Planet be above the Horizon, as at Q, its true Place as ſeen from the Center of the Earth is R, its apparent Place S, and its *Parallax* is RS, which is leſs than before. From whence we ſee, the more the Planet is elevated above the Horizon,

Horizon, the leſs is its Parallax; and when it is got directly over the Place of the Spectator, it has no Parallax at all; for when it is at T, its apparent Place in the Horizon is at V, whether it be ſeen from C or A.

Farther, by how much the more remote a Planet is from the Earth, ſo much the leſs is its Parallax, at the ſame Heights from the Horizon. Thus if the Planet had been at the Diſtance C D from the Center of the Earth, its Parallax, when in the Horizon at D, would have been NO; and when in the Line AQ produced, as at E, its Parallax would have been meaſured by the Arch SF, both which are leſs than the former.

The Angles MPN, and RQS, are called the *parallactic* Angles, ſuppoſing the Planet at P or Q, and are equal, the firſt to the Arch MN, and the other to the Arch RS: They are alſo equal to the Angle APC, or AQC, which are thoſe under which CA, Semidiameter of the Earth that paſſes through the Place of the Spectator, would appear if ſeen from the Planet.

From hence it is obſervable, that the apparent Place of a Planet is always lower in the Heavens than its real one, except when the Planet is vertical, or at T.

As Aſtronomers, by means of the annual Parallax of a Planet, that is, the Parallax of the

the annual Orbit of the Earth, as explained above *(a)*, compute the Diſtance of the Planet from the Sun, with reſpect to that of the Earth from thence; ſo by means of this Parallax, they calculate the Diſtance of the Planet from the Center of the Earth with reſpect to the Diſtance of the Surface of the Earth from the ſame *(b)*.

The Sun is alſo liable to this Kind of Parallax; for it is evident that its Place among the fix'd Stars, when ſeen from the Surface of the Earth, may be different from that in which it would appear if ſeen from the Center of the Earth. Now could this Parallax be determined with Accuracy, the exact

Q Diſtance

(a) See Chap. VI. and VII.

(b) The Parallactic Angle APC being known, and the Angle VAN, which is meaſured by the Arch of the Heavens VN, being taken by Obſervation, which gives its Complement to two right ones PAC, from theſe two the third Angle PCA of the Triangle APC is had. And conſequently by the Relation PC (the Diſtance of the Planet) bears to CA, the Semidiameter of the Earth may be found.

There are ſeveral Methods of finding the Parallactic Angle; one of which is as follows:

Firſt, let it be obſerved when the Planet is between two fix'd Stars which are both in the ſame vertical Circle; and afterwards, when theſe two Stars appear ſituated in a Line that is parallel to the Horizon, obſerve how much the Planet appears below them; for the Planet being depreſſed by the Parallax, will at that time be lower than a Line drawn from one Star to the other; becauſe the fix'd Stars are at too great a Diſtance to ſuffer any Parallax: The Arch of the Heavens intercepted between this Line and the apparent Place of the Planet is its Parallax; allowance being made for the Motion of the Planet between the two Obſervations.

Diſtance of the Sun from the Earth might be known, and from thence, by the Method explained above *(c)*, that of the other Planets from the Sun. But the Sun's Diſtance from the Earth is ſo very great, and of conſequence its Parallax is ſo very ſmall, that the Obſervations neceſſary for taking of it cannot be made with ſufficient Accuracy. Aſtronomers therefore have recourſe to the Parallaxes of *Mars* and *Venus*; which if they could be found, that of the Sun would alſo be had: Becauſe the Horizontal Parallaxes of the Heavenly Bodies, as we have juſt been ſhewing, are proportionably leſs as their Diſtance is greater. Now when *Mars* is oppoſite to the Sun, it is above twice as near to us, and its Parallax is above twice as great as that of the Sun. And *Venus*, when in its inferior Conjunction with the Sun, is almoſt four times nearer than the Sun, and therefore its Parallax is ſo many times greater. Aſtronomers find, that the horizontal Parallax of *Mars*, when it is in Oppoſition to the Sun, does not exceed 25 Seconds, and from thence conclude the Sun's to be about 10 Seconds. With which Parallax of 10 Seconds if a Calculation be made, the Sun's Diſtance will be found about 81 Millions of Miles from the Center of the Earth.

However, Aſtronomers need not be ſollicitous about the Parallax of *Mars*, ſince Dr. *Hal-*

(c) See Chap. VI, VII.

Halley has laid down a Method in the Philosophical Transactions (*d*), whereby the Parallax of the Sun may be had to so great a Nicety, that its true Distance within a 50th Part of the Whole may be determined from it; *viz.* At the next Transit of *Venus* over the Sun's Disk, which will happen on the 26th of *May*, in the Year 1761; when, at five Minutes before Six in the Morning, *Venus* will appear not above four Minutes of a Degree South of the Middle of the Sun's Disk.

(*d*) Numb. 348.

CHAP. XIII.

Of the Refraction *of the* Atmosphere, *and the* Crepusculum *or* Twilight.

WE find by Experience, that when Light enters a transparent Medium obliquely, either denser or rarer than that in which it was before, it does not pass strait on, but its Course is bent at the Point where it enters, which bending is call'd its *Refraction*; and if the Medium into which it enters be denser than the other, the Light is then refracted towards a Perpendicular to the Surface drawn through the Point where it enters; but if the Medium be rarer, it is then refracted or bent

the contrary way (*a*). It is from hence that a Staff, having one End under Water and the other above it, appears bent or broke at the Surface of the Water, the Part below the Water appearing above its natural Situation, because the Light that comes from thence is refracted or bent downwards at the Surface of the Water, where it comes out. Thus also the Stars near the Horizon appear above their true Places, on account of the Refraction which the Light that comes from them suffers in passing through the Atmosphere of the Earth (*b*). The Manner of which is now to be explained.

Let then AB (*Fig.* 22.) represent the Surface of the Earth, C its Center, DF a Portion of its Atmosphere, and GI the Sphere of the fixed Stars, and let AH be the Horizon of a Spectator at A, and let there be a fixed Star or a Planet at K; and let KL be a Ray of Light proceeding from hence. Now this Ray passing out of a Vacuum, or empty Space, into the Earth's Atmosphere at L, will be refracted towards the Line LC, that being a Perpendicular to the Surface of the Atmosphere, at the Point where the Ray enters. And since the superior Air is rarer than the inferior, the Ray is conti-

(*a*) See the Reason of this assigned, and the Manner of it largely explained, Part III. Chap. II.

(*b*) This has been already taken Notice of in Part III. Chap. VII. in the Notes.

continually entring a denſer Medium all the way it goes; and ſo is refracted every Moment downwards towards C, as at firſt, which will cauſe it to deſcribe a Curve, as LA, and to enter the Spectator's Eye at A, as if it came from E, a Point above L; and ſince an Object appears always in that Line in which the Ray paſſes when it enters the Eye *(c)*, the Star will ſeem to be at G, and therefore higher than its true Place, and ſometimes above the Horizon, when its true Place is below it; as in the Caſe before us, where its Situation K is ſuppoſed to be below the Horizontal Line AH.

Thus it has often happen'd, that in an Eclipſe of the Moon, at which Time it is directly oppoſite to the Sun, they have both appeared above the Horizon at the ſame Time.

When a Star is in the Zenith, it is liable to no Refraction; for then the Light that comes from it to a Spectator's Eye, enters the Atmoſphere in a Direction perpendicular to its Surface: But the lower the Star is, the more obliquely the Light enters the Atmoſphere; ſo that the greateſt Refraction of all is when the Star is in the Horizon, and it is then 33 Minutes and 45 Seconds; but when the Star is 50 Degrees or more above the Horizon, its Refraction is inſenſible *(d)*.

From

(c) See the Reaſon of this, Part III. Chap. VIII. at the Beginning.

(d) See Sir *Iſaac Newton*'s Table of Refractions. Philoſoph. Tranſact. Numb. 368.

From hence we see the Reason why the Sun and Moon, when near the Horizon, do not appear of a circular Form, as at other Times, but of an Oval one, having their longest Diameter parallel to the Horizon. For their lower Parts appear elevated by Refraction more than their upper ones, and consequently nearer together than they ought to do; while their Sides are equally elevated, and keep their just Distance from each other.

It is not with regard to Refraction, as in the Parallax: The latter of which, as explained in the preceding Chapter, depresses the Sun only and the Planets; and that according as they are more or less removed from the Earth; while the former elevates all the Heavenly Bodies alike, at whatever Distance they are removed from us.

Farther; were it not for the Atmosphere, which continually reflects the Light of the Sun from one Portion of it to another, and thereby occasions some of it to enter our Eyes, which way soever they are turned, and also throws it upon those Parts of the Bodies about us which are not exposed to the direct Light of the Sun's Beams, the whole Heavens would appear totally dark; and such Bodies, or such Parts of Bodies, as the Sun did not directly shine upon, would be invisible, and the Stars would appear at Noon-day. But the Atmosphere of the Earth being strongly illuminated by the Beams of the Sun,

Sun, they are reflected to us, and to other Bodies from all Parts; ſo that the whole Atmoſphere is enlightened, and every thing therein is render'd viſible; while the fainter Light of the Stars, which would otherwiſe appear, is obſcured and eclipſed thereby.

Beſides, though the Sun immediately before it ſets, would in this Caſe ſhine with the ſame Brightneſs as at Noon, yet the Moment it was ſet, it would be quite dark. Whereas, by means of the Atmoſphere it happens, that although none of the Sun's direct Rays can come to us after it is ſet, yet we ſtill enjoy its reflected Light for ſome Time, and Night approaches by Degrees. For after the Sun is hid from our Eyes, the upper Parts of the Atmoſphere remain for ſome time expoſed to its Rays, and from thence the whole is illuminated by Reflection. But as the Sun grows lower and lower, that Portion of the Atmoſphere which is above our Horizon becomes leſs enlighten'd, till the Sun has got eighteen Degrees below it, after which it ceaſes to be illuminated thereby (*e*), till it

(*e*) Hence it is, that during that Part of the Year in which the Sun is never eighteen Degrees below the Horizon, there is a continued Twilight from Sun-ſetting to Sun-riſing. Now that Part of the Year, in the Latitude of *London*, is while the Sun is paſſing from about the fifth Degree of *Gemini* to the twentieth of *Cancer*, that is, from about the 15th of *May* to about the 7th of *July*.

Some Philoſophers tell us, that the Height of the Earth's Atmoſphere may be determined from the Duration of the Twilight, in the following Manner. Let ABD (Fig. 23.) repreſent the Earth

it has got within as many Degrees of the Eastern Side of the Horizon, at which Time it begins

Earth surrounded by its Atmosphere GEF. Through A, the Place of a Spectator, draw the Tangent AH passing through the Extremity of the Atmosphere at E, from whence draw the Line ES, touching the Surface of the Earth in B, as also from the Center C, the Lines CA, CE and CB. Now, say they, since the Crepusculum is owing to the Reflection of the Sun's Rays, by the Particles of which the Atmosphere consists, it must end at such time as Rays of Light coming from the Sun, and passing close by the Surface of the Earth, are reflected from the Top of the Atmosphere to the Spectator's Eye; and therefore knowing how far the Sun is below the Horizon at that time, we may know the Height of the Atmosphere. Thus let S be the Place of the Sun when the Crepusculum ends, SE a Ray of Light passing close by the Earth at B, and reflected from E (a Particle at the Top of the Atmosphere) into the horizontal Line EA, to the Eye of the Spectator at A; then will the Angle HES be the Measure of the Sun's Distance below the Horizon. And because EB and EA are Tangents to the Earth, the Angle ACB at the Center will be equal to HES, or eighteen Degrees; and half of the former, namely ACE, will be nine, from whence EI, the Height of the Atmosphere, which is the Excess of the Secant EC above the Radius CA or CI, is had by Trigonometry: see *Keil*'s Astronomy, Lect. 20. But this way of arguing is founded on a Supposition, that Rays of Light cannot come to a Spectator's Eye after more than one Refraction in the Atmosphere; the contrary to which seems evident, as I shall shew in the following manner. Let ABD (Fig. 24.) represent the Earth, A the Place of a Spectator, HO his Horizon, MNR a Portion of the Earth's Atmosphere, and TVW an Arch of the Heavens; and let S be the Place of the Sun less than 18 Degrees below the Horizon; and let the Line SAI touching the Earth be drawn. It is well known that in this Case, so much of the Heavens as is not intercepted by the Earth from the Spectator's Eye at A, will still appear enlightened; that is, in other Words, Light will flow to the Spectator's Eye from every Part of that Portion of the Atmosphere that lies between P and Q; some therefore will come from those Particles that lie within the Space PAE. But these Particles cannot be enlightened by the direct Rays of the Sun; for they necessarily

gins to illuminate the Atmosphere again, and in Appearance to diffuse its Light throughout the Heavens, which continues to encrease till the Sun be up. This Light, whether in the Morning before Sun-rise, or in the Evening after Sun-set, is called the *Crepusculum*, or Twilight; and is supposed to begin and end when the least Stars that can be seen by the naked Eye (*viz.* those of the sixth Magnitude) cease or begin to appear.

But as the Twilight depends on the Quantity of Matter in the Atmosphere fit to reflect the Sun's Rays, and also on the Height of it, (for the higher the Atmosphere is, the longer it will be before the upper Parts of it will cease to be illuminated) the Duration of it will be various. For Instance in Winter, when the Air is condensed with Cold, and the Atmosphere upon that account lower, the Twilight will be shorter; and in Summer, when the Limits of the Atmosphere are extended by the Rarefaction and Dilatation of the Air of which it consists, the Duration of the Twilight will be greater. And for the like Reason the Morn-

cessarily pass above the Line AE, these therefore themselves are illuminated by reflected Light. Since then these afford Light, and nearly as much as any of the rest, it is evident, that Rays may come to a Spectator's Eye, and in great Plenty, after having suffered two Reflections. And therefore that Demonstration, which is built upon a Supposition that such Rays can suffer but one, must fall to the Ground.

ing Twilights, the Air being at that Time condenſed and contracted by the Cold of the preceding Night, will be ſhorter than the Evening ones, when the Air is more dilated and expanded.

Another Effect that the Atmoſphere has upon the Rays of the Sun is, that it intercepts a conſiderable Part of them in their Progreſs through it. By which Means it happens, that when the Sun is near the Horizon, it appears leſs luminous and bright than when it is more elevated; for the nearer the Sun is to the Horizon, the greater is the Portion of the Atmoſphere through which the Rays of the Sun muſt paſs to come to a Spectator's Eye. For Inſtance, in Fig. 23, where ABD repreſents the Earth, ſurrounded by its Atmoſphere GEF, and A the Place of a Spectator; it is obvious, that when the Sun is in or near the Horizon, ſuppoſe at H, its Rays muſt paſs through a greater Portion of the Atmoſphere to come to the Spectator's Eye at A, than they would do if the Sun were ſituated in any Part of the Heavens above the ſaid Point.

CHAP.

CHAP. XIV.

Of the Doctrine *of the* Sphere.

THE Subject of this Chapter relates chiefly to the Explanation of those Circles which Astronomers, for the better distinguishing the Places and Motions of the heavenly Bodies, conceive to be described within the Sphere of the Heavens. Some of these have already been defined; but that the Reader may have a compleat View of their Definitions all together, I shall repeat them here in their proper Places.

These Circles are of two Kinds, *viz.* the *greater* and the *smaller*.

A great Circle is that which divides the Sphere into two equal Parts, having the same Center therewith. A great Circle.

A less Circle divides the Sphere into unequal Parts, having a different Center from that of the Sphere. A less Circle.

These are sometimes called parallel Circles, and receive their Denominations from some great Circle to which they are parallel. Thus, lesser Circles Parallel Circles.

running parallel to the Equator, are called Parallels of the Equator; and so of the rest.

The Poles of a Circle.

The Poles of a Circle are two Points in the Surface of the Sphere, every where equally distant from the Circle.

Axis of a Circle.

The Axis of a Circle is an imaginary Line supposed to pass through both its Poles: Or, that passes through its Center, and is perpendicular to its Plane.

Secondaries of a Circle.

The Secondaries of a Circle are such as pass through both its Poles. These therefore cut it at right Angles in two Points opposite to each other, and are therefore always great Circles.

The Circles of the Sphere are also distinguished into fixed and moveable.

A fixed Circle.

A fixed Circle is that whose Situation depends not upon that of the Spectator.

A moveable Circle.

A moveable Circle is that whose Situation depends on that of the Spectator.

Of the former Sort are the Ecliptic and the Equator, with their Secondaries and Parallels.

Of the latter are the Horizon, with its Secondaries and Parallels.

I. Of

I. *Of the* Ecliptic, *and other Particulars relating to it.*

The Ecliptic is that Circle which the Plane of the Earth's Orbit would mark out, were it ſuppoſed to be extended to the Heavens. This therefore is that which the Sun ſeems to deſcribe in its annual Courſe. The Ecliptic.

The Axis of the Ecliptic is a right Line ſuppoſed to paſs through the Centre of the Sun, and to be perpendicular to the Plane of the Ecliptic; and the Points in the Heavens, to which this Axis points, are called the Poles thereof; and the great Circles paſſing through theſe Poles, will be perpendicular to the Ecliptic, and are therefore called its Secondaries, and ſometimes Circles of Longitude. By Means of theſe the Place of any Star is referred to the Ecliptic, and its Longitude and Latitude eſtimated. For the Longitude of a Star is an Arch of the Ecliptic, intercepted between the firſt Point of Aries, and that where one of theſe Secondaries that paſſes through the Star cuts the Ecliptic. And the Latitude of a Star The Axis of the Ecliptic. Its Poles. Secondaries of the Ecliptic. Longitude of a Star. Latitude of a Star.

a Star is the Arch of the ſame Secondary, intercepted between the Star and the Ecliptic. Upon this account all Circles conceived to be drawn parallel to the Ecliptic, that is, Parallels of the Ecliptic, are ſometimes called Parallels of Latitude. The Latitude of a Star is either Northern or Southern, as the Star is placed on this or that Side the Ecliptic.

Parallels of Latitude.

The Signs of the Ecliptic.

The Ecliptic is divided into twelve equal Parts, called Signs. They begin at that Point in which the Sun appears to be at the Vernal Equinox, and are reckoned from Weſt to Eaſt. The Names of the ſaid Signs, together with the Characters whereby they are commonly expreſſed, are theſe that follow:

Aries,	Taurus,	Gemini,	Cancer,	Leo,	Virgo,
♈	♉	♊	♋	♌	♍
Libra,	Scorpio,	Sagittarius,	Capricornus,	Aquarius,	Piſces.
♎	♏	♐	♑	♒	♓

The Zodiac.

The firſt ſix of theſe are called the Northern Signs, as poſſeſſing that Half of the Ecliptic which lies neareſt the North Pole of the Earth; the latter, the Southern, as being in that which is neareſt the South Pole. The Sun is always ſeen in ſome Part of this Circle; but

but the Planets, by reaſon of the Inclination of the Plane of their Orbits to that of the Ecliptic, appear ſometimes above it, and ſometimes below it, but never deviate from it above ſeven or eight Degrees on either Side: So that if we imagine a Space ſurrounding the Heavens ſixteen Degrees in Breadth, and extended equally above and below the Ecliptic, this will include all their Ecliptics, if I may ſo ſay; that is, all the Circles they would appear to move in when ſeen from the Sun, and is called the Zodiac.

The Signs of the Ecliptic took their Names, as was obſerved above, from Twelve Conſtellations ſituated in the Heavens near thoſe Places. But it muſt be remembred, that the Signs are different from the Conſtellations which bear the ſame Name. For the *Sign* Aries for Inſtance, is not in the ſame Part of the Heavens with the Conſtellation Aries. The Sign Aries is only thirty Degrees of the Ecliptic, counted from one of the Equinoctial Points; whereas the *Conſtellation* of that Name is a Syſtem of Stars, the moſt of which are now ſituated between the firſt and laſt Degree of Taurus. And ſo of the other

other Signs. These Constellations were in the Infancy of Astronomy situated within the Signs which now bear their Names; but by a slow retrograde Motion of the Equinoctial Points, called the *Precession of the Equinoxes*, as explained above (*a*), the Constellation Aries has got into the Sign Taurus, and that of Pisces into the Sign Aries; and so of the rest.

II. *Of the* Equator, *and other Particulars relating to that.*

The Axis of the Earth. The Axis of the Earth is a right Line supposed to pass through its Center, about which its daily Rotation is performed.

The Poles of the Earth. The Poles of the Earth are two Points in its Surface, where its Axis passes through; the one the North, the other the South Pole.

The Poles of the Heavens. The Poles of the Heavens are two Points therein, where the Axis of the Earth, if produced, would pass through them.

The Equator. The Equator is a great Circle on the Surface of the Earth, equally distant from either Pole thereof. This is by Mariners called the *Line*.

A

(*a*) See Chap. V.

Fig. 20. *P.*

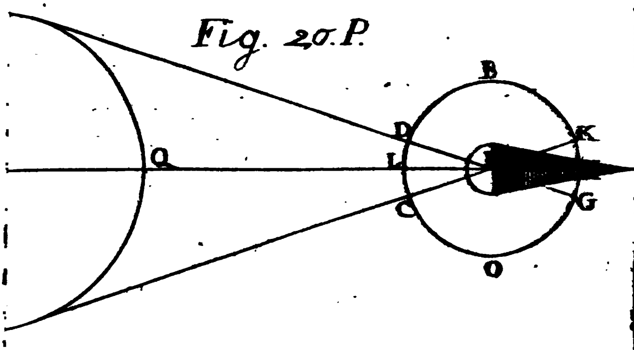
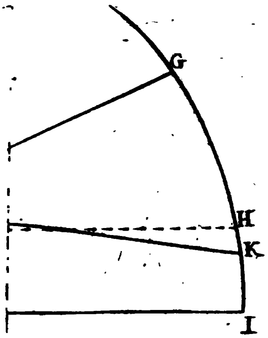
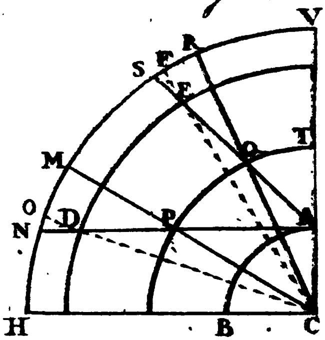


Fig. 21.





.*P.* 121.

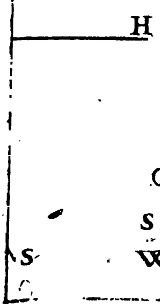
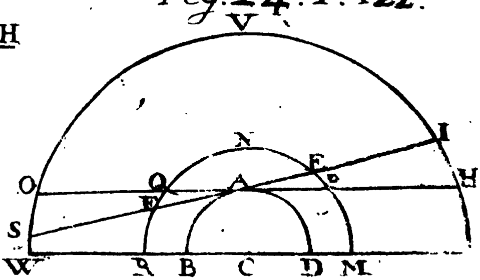


Fig. 24. *P.* 122.



A Secondary of the Equator passing through any particular Place upon the Surface of the Earth, is called the *Meridian of that Place*. Meridian.

And as the Longitude and Latitude of the heavenly Bodies are determined by the Ecliptic and its Secondaries; so by the Equator and its Secondaries, Places upon the Earth's Surface are distinguished as to their Longitude and Latitude.

The Longitude of a Place is then an Arch of the Equator, intercepted between the first Meridian (*b*), and that which passes through the Place. The Longitude of a Place.

S The

(*b*) By the *first Meridian* is meant that which passes through some remarkable and well-known Place, which Geographers pitch upon to estimate the Longitude of other Places from. The ancient Geographers called that the first Meridian which passed through the most Western Part of the then known World; and so reckoned the Longitude Eastward from thence. And as they look'd upon the World to be longer from West to East than from North to South, they gave the Name of Longitude to the longer Dimension, and that of Latitude to the shorter. But when it was discovered that the Earth was round, and consequently that there was no extream Point of Longitude, Geographers began to neglect this Method, and every one chose a first Meridian according to his Fancy and Inclination. Thus, an *English* Geographer shall tell you the first Meridian passes through *London*; a *Frenchman*, that it passes through *Paris*, &c.

This

The Latitude of a Place.

The Latitude of a Place is an Arch of the Meridian, intercepted between the Place and the Equator; and is either Northern or Southern, as the Place lies on this or that Side the Equator.

The Equinoctial.

The Equinoctial and the Equator are different, the former being a Circle in the Heavens, which the latter would mark out if supposed to be extended thither.

Northern and Southern Hemispheres.

The Equinoctial divides the Heavens as the Equator does the Earth, into two equal Portions, called the Northern and Southern Hemispheres.

Right Ascension and Declination.

And as by the Secondaries of the Equator, Places upon the Earth are referred to the Equator; so by these of the Equinoctial, the Places of the heavenly Bodies are referred to the Equinoctial. But as an Arch of the Equator, intercepted between the first Meridian and that which passes through any Place, is the Longitude of that Place; an Arch of the Equinoctial intercepted between the first Point of Aries and that where it is cut by a Secondary

This arbitrary Way of reckoning the Longitude from different Places, makes it necessary whenever Mention is made of the Longitude of any Place, to mention also the Place from whence that Longitude is counted.

dary of the Equinoctial that passes thro' the said Body, is called its Right Ascension. And an Arch of the same Secondary intercepted between the same Body and the Equinoctial, is called its Declination, which is also either Northern or Southern, as it happens to be in the one or the other Hemisphere. Declination.

From hence the Secondaries of the Equinoctial are sometimes called Circles of Ascension and Declination. Circles of Ascension and Declination.

The Equinoctial Points are those where the Equator cuts the Ecliptic; and are so called, because when the Sun is in these Points, it is also in the Equator, and therefore makes the Days and Nights of an equal Length all the World over. The Solstitial Points are those Points in the Ecliptic where the Equator is most distant from it; and are so called, because when the Sun is in them, it can scarce be perceived to alter its Distance from the Equator for some time. Equinoctial and Solstitial Points.

There are two of the Secondaries of the Equinoctial more remarkable than the rest, called the *Colures*; the one passes through the Equinoctial Points, and is called the Equinoctial Colure; the other is placed at right Angles with this, and passes through both the Poles

of the Equator and of the Ecliptic too, and is called the Solstitial Colure.

Hour Circles. Secondaries of the Equinoctial drawn through each 15th Degree thereof, will divide it into 24 Parts; each of which, since the Earth revolves once round its Axis in 24 Hours, will answer to one Hour; that is, each of them (supposing the Heavens to revolve as they appear to do) will succeed into the Place of a foregoing one in the Space of one Hour, and are for that Reason called Hour Circles. And the Inhabitants of any Part of the Earth, whose Meridian coincides with any one of these, will differ in their reckoning of Time one Hour from those whose Meridian is coincident with the next; and so on.

Tropics and Polar Circles. Of the Parallels of the Equator, four are more remarkable than the rest, and distinguished by particular Names, *viz.* the Tropics and Polar Circles.

The Tropics. The Tropics are those Circles which the Sun in its greatest Declination seems to describe by its diurnal Motion; and are therefore so far removed from the Equator, the one on the one Side, and the other on the other, as is the Inclination of the Equator to the Ecliptic, *viz.* 23 Degrees and an half. They are called Tropics, because when the Sun ar-

rives

rives at either of them, it returns back again towards the Equator.

That which lies in the Northern Hemisphere is called the Tropic of Cancer; because it touches the Ecliptic in the first Degree of that Sign. The other, the Tropic of Capricorn, for a like Reason. Tropic of Cancer, and Tropic of Capricorn.

The polar Circles are those which the Poles of the Ecliptic seem to describe by the diurnal Motion of the Earth; and are therefore each of them at the same Distance from the Poles of the Earth that the Tropics are from the Equator. That which lies in the northern Hemisphere is called the Arctic Circle, from *Arctus*, the Bear, a Constellation near that Place; the other being in Position opposite to this, is called the Antarctic Circle. For the same Reason the North Pole of the Earth is sometimes called the *Arctic Pole*, the other the *Antarctic*. The Polar Circles. Arctic and Antarctic Circles. Arctic and Antarctic Poles.

By means of the Tropic and Polar Circles (or rather by certain imaginary Circles upon the Surface of the Earth corresponding to them, and called by the same Names) the Surface of the Earth is divided into five Parts, called *Zones*. The one of which is called the torrid Zone, and is that Space or Tract Torrid Zone.

of the Earth which is comprehended between the two Tropics. The Ancients imagined this Tract of the Earth to be uninhabitable, on Account of the Heat of the Sun there. There are two frigid Zones, the one is that Portion of the Earth's Surface which is included within the arctic Circle, the other within the Antarctic. The remaining two are the temperate ones, lying one on one Side the Equator, and the other on the other, between the frigid and the torrid ones.

Frigid Zone.

Temperate Zones.

The Inhabitants of these Zones are distinguished by the different Direction of their Shadows. Thus, they that live in the torrid Zone are called Amphiscii; because their Noon Shadows are cast sometimes towards the North, and sometimes towards the South, according as the Sun at Noon is to the Northward or the Southward of them. But when the Sun passes directly over their Heads at Noon, they cast their Shadows neither the one way nor the other, and are called Ascii.

The Amphiscii.

The Ascii.

The Inhabitants of the temperate Zones are called Heteroscii, because they never cast their Meridian Shadow but one way. Such as inhabit the frigid Zones are called Periscii, because, by

The Heteroscii.

The Periscii.

by reaſon that the Sun is ſometimes above their Horizon for a Day or more without ſetting, their Shadows turn quite round them.

The Inhabitants of the Earth have alſo been diſtinguiſhed into three Sorts, with regard to their Situation with reſpect to each other. They who live under the oppoſite Points of the ſame Parallel of the Equator, are called with reſpect to each other, Periœci. Periœci.

Theſe have the ſame Seaſons of the Year, but it is Midnight to the one when it is Noon or Mid-day to the other. Others are called Antœci, as Antœci. living under the ſame Meridian, but in oppoſite Parallels. Theſe have Day and Night at the ſame Time, but different Seaſons, it being Summer with one when it is Winter with the other. Others are Antipodes, with reſpect to Antipodes. each other, that is, they live in oppoſite Parallels and Meridians too; ſo that they are on the oppoſite Points of the Globe; and therefore have both their Days and Nights, and their Seaſons of the Year reverſed, the one with reſpect to the other.

III. Of

III. *Of the* Horizon, *and other Particulars relating to it.*

The Horizen is a Circle in the Heavens, whose Plane touches the Surface of the Earth, in the Place of the Spectator. This Circle separates that Part of the Heavens which is visible from that which is not; and is called the sensible Horizon, in contradistinction to another Circle parallel to this, whose Plane passes through the Center of the Earth, and is called the rational Horizon. The Distances between these two Circles is so small with respect to that of the fixed Stars, that it is usually neglected, and the Circles are supposed to coincide.

The sensible Horizon.

The rational Horizon.

The Poles of the Horizon are two Points, the one of which is exactly over the Head of the Spectator, and is called the Zenith; the other directly under his Feet, and is called the Nadir.

Zenith.

Nadir.

The Secondaries of the Horizon pass through the Zenith and Nadir, and are called Vertical Circles or Azimuths: Its Parallels, Almicantaraths, or commonly Almicantars (*c*).

Vertical Circles, or Azimuths.

Among

(*c*) These are Terms introduced into Astronomy by the ancient *Arabian* Philosophers, and still retained.

Among the Vertical Circles of the Horizon there are two more remarkable ones, *viz.* the Meridian, and the prime Vertical, (for the Meridian of any Place is a Secondary of the Horizon of that Place, as well as of the Equator.) The Meridian, as observed above, passes through the Zenith and the Poles of the Equator. This cuts the Horizon in two Places, which are called the North and South Points thereof. The prime Vertical cuts the Meridian at right Angles in the Zenith and Nadir, and by intersecting the Horizon marks out the East and West Points; and these two Circles cut the Horizon into four Quarters, each of which is supposed to be divided into eight, the whole therefore into thirty-two, which are called the Points of the Compass, of which the four first mentioned, *viz.* East, West, North, and South, are called by way of Eminency, the Cardinal Points.

Meridian and prime Vertical.

Points of the Compass.

Cardinal Points.

The Altitude of a Star is an Arch of a Vertical Circle that passes through it, intercepted between the Star and the Horizon. If the Star be below the Horizon, it is then called its Depression.

Altitude and Depression of a Star.

When a Star is upon the Meridian of any Place, it is said to culminate, or to be in its Meridian Altitude.

Culmination of a Star.

T The

The Azimuth of a Star. The Azimuth of a Star is an Arch of the Horizon, intercepted between the North or South Point thereof, and the Vertical that passes through the Star; and is called Eastern or Western according to the Situation of the Star.

The Amplitude of a Star. The Amplitude of a Star is an Arch of the Horizon, intercepted between that Point where the Star rises or sets, and the East or West Point of the Horizon; and is either Northern or Southern. If the Amplitude be taken from the rising of the Star, it is then called its rising Amplitude; if when it sets, its setting Amplitude.

Rising and setting Amplitude.

The Mid-heaven. That Point of the Ecliptic through which the Meridian of any Place passes, is called the Mid-heaven. And that Point of the Ecliptic which is distant a quarter of a Circle from the Points where it cuts the Horizon, is called the *Ninetieth Degree*.

Ninetieth Degree.

The Elevation of the Pole. The Height of the Pole above the Horizon is called its Elevation. And this is equal to the Latitude of the Place; for either of them being added to the Distance between the Zenith and the Pole, makes just a quarter of a Circle (c).

The

(c) From hence arises the Method of measuring the Cir-

The Inhabitants of the Earth are again distinguished by the Sphere they are said to live in. Thus, such as live under the Equator, have their Horizon passing through both the Poles of the Earth, and cutting the Equator and all its Parallels at right Angles, and from thence are said to live in a right Sphere. The Property of which Sphere is this, that the Days and Nights are of an equal Length all the Year long, and the Stay which any of the heavenly Bodies make

Right Sphere.

T 2 above

Circumference of the Earth. For, let two Places be pitched upon, lying North and South of each other, such that the Elevation of the Pole in one shall exceed its Elevation in the other by one Degree suppose, then measure the Distance between the two Places in Miles: then as one Degree is to the Circumference of a Circle, or 360 Degrees, so is that Number of Miles to the Circumference of the Earth. By this Means the Circumference of the Earth is found such, that the Length of one Degree therein is equal to 69 *English* Miles. This Measure was first found out by our Countryman *Norwood*, and afterwards attempted by the *French* Mathematicians, who twice applied themselves to it by the King's Command, and discovered almost the same Measure that *Norwood* had done before. They took the Length of a paved Causey of near seven Miles, that lies between *Jews Village* and *Juvinum* in *Picardy*, by actual Mensuration; and then by a Survey from thence with the most accurate Instruments, they took the Distance between *Malcivin* and *Surdon*, which lies almost in a strait Line from North to South; in both which Places they took the Height of the Pole, and from thence computed the Circumference of the Globe. The Mathematicians set over this Work were Monsieur *Picard* the first time, and Monsieur *Cassini* the second.

above the Horizon is equal to that which they make below it. The Reason is, that all the Parallels of the Equator, in which the heavenly Bodies seem to perform their diurnal Revolution, are cut into two equal Parts by the Horizon.

Such as live between the Equator, and either Pole, have their Horizon crossing the Equator and all its Parallels, less or more obliquely, as they live less or more distant from the Equator. Hence those are said to live in an oblique Sphere, and have all the Parallels of the Equator cut into unequal Parts by their Horizon, so that neither the Sun nor any of the heavenly Bodies, unless they be in the Equator, make an equal Stay above and below their Horizon.

Oblique Sphere.

To those who live at the Poles of the Earth, if any such there be, the Equator is parallel to, or rather coincident with the Horizon, and all its Parallels are parallel to it; from whence this is called a Parallel Sphere. The Property of this Sphere is, that it is Day for half the Year together, and Night for the other half. For the Equator and Horizon being parallel, so long as the Sun continues on the same Side the Equator,

Parallel Sphere.

so

ſo long muſt its Stay be on the ſame Side the Horizon, and conſequently ſo long is it Day at one of the Poles, and Night at the other. And the fixed Stars do not riſe or ſet in this Sphere, becauſe they do not remove their Places from one Side of the Equator to the other, as the Sun and Planets are obſerved to do.

It is obvious, that in a right Sphere, the ſame Point of the Equator which comes to the Horizon with any Star, will come to the Meridian with the ſame; becauſe in this Sphere both the Horizon and the Meridian become Secondaries of the Equator. Now an Arch of the Equator intercepted between this Point and the firſt of Aries, is called the right Aſcenſion of a Star. But in an oblique Sphere it will not be ſo, but ſome other Point different from that which comes to the Meridian with it, will riſe with it, becauſe of the Obliquity of the Horizon to the Secondaries of the Equator. And the Diſtance between this Point and the firſt of Aries, is called the oblique Aſcenſion of a Star. And the Difference between right and oblique Aſcenſion is termed its Aſcenſional Difference.

Right Aſcenſion of a Star.

Oblique Aſcenſion.

Aſcenſional Difference.

In an oblique Sphere there is a Parallel of the Equator, so far distant from the Pole as is its Elevation above the Horizon, the Stars that are comprehended within which seem to revolve about the Pole without ever rising or setting. This is called the Circle of perpetual Apparition. There is another opposite to this, and at the same Distance from the other Pole, called the Circle of perpetual Occultation; because the Stars that are included therein never appear above the Horizon.

Circle of perpetual Apparition.

Circle of perpetual Occultation.

The ancient Geographers were wont to distinguish the different Regions of the Earth by Parallels and Climates, in such manner, that in going from one Parallel to another, the Days at the Summer Solstice should be encreased a quarter of an Hour. So that supposing the Equator to be the first Parallel, under which the Days are always 12 Hours long, the next should be where the longest Day of the Year should be 12 Hours and a Quarter long, *&c.* And two such Parallels made one Climate, till we come to the Polar Circles. And from thence to the Poles they reckoned the Climate such, that the Stay of the Sun above the Horizon was a whole Month

Parallels and Climates.

Month different in one from what it was in the next contiguous one.

As the Year of the Ancients was not precisely adapted to the annual Motion of the Sun, as we shall see hereafter, it happened that the Seasons did not always fall on the same Part of their Year, so that it was not accurate to denote the Time of the Year by the Day of the Month. It was therefore usual with them to distinguish the Seasons by the Cosmical, Achronical, and Heliacal rising and setting of the Stars. And because we meet with these Distinctions most frequently among the Poets, they are called the Poetical Risings and Settings of the Stars.

The poetical rising and setting of the Stars.

A Star is said to rise or set cosmically, when it rises or sets at Sun-rising; and when it rises or sets at Sun-setting, it is said to rise or set achronically. A Star rises heliacally, when it emerges out of the Sun's Rays, and is got so far from it as to become visible. And it is said to set heliacally when it first immerges into the Rays, and becomes invisible (d).

(d) *Kepler* will have it, that these Words are to be taken in another Sense, so as that to rise or set cosmically shall signify the same as to ascend above, or descend below the Horizon; but that to rise or set achronically, is the same as to rise or set in the Sun's Oppo-

Opposite, or in the other Achron, or Extreme of the Night: In which Sense *Ptolemy*, and to this Day most Astronomers say, a Planet is achronical, when it is opposite to the Sun, and shines all Night; so that to rise achronically is, as above, the same as to rise when the Sun is setting; but to set achronically, is to set when the Sun is rising; which is commonly called the Cosmical Setting.

A
COMPENDIOUS SYSTEM
OF
Natural Philoſophy:
With NOTES,
Containing the
MATHEMATICAL DEMONSTRATIONS,
And ſome Occaſional REMARKS.

In FOUR PARTS.

PART IV. Continued.
Containing the DESCRIPTION of the
ORRERY *and* GLOBES,
The PRINCIPLES of
CHRONOLOGY;
Together with the Phyſical Cauſe of the Motion of the
HEAVENLY BODIES.

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CHAP. XV.

The Description *and* Use *of the* Orrery *and the* Globes.

I. THE Orrery is a Machine designed to represent at one View the real Motions of the heavenly Bodies. In order to this, in the middle of a large Circle designed to represent the Ecliptic, is fixed a Globe representing the Sun. Next to the Sun a small Ball representing *Mercury*. Next to this is *Venus* represented by a larger. At a greater Distance still from the Sun, is the Earth represented by an Ivory Ball. Round the Earth, at some Distance from it, is a Ring to express the Orbit of the Moon, making an Angle with the Circle that represents the Ecliptic, and thereby shewing the Inclination they have to each other in the Heavens, and also the Line of the Nodes. Within this Ring is a small Ivory Ball representing the Moon, and having a black Cap or Case, which always covers that Hemisphere which is turned from the Sun; and thereby distinguisheth the enlightened Part from the dark one. Beyond these is seen *Mars*, then *Jupiter* and its four Moons, and outermost of all is *Saturn* with its Ring and five Moons.

All thefe are fixed upon fmall Stems which feverally reprefent their Axes, each of which has its peculiar and próper Inclination to the Plane of that Circle which we faid was to reprefent the Ecliptic: And when the Machine is put in Motion, all thefe Bodies move round that which reprefents the Sun, and at the fame time both that and all thofe which reprefent fuch of the Planets as have been obferved to have a Rotation about their Axes, turn round upon the faid Stems, and in their proper Times. The Moons alfo revolve about their Primaries at the fame Time, and the Ring that reprefents the Orbit of the Moon has likewife its proper Motion, whereby that of its Nodes is alfo expreffed.

Thefe Bodies are all kept in Motion by Wheelwork concealed in a Cafe underneath, and turned by a fmall Winch in the fide of the Cafe; one Turn anfwering to a Revolution of the Ball which reprefents the Earth, about its Axis; and confequently to 24 Hours of Time. So that 27 Turns caufes the Sun to turn once about its Axis, 365 carries the Earth about the Sun, 27 the Moon about the Earth, *&c.* So that not only the Motion of the Heavenly Bodies may be reprefented, but their Situations with refpect to each other may be fhewn for any Time either paft or to come; (*a*)

(*a*) This and the next Paragraph muft be underftood with fome Limitation, Machines of this kind being fufficiently adapted to common Ufe, though not accurate enough for this purpofe; none of the Machines of this kind which have yet been made, being fo contrived as to fhew this with Accuracy.

2 by

by giving the Winch a proper Number of Turns backwards or forwards as the Caſe requires.

There is uſually alſo a Lamp in a Caſe, with one or more convex Glaſſes in the ſide of it, to throw a ſtrong light directly forwards; this Lamp being put in the Place of the Sun, and that ſide of the Caſe which has the Glaſs in it directed towards the Earth, the Lamp being alſo contrived to turn according to the Motion of the Earth, a ſtrong Light is thereby continually thrown upon the Earth and the Moon, wherever they are, and ſo not only the Times in which the Eclipſes of the Sun and Moon, that is, properly of the Earth and Moon, will happen, are ſhewn, but they themſelves are duly repreſented. (*a*)

II. As

(*a*) Artificers generally erect upon the Ecliptic ſome Semicircles, to repreſent ſome of the principal Circles of the Heavens. But this is wrong, and tends to Confuſion; becauſe theſe Circles being only imaginary, and ariſing from the *apparent* Motions of the heavenly Bodies, ought to have no Place in the Orrery: Which as it expreſſes their *real* Motions, ſerves to *explain*, yet is by no means intended to *repreſent* their *Phænomena*; the Phænomena and real Motions being in ſeveral Caſes directly contrary to one another. To be particular, they have in ſome affixed the two Extremities of a Semicircle, which they call a moveable Horizon by two Hinges to the Ecliptic, at the firſt Degree of *Aries* and *Libra*; this is as if all Horizons neceſſarily paſſed through theſe Points, which is impoſſible, becauſe there are as many Horizons as Points upon the Earth. So that the putting on this Circle is not only an Impropriety, but is attended with a neceſſary Blunder. A learned Expoſitor of the Orrery ſays, 'when this Machine is ſet to any Latitude, (for 'they have a Contrivance for that too) ſet the moveable Ho- 'rizon to the ſame Degree upon the Meridian, and you may

 'form

II. As the Orrery is a Machine proper to represent the real Motions of the Heavenly Bodies, so the Globes are adapted to their apparent Motions: And that I may give the Reader a just and adequate *Idea* of them, I shall premise the following Description of a Machine, which though imaginary, will greatly tend to the true Conception both of their Nature and Uses.

In order to this, let us conceive a large hollow Sphere representing the inside of the Heavens, and on the inner Surface of this Sphere let the Sun, Moon, and Stars, be supposed to be delineated in their proper Places; and let the Sphere be put into a Frame, and hang there in such a Manner, as that it may be capable of turning about an Axis: And let it be so contrived, that either end of the Axis may be raised or lowered at Pleasure.

'form an Idea of the respective Altitudes, or Depressions of the 'Planets above or below the Horizon, according to their respective Positions, with regard to the Meridian.'

Now the Idea which the Motion of the Orrery in this Position gives me, is that the whole Earth is one half of the Year above the Horizon, and the other half below it. For the Ball which represents it in the Orrery is manifestly so, while it performs its Revolution about the Sun.

Again, some Artists, in order to make the Orrery represent the several Phases of the Moon, make one half of the Ball that represents her black; in which case they are obliged to contrive it so, that the same side of the Moon shall not be always turned towards the Earth; which is contrary to her real Motion, or that the dark side shall sometimes be turned towards the Sun, which is absurd.

In

In the midst of this Sphere, let there be a small Ball to represent the Earth, but so fixed there, as not to turn round with the Sphere, and let a Spectator be supposed to be placed on the uppermost Point of this Ball; and to prevent any Part of the lower half of the Sphere from being visible to him, let a Plane, that is, a thin broad Substance, a Plate of Brass for Instance, be supposed fixed on that Point of the Ball where the Spectator stands, and to reach on all sides as far as the inner Surface of the Sphere: This will represent the Horizon of that Spectator; because it separates the upper half of the Sphere, which we are to suppose visible to this Spectator, from the lower, which we must suppose to be out of his Sight. Then, as is most obvious, by turning this Sphere once round, while the Ball, the Spectator, and the aforesaid Plane remain unmoved, the rising and setting of the Sun, Moon, and Stars, by their passing by the Edges of this Plane, will be duly represented (*a*): And those Stars which do not go down so low as the Plane (as is evident if one end of the Axis of the Sphere be above it, some will not,) will be those which never set to a Spectator on that Part of the Earth, which corresponds with the

(*a*) The Reader is to suppose here, that in this case the Sphere is not hung in its Frame in such manner, that one Extremity of its Axis shall be directly over the Spectator; because in that case none of the Heavenly Bodies in turning the Sphere, will pass by the Edge of the Plane.

uppermost Point of the Ball; those which during the whole Revolution, do not rise so high as to come above it, will be those which never rise.

Again, if we suppose a Circle as large as the inside of the Sphere will admit of, to be placed within it, in such manner as to have the Axis of the Sphere passing through two opposite Points of it, and also to have one Point of it directly over the Head of the Spectator, and the opposite one directly under his Feet, and to remain unmoved while the Sphere turns: This will represent the Meridian of that Spectator; and so the Culminations of the Heavenly Bodies, that is, their passing the Meridian in the Heavens, will be exhibited by their passing by this Circle.

Farther, if we also imagine an Index like the Hand of a Clock, to be fixed upon one of the Extremities of the Axis of the Sphere, in such manner as to turn with the Sphere, and to point continually to a Circle fixed on the Frame, with the 24 Hours delineated upon it; and if this Index be set to point to the Hour 12, when the Sun's Place upon the Sphere for any given Day, is at the Circle that represents the Meridian, then by turning the Sphere this way or that, till the Place of the Sun or any of the Stars comes to the Plane abovementioned, the Index moving along with the Sphere, will point to the Hour at which they rise or set that Day: And if we observe to what Hour

Hour the Index points, when any of them crosses the Meridian within the Sphere, the Time of their Culmination, or Southing, will be expressed: And so by this Machine all Problems relating to the rising, setting, and southing of the Sun, Moon, and Stars, may be solved. The Reason why the Index must be set to point to the Hour 12, when the Place of the Sun is brought to the Meridian, is that it may point to that Hour, which the real Sun indicates, when the Heavens stand in a Position corresponding with that of our imaginary Sphere.

But since by means of the annual Motion of the Earth round the Sun, the Sun continually shifts its apparent Place in the Heavens in such manner, as to seem to describe a great Circle once a Year; it is evident that its Place cannot be delineated on the inside of this Sphere as we have supposed, unless it be for a Day or a few Hours, during which Time its change of Place is so small that it may be neglected. In like manner the Planets are continually changing their apparent Places in the Heavens, by means of their Motions in their respective Orbits; neither therefore can their Places be assigned them, unless for a very small interval of Time. Let us then imagine both the Sun and the Planets to be represented by little Balls adhering to the inner Surface of the said Sphere; but let it be so contrived, that each Ball shall shift its Place with every Turn of the Sphere,

juft fo much, and in the fame Direction that the Sun or other heavenly Body the Ball reprefents, appears to fhift its Place in the Heavens in one Day: And let the Index fhift with the Sun, fo that it may always point to 12, when the Place of the Sun comes to the Meridian.

Then, as is moft obvious, will our Machine be fo adapted, that upon turning it round continually, the Change of the Time of the rifing and fetting of the Sun, its different Meridian Altitude, the viciffitude and length of Day and Night, the alteration of the Time of the rifing and fetting of the fixed Stars and of the Planets, for fo many Days running, as is the Number of Turns given to the Globe, will all be exhibited to View: And confequently by turning the Sphere 365 Times round, the Succeffion of the Seafons will be reprefented throughout the Year with abundance more, or rather will all the remaining Phænomena of the Heavens.

This Machine, as hitherto defcribed, ferves only to exhibit the Phænomena of the Heavens as they appear on one Point of the Earth's Surface, *viz.* that which is reprefented by the uppermoft Point of the Ball, or that where the Spectator was fuppofed to ftand. But if we imagine the Plane which was put as a Boundary to the Spectator's Sight, to be applied to fome other Point of the Ball and fixed there, then the Motion of the Sphere will exhibit the Phænomena, as they would appear to a

Spectator

Spectator on that Part of the Earth, and so for the rest; or which comes to the same thing, if while the Plane remains unmoved, one of the Poles of the Sphere be elevated above it as many Degrees as the corresponding Pole in the Heavens is above the Horizon of a Spectator, on any Part of the Earth; then will the Revolution of the Sphere represent the Phænomena as they appear to that Spectator. For it is the same Thing whether the Pole be lifted up from the Horizon, or the Horizon put down below the Pole. (*a*)

Now this Machine (except in some few Particulars, in which it vastly excels the celestial Globe) is exactly the same with it as to its Uses. Thus, the Globe turns upon an Axis, either End of which may be raised or depressed at Pleasure; and on one of them is an Index that turns with the Globe, and points to a fixed Circle that has the Hours upon it. There is no Ball is the middle, as we supposed in the Machine, but the Place of the Spectator is al-

(*a*) By means of an Orrery to represent the real Motions of the heavenly Bodies, and such a Machine as above described, to shew afterwards how the apparent Motions of those Bodies depend on, and are connected with their real ones, the Elements of Astronomy might be taught much better than by any thing that has as yet been contrived; or rather, if I may so say, the Science itself might thereby be exhibited to View. For the Orrery, though it admirably well illustrates some Things, particularly the Eclipses, and in general what relates to the Motions of the Planets about the Sun, yet it shews the Appearances or Phænomena arising from thence to a Spectator, supposed to be at rest upon the Surface of the Earth, but in a forced and unnatural Way.

ways ſuppoſed to be there; and inſtead of the Plane we ſuppoſed to touch that Part of the Ball on which the Spectator ſtands, and to be extended every way as far as the inner Surface of the Sphere, there is a broad wooden Circle on the outſide of the Globe, (which is Part of the Frame of it) exactly where that Plane would paſs, was it continued beyond the Surface of the Globe, and ſo anſwers the Uſes of that Plane. In the next Place, inſtead of the Circle we ſuppoſed fixed within the Sphere and paſſing through the *Zenith* and *Nadir* of the Spectator, and through which the Axis of the Sphere paſſes, there is a brazen Circle on the outſide of the Globe, and in which it ſwings. And inſtead of the Stars being on the inner Surface, they are here on the outer one, and the Conſtellations are delineated in their proper Forms. Then as to the Places of the Sun and Planets, Artificers are content to leave them quite out, and only draw a great Circle repreſenting the Path of the Sun in its annual Courſe through the Heavens, *viz.* the Ecliptic, and diſtinguiſh it into 12 Parts by the Signs thereof, and each Sign into 30 leſs, which are the Degrees thereof. When therefore a Problem is to be determined by the Globe, requiring that the Sun ſhould be delineated in its proper Place, you are to look out the Place of the Sun for the given Time in ſome aſtronomical Tables, Ephemeris, or Almanack for that Purpoſe, or on the broad Surface of the

Horizon

Horizon of the Globe, where there is usually a Table shewing the Sun's Place in the Ecliptic for every Day of the Year, and then to mark its Place on the Line which we said represented the Path of the Sun upon the Globe. In like manner, if a Problem is to be solved requiring the Place of any of the Planets, the Place of the Planet for the Time given must be looked out in some Ephemeris or Almanack, of which there are many published yearly by *Parker*, and others, for that Purpose, containing Tables expressing the Longitude and Latitude of each Planet for every Day of the Year; and marking the Place of the Planet upon the Globe, by sticking on a Patch, or the like, and setting the Index according to the Place of the Sun for the given Time, and turning the Globe about, the Index will shew the Time of that Planet's coming to either Side of the Horizon, to the Meridian, or to any Height above or below the Horizon, *&c.*

If then the Reader will form to himself a true Idea of the Machine I have described, and of the manner in which such a Machine is capable of representing the apparent Motions of the heavenly Bodies, he cannot fail of having a thorough Notion of the Nature of the celestial Globe; the main Difference between them being only this, *viz.* that what we have supposed in the one to be on the Inside, is either actually in the other on the Outside, or

to

to be fixed on, or to have its Place marked there, as Occaſion requires. The uſes both of the celeſtial and terreſtrial Globe, as to particular Caſes, will be further illuſtrated by the Inſtances at the End of this Chapter, after we have conſidered ſome other Particulars relating to each.

The Circles delineated on the celeſtial Globe are uſually, beſides the Ecliptic abovementioned, the Equinoctial Circle at an equal Diſtance from either Pole, and two Parallels of it, *viz.* one on either ſide, to repreſent the apparent diurnal Motion of the Sun, when at its greateſt Diſtance from the Equator, called the Tropics; and at the like Diſtance from either Pole, two more Parallels called the polar Circles. Both the Ecliptic and the Equinoctial are divided into Degrees, beginning at the Point that repreſents the vernal Equinox, which is one of thoſe where they croſs each other; the Equinoctial into 360, but the Ecliptic into 12 Signs, diſtinguiſhed by their proper Characters, every one of which is ſubdivided into 30 Degrees (*a*). There are alſo ſix Secondaries of the Equator, and as many

(*a*) As to the Diviſions of the Ecliptic, it is to be obſerved, that the Degrees belonging to each Sign are not to be found where the Conſtellation whoſe Name it bears is delineated, but where the Character peculiar to that Sign is placed. The Reaſon of which is, becauſe the firſt Degree of *Aries*, from whence all the reſt are counted, being where the Equinoctial cuts the Equator, is not in this Age in any Part of the Conſtellation that bears that Name, as was accounted for above.

of

of the Ecliptic cutting it in the first Degree of each Sign; two of which are also Secondaries of the Equator, *viz.* the Equinoctial and Solstitial Colures. Further, the Globe is so hung in its Frame, that that Side only of the brazen Meridian which has the Degrees marked upon it, passes through the Axis of it, which Side only, in all Cases where that Meridian is considered, is therefore supposed to represent it. Upon the broad Surface of the wooden Horizon, besides the innermost Circle divided into its proper Degrees, there is a Table shewing what Degree of the Ecliptic the Sun is in for every Day of the Year, as was observed above; in order to which the 12 Signs of the Ecliptic, with the Degrees of each, and the 12 Calendar Months, are so placed over-against one another, both according to the *Julian* and *Gregorian* Account, that is, according to the old and new Stile, as that the Days of the one may duly answer to the corresponding Places of the Sun upon the other. There is also a Circle, representing the Winds or Points of the Compass.

There is also belonging to the Globe a *Quadrant of Altitude*. It is a long and narrow Plate of Brass, made so thin and pliant, that the whole Length of it may be applied close to the Surface of the Globe, and having the Degrees upon it, (*a*) serves to measure the Distances

(*a*) That Edge which has the Degrees nearest it, is called its *graduated Edge*, and is the only part of it that is considered in the using it.

of

of Places on the Globes; or being fixed on to the uppermoſt Point of the Meridian of the celeſtial Globe, (for which Purpoſe it has a Screw at one End) and having the other End thruſt in between the Horizon and the Globe, meaſures the Height of a Star, or other Heavenly Body above the Horizon, *&c.* From which latter Uſe it has its Name.

The terreſtrial Globe differs from the celeſtial principally, in that inſtead of the Conſtellations, the ſeveral Countries of the Earth are delineated thereon; and the Circles on this Globe are uſually the Ecliptic, and the Equator, on which are numbered the Degrees of Longitude (*a*), the two Tropics, the two polar Circles, Secondaries of the Equator cutting it at every tenth Degree, and Parallels alſo of it at every tenth Degree on each Side. There are alſo drawn upon it Rhumb Lines, or thoſe which a Ship, keeping the ſame Courſe, that is, ſteering continually in the ſame Point of the Compaſs, deſcribes upon the Surface of the Sea. The Property of a Line of this kind, is that it cuts all the Meridians it paſſes through, under the ſame Angle, and therefore is a kind of Spiral continually approaching towards ſome Point. This is called a *Loxodromic* Curve.

(*a*) On ſome Globes, the Degrees of Longitude are numbered both ways from the firſt Meridian, till you come to the oppoſite Point, and the Longitude is accordingly diſtinguiſhed into Eaſt and Weſt.

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The Lines of Variation described upon the terrestrial Globe, shew the Direction of the magnetic Needle, in those Places in which it does not point directly towards the North. For it is observed, that the Needle in some Parts of the World declines from the North to the East, and in others to the West.

Further, on the Pedestal of some Globes there is a Mariner's Compass fixed, by the help of which, the brazen Meridian of the Globe may be set North and South. (*a*)

After this Description of the Globes, and general Idea of the Use thereof, it may now be proper to exemplify the latter by the following Instances.

And first of the celestial Globe.

Problem I. *To rectify the* Globe *to the Latitude of any Place upon the Surface of the Earth.*

If the Place be in the Northern Hemisphere of the Earth, slide the Meridian within the

(*a*) The Armillary Sphere comes the nearest of any we have to that imaginary Machine described above, for it is hollow, and has a Ball in the middle to represent the Earth; but then in all other Respects it is like a Globe, except that the Surface is cut away every where, (unless where a Circle is described) that you may see the Ball in the middle. There is one Circle left broad enough to represent the Zodiac, on which the Places of the Sun and Planets may be marked, as Occasion requires.

Notches

Notches of the Horizon, till there be as many Degrees of the Meridian between the North Pole and the Horizon, as are equal to the Latitude of the Place (*a*). If the Place be in the Southern Hemisphere, the Southern Pole must be raised to a like Height above the Horizon.

Prob. II. *To find what* Stars *never rise or never set in any* Place *or* Latitude *given.*

The Globe being rectified to the given Latitude of the Place by Problem the first, those Stars which pass not under the wooden Horizon of the Globe, during its whole Revolution, never set to a Spectator in that Latitude: And such as do not come up above it, never ascend above that of the Spectator, and so never are seen to rise.

(*a*) The Reason of this is, because to a Spectator, upon any Part of the Earth, the Elevation of the Pole is equal to the Latitude of the Place; and the Globe is said to be *rectified* by this Operation, because, by this means, that Point of the little Ball, (for such we must always suppose to be in the middle of the Globe, to represent the Earth) that corresponds to the Place of the Spectator upon the Earth, is brought to the uppermost Point thereof; and so the Horizon thereof becomes coincident with the Plane of the wooden Horizon that surrounds the Globe, and the Poles of the Globe having the like Situation with respect to the uppermost Point of the Ball, that the Poles of the World have with respect to the Place of the Spectator; it is evident that the Revolution of the Globe is fitly disposed to exhibit that of the Heavens, as it would appear to that Spectator.

PROB. III. *To find the* Sun's *Place in the Ecliptic, for any Day given.*

If the Day given be according to the *Julian* Account, or old Stile, then in the wooden Horizon of the Globe, in the *Julian* Kalendar, otherwise in the *Gregorian*, find the Day given, over-againſt which in a Circle, which contains the ſeveral Signs and Degrees of the Ecliptic, is that Degree in which the Sun is on that Day. Though it may be found more accurately by an Ephemeris calculated for that Year. Then in the Ecliptic, delineated on the Surface of the Globe, look for the ſame Degree, and that is the Place of the Sun upon the Globe for that Day.

PROB. IV. *The Latitude of a Place being given, to find the Time of the* Sun's *riſing and ſetting, and conſequently the Length of Day and Night in that Place.*

Having rectified the Globe according to the Latitude, (by *Prob.* 1.) and found the Sun's Place in the Ecliptic (by *Prob.* 3.) bring the latter to the graduated Edge of the brazen Meridian, and holding the Globe in that Poſition,

set the Hour Index to 12 at Noon; (*a*) then turn the Globe till the Sun's Place in the Ecliptic comes to the eastern Part of the Horizon, and the Index will point to the Hour at which the Sun rises; and if you turn the Globe till the Sun's Place cuts the western Side of the Horizon, the Index will shew the Time of its setting.

PROB. V. *To find the Hour of the Day, by the Height of the* Sun.

First, by the Help of a Quadrant or some other Instrument for that Purpose, observe how many Degrees the Sun is elevated above the Horizon. Then the Globe being rectified to the Latitude of the Place, and the Sun's Place in the Ecliptic found, and the Hour Index duly placed, that is, so as to point to 12 at Noon, when the Sun's Place is under the Meridian; and screwing the Quadrant of Altitude on to the Meridian at the Zenith, *(i. e.)* at so many Degrees from the Equator, as is the Latitude of the Plate. Turn the Quadrant of Altitude and the Globe backwards and for-

(*a*) Because when the Place of the Sun is in the Meridian, the Globe represents the Position of the Heavens, in respect of the given Place, as they are at Noon, or 12 o'Clock of the Day given. And consequently, if the Hour Index be put to 12, when the Globe is in this Position, as it turns with the Globe, it will point to the Hour that corresponds with any other Position of the Globe that Day.

wards,

wards, till the Sun's Place in the Ecliptic lies under that Degree on the graduated Edge of the Quadrant of Altitude, which answers to the Sun's Height found as above, then will the Hour Index point out the Time of the Day. (*a*)

Prob. VI. *To find the Hour of the Night, by the Height of a* Star.

The Height of the Star being found by an Instrument, the Globe rectified, the Quadrant of Altitude fixed, and the Hour Index duly placed, as in the foregoing Problem, move the Globe and the Quadrant, till the Star comes under its graduated Edge, at that Degree which expresses the Height of the Star above the Horizon, then will the Hour Index point to the Hour of the Night.

Prob. VII. *To find the Place of a* Star *or* Planet *upon the* Globe, *its Longitude and Latitude being given.*

Lay the first Degree of the Quadrant of Altitude, upon that Degree of the Ecliptic, which expresses the given Longitude, and the ninetieth Degree thereof on the Pole of the

(*a*) Because when the Sun's Place is brought to this Point of the Globe, it is so many Degrees above the Horizon, as the Sun itself is above that of the Spectator.

Ecliptic, (that is, where its Secondaries cross each other upon the Globe) either Northern or Southern, as the given Latitude is North or South: Then look upon the Quadrant for the Degree of Latitude given, and the Point of the Globe which is under that, is the Place of the Star or Planet sought. (*a*)

PROB. VIII. *To represent the Appearance of the* Heavens *at any Time of the Night; that is, to shew the Situation of the fixed* Stars *at that Time.*

Rectify the Globe, and set the Hour Index to the Sun's Place in the Ecliptic, and turn the Globe till the Index points to the Hour of the Night. (*b*)

PROB.

(*a*) The Reason of this is, because the Quadrant of Altitude in this Situation, represents a Secondary of the Ecliptic, an Arch of which intersected between the Star and the Ecliptic is its Latitude; and an Arch of the Ecliptic between the Point where it is intersected by a Secondary passing through the Star, and the first Degree of *Aries*, is its Longitude.

(*b*) For then the Situation of the Stars upon the Globe, with respect to the Horizon and Meridian, will be similar to that of the Stars in the Heavens; *viz.* those which appear on the Eastern Side the Globe in the Horizon will then be rising, and those in the Horizon at the other Side will be setting, those which are under the Meridian will then be culminating, &c.

And if by means of a Compass upon the Pedestal of the Globe, you set the brazen Meridian North and South, so that the South Part

Prob. IX. *The Day of the Year and the Latitude of the Place being given, to find the Beginning of the Morning, and End of the Evening* Twilight.

The Globe being rectified to the Latitude of the Place, and the Hour Index set to the Place of the Sun in the Ecliptic, elevate that Degree of the Ecliptic which is diametrically opposite to the Sun's Place, 18 Degrees above the West Side of the Horizon (*a*), and the Hour Index will shew the Time of Day-break or the Beginning of Morning Twilight. And

Part of it shall respect the South Part of the Heavens, the Situation of the Stars upon the Globe, with respect to the Points of the Compass, will also be similar to that of the Stars in the Heavens at that Time; so that by comparing the Situation of the Stars upon the Globe, with those in the Heavens, you may easily make yourself acquainted with all the Stars, that are visible in this Part of the World.

If the Situation of the Moon or the other Planets is also to be represented, their Places in the Zodiac for that Time must be assigned them by Problem the last, their Longitude and Latitude being first found by an Ephemeris for that Purpose.

(*a*) Because then the Place of the Sun will be so many Degrees below on the other Side, in which Situation of the Sun, we observ'd above, that the Twilight begins. The Quadrant of Altitude will shew when the Point opposite to that of the Sun is 18 Degrees above the Horizon. If this be tried for any Time while the Sun is passing from about the 5th Degree of *Gemini*, to the 20th of *Cancer*, that is, from about the 15th of *May* to about the 7th of *July*, it will be found that there is no Beginning or End of Twilight, the Sun never being 18 Degrees below the Horizon (in the Latitude of *London*) during that Time.

if the same Point of the Ecliptic be raised 18 Degrees above the East-Side of the Horizon, the Index will point out the Time when the Evening Twilight ends.

PROB. X. *To account for the* Phænomena *of the* Harvest Moon.

About the autumnal Equinox, when the Moon is at or near the *Full*, it rises almost at the same Hour for several Nights together; which Phænomenon is called the *Harvest Moon* (*a*). In order therefore to shew by the Globe how this comes about, let us suppose the Place of the Sun to be in the first Degree of *Libra*, as it always is at that Equinox, and that of the Moon to be in the Point opposite to the Sun, or the first Degree of *Aries*, because the Moon must be considered as being at or near the *Full*. In this Case it is obvious that the Moon will rise when the Sun sets, which will be at Six of the Clock; because both Luminaries are in the Equator and at opposite Points. Now since the Moon performs an entire Revolution in its Orbit, (which we will at present consider as coincident with the Ecliptic) in the Space of one Month, it therefore advances forwards 12 Degrees each Day, and consequently

(*a*) Mr. *Johnson* calls this Phænomenon *Luna autumnalis*, and is the only Author that I know of that has accounted for it. Quæstiones Philosoph. Edit. 2da. Page 124.

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the next Night it will be in the 12th Degree of *Aries*. Let us then enquire at what Time the Moon will rise this Night. To this End, rectify the Globe to the Latitude of the Place, bring the Sun's Place, which is now the second Degree of *Libra*, to the brazen Meridian; and put the Hour Index to 12; and if the Globe be turned till the Place of the Moon, *viz.* the 12th Degree of *Aries*, cuts the eastern Side of the Horizon, we shall find the Hour Index will point nearly to the Hour of Six, which, as observ'd above, was the Time of the Moon's rising the Evening before. And thus if we investigate the Time of the Moon's rising for several Nights together, both before and after the Full, it will be found much the same in all.

The Reason of this is, that about the Time of the full Moons, which happen at this Part of the Year, the Moon being in the ascending Signs of the *Zodiac*, appears in its nocturnal Course to describe a Parallel of the Equator one Night, much nearer the North Pole than another, and so rises every Night more and more to the Northern Parts of the Horizon, and that considerably, as passing quite from the Southern to the Northern Tropic in a Fortnight's Time. Now it is easy to conceive that the nearer any of the Heavenly Bodies is to the North Pole, the sooner that Body (*cæteris paribus*) ascends the Horizon: Thus, if the Body be within a little of the *Circle of perpetual Apparition*, that Body is no sooner set but it rises again. In every Lunation there-

fore, while the Moon is passing from the Southern to the Northern Tropic, some Minutes are on that Account to be deducted from those by which it would rise later and later each Night, did the Plane of its Orbit coincide with that of the Equinoctial; and as much is to be added while she passes from the Northern to the Southern Tropic. And for the like Reason in the former Case something is to be added to the Time it would set at; and in the latter a like Quantity is to be deducted.

Now about the Time of the autumnal Equinox, the Sun being then in *Libra* or thereabouts, the Moon is in the midst of the ascending Signs, when at the *Full*. Hence it is that the *full* Moons at that Time of the Year, rise so little later the one than the other.

In like manner, if any one thinks it worth while to observe it, he will find that the *new* Moons in the Spring, rise nearly at the same Hour for several Nights successively, while the full Moons shall rise one later than another, by a greater Difference than at any other time of the Year. The Reason is, because at the time of the new Moons, which happen at that Part of the Year, the Moon is in the ascending Signs, but at the full, in the descending ones. (*a*)

(*a*) It is here to be observed, that this Phænomenon is not the same as to Degree, in different Years; because as the lunar Orbit is inclined to the Ecliptic 5 Degrees, and the Line of its Nodes is continually revolving round, the Inclination of that Orbit to the Equator will be sometimes 10 Degrees greater than at others. So that the Moon does not hasten to the North, or descend to the South in each Revolution with equal Pace.

PROB.

Prob. XI. *To determine the Time of the Year in which a Star riſes or ſets* Coſmically, *or* Acronically.

Having rectified the Globe to the Latitude of the Place, bring the Star to the Horizon on the Eaſtern Side of the Globe, and obſerve what Degree of the Ecliptic riſes with it. Look for that Degree in the wooden Horizon, and right againſt it in the Kalendar, you will find the Month and Day when that Star riſes coſmically (*a*). And if you bring the Star to the Weſtern Side of the Horizon, the Degree of the Ecliptic which cuts the Eaſtern Side of the Horizon in that Situation of the Globe, will give the Day of the Month when the ſaid Star ſets coſmically (*b*). So likewiſe againſt the Degree which ſets with the Star, you will find the Day of the Month of its *Acronical* ſetting; and if you bring it to the Eaſtern Part of the Horizon, the Degree of the Ecliptic which then cuts the Weſtern Part of the Horizon will be the Sun's Place when the Star riſes *Acronically*.

(*a*) For that will be the Day when the Sun and the Star riſe at the ſame time, which is its coſmical Riſing.

(*b*) The coſmical ſetting of a Star being its ſetting at Sun-riſe.

PROB.

PROB. XII. *To find the Time of the* Heliacal *rifing and fetting of a* Star.

Having rectified the Globe to the Latitude of the Place, bring the Star to the Eaftern Side of the Horizon, and apply the Quadrant of Altitude to the Weftern Side in fuch manner that its twelfth Degree may cut the Ecliptic, if the Star be of the firft Magnitude (*a*). Then will the Point of the Ecliptic oppofite to that which is cut by the Quadrant, be 12 Degrees below the Horizon. Look for this Degree on the wooden Horizon, and over-againft it you will have the Day of the Year, when the Star rifes Heliacally. To find the Heliacal fetting, bring the Star to the Weftern Side of the Horizon, and turn the Quadrant of Altitude about to the Eaftern Side, till the 12th Degree of it cuts the Ecliptic; then that Degree of the Ecliptic which is oppofite to this Point, is the Sun's Place at that Time of the Year when the Star fets Heliacally.

(*a*) Becaufe a Star of the firft Magnitude being above the Horizon, may be feen when the Sun is 12 Degrees below it. If the Star be of the fecond Magnitude, that Degree of the Ecliptic muft be taken, which is 13 Degrees below the Horizon when the Star rifes; if of the third, then the 14th, &c.

PROB.

PROB. XIII. *The Day of the Month, the Time of the Day, and the Latitude of the Place being given, to find thereby the Hour of the Day, according to the* Babylonic, Italic, *and the* Judaical *Way of reckoning.*

1. To find the *Babylonic* Hour (which is reckoned from the Time of the Rising of the Sun) having found the Time of the Sun's rising in the given Latitude, the Number of Hours between that and the Time given, is the Hour of the Day according to the *Babylonic* Way of reckoning.

2. To find the *Italic* Hour (which is reckoned from Sun setting) first find the Time at which the Sun sets on the given Day. The Time elapsed from that to the Time given, will give the *Italic* Hour sought.

3. To find the Hour of the Day according to the *Jewish* Way of reckoning (one of which Hours is a twelfth Part of the Time that the Sun continues above the Horizon, and they are reckoned from Sun rising,) first find the Hour the Sun sets at, this being doubled will give the Number of Hours the Sun continues above the Horizon on the Day given. Then as that Number of Hours is to 12, so is the Number of

of Hours ſince Sun riſing, to the *Judaical* Hour required. To find the Hour of the Night, double the Hour the Sun riſes at, and ſay, as that Number is to 12, ſo is the Number of Hours ſince Sun ſetting to the Hour required.

Thus in the Latitude of *London*, when the Sun riſes at 4 of the Clock and ſets at 8, if the Hour given be 5 in the Evening, the *Babylonic* Hour will be the 13th, the *Italic* the 21ſt, and the *Jewiſh* Hour will be Nine and three Quarters.

Prob. XIV. *To find the Place of any of the Heavenly Bodies upon the* Globe, *from their right Aſcenſion and Declination.*

Bring that Point of the Equator which expreſſes the right Aſcenſion given, to the brazen Meridian, and look upon the Meridian towards the North or South Pole, according as the Declination is North or South, for the Degree of Declination given, under which is the Place ſought (*a*).

(*a*) For in this Caſe the Meridian is made to repreſent a Secondary of the Equator paſſing through the Place, an Arch of which lying between the Place and the Equator is the Declination, and the Diſtance between the Place of the Equator through which it paſſes, and the firſt Degree of *Aries*, is the right Aſcenſion.

Of the Terreſtrial Globe.

Prob. XV. *From the Longitude and Latitude of a Place, upon the Surface of the Earth being given, to find the Place upon the* Globe.

This Problem is much the ſame with the laſt, only inſtead of the celeſtial, we uſe the terreſtrial Globe; and inſtead of reckoning the Longitude from the firſt Degree of *Aries*, the Point from which right Aſcenſion is reckoned, it muſt be counted from the firſt Meridian.

Bring the Degree of the Equator, which numbers that of the Longitude given, to the brazen Meridian, upon which count towards the North or South Pole, according as the Latitude is North or South, till you come to the Number of the Latitude, under which is the Place ſought. (*a*)

(*a*) In ſome Globes the Degrees of Longitude are numbered both Ways from the firſt Meridian, till they meet in the oppoſite Point, and the Longitude is diſtinguiſhed into Eaſt and Weſt, as obſerved above: In ſuch Caſe it muſt be known which Kind the given Longitude is of.

Prob.

PROB. XVI. *To determine the Difference of Time in different Places.*

Find the Longitude of each Place, by the Converse of the last Problem, and reduce the Difference into Time; which is done by allowing an Hour for every 15 Degrees, and so proportionably for lesser Parts, the Number of Hours, &c. thus found, will be the Difference between the Time of the Day at one Place, and that at the other. (*a*)

PROB. XVII. *To find the Distance between two given Places, and the Angle of Position, that is, the Bearings of them, or the Situation of one from the other, with respect to the Points of the Compass.*

Rectify the Globe to the Latitude of one of the Places, and bring that Place to the brazen Meridian; then fix the Quadrant of Altitude

(*a*) The Reason of this is, because as the Earth by turning round its Axis, brings all Places to the Sun once in 24 Hours; that Place which lies 15 Degrees to the Westward of another, comes to it an Hour later than the other; and so the Inhabitants of this Place are an Hour later in their reckoning of Time than at the other Place. And the contrary holds of such as live towards the East.

to

to the uppermoſt Point of the Meridian, and putting the lower End of it between the Horizon and the Globe, ſlide it about till it paſſes through the other Place, and the Number of Degrees upon the Quadrant between Place and Place, turned into Miles by reckoning 60 to each Degree, will give the Diſtance between them; and the Number of Degrees upon the Horizon, between the Meridian and the Foot of the Quadrant, will give the Bearing of the Second, or its Situation from the firſt, with reſpect to the Points of the Compaſs.

PROB. XVIII. *To find a Meridian Line partly by the Help of the celeſtial Globe.*

Set the Hour Index to the Place of the Sun in the Ecliptic, and turn the Globe till the Pole Star, that is, the Star which is the neareſt the Axis of the Globe, comes under the Meridian, either above or below the Pole; above, if you find that the Index in that Situation of the Star, points to one of the Hours of the Night, otherwiſe below; and mark what Hour the Index points to. Then at that Hour of the Night hang up two Plummets, letting their Weights hang in Water to prevent their vibrating, in ſuch manner that the String of one of them may be directly between the Pole Star and the String of the other. Then will a Line drawn

drawn from one String to the other, or the Shadow of the one when it falls upon the other, be a Meridian Line. And

This I take to be a much more accurate, as well as an easier Method, than the common one of fixing up a Pin upon a Plane, drawing a Circle about the Foot of the Pin, and observing where the Extremity of the Shadow of that Pin enters the Circle in the Morning and goes out of it in the Afternoon, and drawing a Line from the Foot of the Pin through the middle Point between those Places for a Meridian Line. For this latter Method requires more Care and Trouble to do it accurately, than they who have not tried it would imagine.

CHAP. XVI.

Of the Equation *of* Time.

WHEREAS Time constantly passes on with an uniform and regular Flux, it is evident there is no possible Way accurately to measure and compare the several Intervals of it one with another, but by the Motion of some Body whose Progress is as uniform and regular as itself. To this End it was natural to pitch upon the Sun, whose Motion is obvious to all, and seemingly regular enough for that Purpose. But by the Circumspection and accurate Observations

ſervations of Aſtronomers, it is diſcovered not to be ſo; and that neither the Days, nor even the Hours themſelves, as meaſured by the Motion of the Sun, are of an equal Length. And this upon two Accounts, as will appear from the following Conſiderations.

I. A natural Day, or 24 Hours, is that interval of Time, in which the Sun ſeems to paſs from the Meridian of any Place to the ſame again: Had therefore the Earth no Motion but about its own Axis, and conſequently the Sun no apparent one along the Ecliptic, all the Days would be of an equal Length, as depending only on the Motion of the Earth about its Axis, which is uniform: But ſince the Earth is conſtantly moving in its Orbit, the ſame Way it turns about its Axis, the Sun is always in appearance advancing forwards. When the Earth therefore is turned round, and a Spectator upon its Surface brought again to the Place he was at the foregoing Day at Noon, it is not yet Noon with him, the Sun in appearance having advanced to the Eaſtward in the mean time; ſo that the Spectator muſt be carried farther ſtill, *viz.* ſo many Degrees as the Sun has advanced that Way, before he can be brought to the Meridian the Sun is in (*a*). Now by Reaſon of the unequable and

(*a*) This is the Reaſon that the Length of the Day is not exactly equal to the Time, in which the Earth performs its Rotation about its Axis, which is, as was obſerved in Note (*f*), Chap. I. but 23 Hours 56 Minutes and 4 Seconds.

irregular Motion of the Earth in its Orbit, or, the irregular apparent Motion of the Sun along the Ecliptic depending thereon, the Number of Degrees the Sun advances Eaſtwards each Day, is ſometimes greater and ſometimes leſs; and conſequently upon this Account, the Days muſt differ one from another in Length; and therefore the Hours alſo, ſince each Day is ſuppoſed to conſiſt of 24, be it long or ſhort.

II. Was the apparent Motion of the Sun through the Ecliptic uniform and regular, yet as the Ecliptic is oblique to the Equator, and therefore equal Portions of the one do not correſpond to equal Portions of the other; the Motion of the Sun, when referred to the Equator, would not be uniform; or, which is the ſame Thing, thoſe Points of the Equator which come to the Meridian with the Place of the Sun on different Days would not be at equal Diſtances one from another (*a*); which as to the Time of the Sun's appearing in the Meridian, or its ſhewing the Hour of the Day upon a Dial, is the ſame as if it had an irregular Motion along the Equator; and therefore muſt cauſe it to render the Days unequal among themſelves.

As theſe Cauſes are independent of each other, they ſometimes conſpire together; ſometimes they are contrary the one to the other.

(*a*) This is eaſily tried upon a Globe, where if we bring every tenth Degree of the Ecliptic to the brazen Meridian, we ſhall find that each tenth Degree of the Equator will not come thither with it.

The Time which an uniform Motion would mark out, is called *true Time*; and that which is indicated by the Sun, is *apparent Time*; and the Difference between these is termed *the Equation of Time*. When the abovementioned Causes have continued for some time, to render the apparent Motion of the Sun too quick or too slow, this Difference is considerable. About the 13th of *October*, it is the greatest of all; at which Time the Sun is 16 Minutes and 11 Seconds too fast.

CHAP. XVII.

Of the Division *of* Time.

THE several Parts of Time are principally distinguished by Days, Hours, Weeks, Months, Years, Cycles, Periods, and Æras. Of which in their Order.

By a Day, according to the most natural and obvious Sense of the Word, is meant that Space of Time during which it continues to be *Light*, and thus is opposed to that wherein it is *Dark*, which is called the Night. But this Space of Time being somewhat vague and indeterminate, the Time between the Sun's Rising and Setting is usually looked upon as the Day, and the Time which lapses from its setting to its rising again, the Night.

But the Word Day is frequently taken in a larger Senſe, ſo as to include alſo the Night, and to denote the Time of an whole apparent Revolution of the Sun round the Earth. In this Senſe it is called by ſome a natural Day, and by others an artificial one; but what ſome call the Artificial, others call the Natural. To avoid which Confuſion, it is uſual to call it in the former Senſe, ſimply the Day, and in the latter a *Nuchthemeron*; by which Term that Acceptation of it is aptly denoted, as being a Word that implies both Day and Night (*a*).

The *Nuchthemeron* is divided into 24 Parts, called Hours, which are of two Sorts, *Equal*, and *Unequal* or *Temporary*. The common Diviſion of the equal Hour is into Halves and Quarters: But Aſtronomers, and thoſe who are more accurate in their Account of Time, divide the Hour into ſixty Parts, called Minutes, and theſe again into ſixty Parts called Seconds, *&c.* And here we may obſerve, that the Word Minute is taken in a double Senſe, either to denote the ſixtieth Part of an Hour, and is therefore by Way of Diſtinction ſometimes called an *Horary* Minute; or elſe to ſignify the ſixtieth Part of a Degree, in which Caſe it may be called a *graduary* Minute, or Minute of a Degree.

Different Nations and People begin their Day at a different Hour. Thus the *Egyptians*

(*a*) A Greek Compound of νὺξ *Night*, and ἡμέρα *Day*.

began

began their Day at Midnight; from whom *Hipparchus* introduced that Way of reckoning into Aftronomy, and *Copernicus* and others have followed him. But the greateft Part of Aftronomers reckon the Day to begin at Noon, and fo count 24 Hours till the Noon of the next Day, and not twice twelve, according to the vulgar Computation.

The Method alfo of beginning the Day at Midnight prevails in *Great-Britain*, *France*, *Spain*, and moft Parts of *Europe*.

The *Babylonians* began their Day at Sun-rifing, reckoning the Hour immediately before its rifing again, the 24th Hour of the Day. From whence the Hours reckoned in this Manner are called the *Babylonic*.

In feveral Parts of *Germany*, they begin their Day at Sun-fetting, and reckon on till it fets the next Day, calling that the 24th Hour. Thefe are ufually called the *Italian* Hours.

The *Jews* alfo began their *Nuchthemeron* at Sun-fetting, but then they divide it into twice twelve Hours as we do, reckoning 12 for the Day, be it long or fhort, and 12 for the Night (*a*). So that their Hours continually

(*a*) According to this Way of reckoning, the Hours of the Day mentioned in Scripture are to be underftood. The *Jews* alfo divided the Night into four Quarters, called *Watches*, each Watch containing three of their Night Hours. And diftinguifhed them fometimes according to the Order of their Succeffion. Thus we find in Scripture Mention of the *fecond*, *third*, and *fourth* Watch, fometimes the firft Watch was called the *beginnning of the Watches*; the fecond, the *middle Watch*, as not being ended till Midnight; and the fourth, the *Morning Watch*. The firft alfo was fometimes termed the *Evening*; the fecond *Midnight*; the third, the *Cock-crowing*; and the fourth, the *Dawning of the Day*.

 varying

varying with the Length of the Day and Night, the Hours of the Day were longer than those of the Night one half of the Year, and the contrary the other, From whence their Hours are called *Temporary:* These at the Time of the Equinoxes become equal, because the Days and Nights are so then. The *Romans* also reckoned their Hours after this Manner, as do the *Turks* at this Day (*a*).

A Week consists of the seven Days in most Countries, called after the Names of the Planets as observed in the foregoing Note. But because Easter Week was formerly esteemed the

(*a*) This kind of Hours are also called *Planetary*, because the seven Planets were anciently looked upon as presiding over the Affairs of the World, and to take it by turns each of these Hours, according to the following Order, *Saturn* first, then *Jupiter*, *Mars*, the *Sun*, *Venus*, *Mercury*, and last of all the *Moon*. Hence they denominated each Day of the Week from that Planet whose Turn it was to *reign* or preside the first Hour of the *Nuchthemeron*. Thus assigning the first Hour of *Saturday* to *Saturn*, the second will fall to *Jupiter*, the third to *Mars*, and so the twenty-second of the same *Nuchthemeron* will fall to *Saturn* again, and therefore the twenty-third to *Jupiter*, and the last to *Mars*. So that on the first Hour of the next Day, it will fall to the Sun to preside. And by the like Manner of reckoning the first Hour of the next will fall to the Moon, of the next to *Mars*, of the next to *Mercury*, of the next to *Jupiter*, of the next to *Venus*. Hence the Days of the Week came to be distinguished by the Latin Names of *Dies Saturni*, *Solis*, *Lunæ*, *Martis*, *Mercurii*, *Jovis*, and *Veneris*; and among us by the Names of *Saturday*, *Sunday*, *Monday*, *Tuesday*, *Wednesday*, *Thursday*, and *Friday*. For, as *Saturday*, *Sunday*, and *Monday* plainly denote the Day of *Saturn*, the *Sun*, and the *Moon*; so *Tuesday*, *Wednesday*, *Thursday*, and *Friday*, denote the Day of *Tuisco*, *Woden*, *Thor*, and *Friga*, which are the *Saxon* Names respectively answering to *Mars*, *Mercury*, *Jupiter*, and *Venus*. *Wells*'s Chronol. Chap. II.

first

first or principal Week of the Year, and each Day thereof was a *Feria* or Holiday; hence the several Days were distinguished among the primitive Christians, by the Names of *Feria prima*, *secunda*, &c. that is, the first, second Holiday, *&c.* But the Sunday, or *Feria prima*, was otherwise stiled by them the *Lord's Day*, as being the Day of our Lord's Resurrection (*a*).

Months are distinguished principally into two Sorts; *viz.* *Solar* and *Lunar*. The Solar Month is either *Astronomical* or *Civil* (*b*). The Lunar Month is divided into *Periodical*, *Synodical*, and *Civil*. The *Astronomical* Solar Month is the Time the Sun takes up in passing through one of the Signs of the Ecliptic.

The *Periodical* and *Synodical* Lunar Months have already been observed; the former to be equal to the Revolution of the Moon about the Earth, *viz.* 27 Days, 7 Hours, and 43 Minutes; the latter, the Interval of Time that lapses between two succeeding Conjunctions of the Moon with the Sun, *viz.* 29 Days, 12 Hours, 44 Minutes, and 3 Seconds.

(*a*) The Hebrew Word which in the old Testament is rendered *Week*, signifying a Collection of seven, is there sometimes used for seven Years; and in Conformity thereto, our *English* Word *Week* in those Places must be understood to imply not a *Week of Days*, but a *Week*, or *System of seven* Years.

(*b*) A Month or Year is denominated *Civil*, from its being of common Use in any Republic or Society of People.

Now because in common Use it would be inconvenient to have the Months to consist of odd Pieces of Days; it was usual among the *Jews*, *Greeks*, and *Romans*, (till *Julius Cæsar* made an Alteration) in order to adapt their Months to the apparent Motion of the Sun, to have their Months consist of 29 and 30 Days alternately, one of these Numbers being about half a Day above, and the other about half a Day less than the Synodical Month; so that by this Means the same Appearances of the Moon would happen nearly on the same Day of the Month for a long Time together. This Month thus adapted to the last mentioned lunar Month, is called the *Civil* lunar Month; and is still made use of by the *Turks*.

The *Egyptian* Months consisted of 30 Days each, 12 of these make a Year wanting 5 Days, which they added to the End of the Year, and from thence called them Epagomenæ, or Supplemental.

Now because 12 civil lunar Months want 11 Days of a Year, it is evident that the several Seasons of the Year must in Time fall upon different Months; to prevent which, *Julius Cæsar* ordained, that the Month should not be taken from the Motion of the Moon, but from that of the Sun; that they should consist alternately of 30 and 31 Days, and that *February* should contain 29, and every fourth Year 30. But it happened afterwards that the Name of the sixth Month, which was formerly

Sextilis,

Sextilis, was in Honour of the Emperor *Augustus* changed for that of his, as that of the foregoing Month *Quintilis* had before been, in Honour of his Father *Julius*; and therefore since this Month consisted but of thirty Days according to the Institution of *Julius Cæsar*, one more was added to it, to make it equal to the foregoing one, that the Honour paid to the Son might not seem to fall short of that which was paid to the Father. Upon this, the next Month, *viz. September*, was reckoned but 30, the next 31, and so on, that the alternate Order abovementioned might not be destroyed, till *January*, which, according to that Order, ought to have had but 30. But this Month being so named in Honour of *Janus*, the God of Time, it was thought improper to rob the God of a Day, wherefore *February* was reduced to 28, and every fourth Year to 29. And so the Months came to stand as they are in our Calendar, from whence they are called *Calendar* Months. These are properly *civil Solar Months*.

The principal Division of the Year is also into two kinds, *viz. Solar* and *Lunar*; each of which are again divided into *Astronomical* and *Civil*.

The Astronomical Solar Year is of two kinds, *viz. Tropical* and *Sydereal*. The former is the Space of Time which lapses while the Sun appears to move from either of the Solstitial or

Equinoctial

Equinoctial Points to the same again. Now by Reason of the continual Precession of the said Points, whereby they do as it were meet the Sun, this kind of Year is something less than the *Sydereal* one; which is the Time in which the Earth performs an entire Revolution in its Orbit, or in which the Sun seems to pass from any fixed Star to the same again. The former consists of 365 Days, 5 Hours, 48 Minutes, and 57 Seconds; the latter of 365 Days, 6 Hours, 9 Minutes, and 14 Seconds, and is also called *Periodical*, or *Anomalistical*. The Astronomical Lunar Year consists of 12 Synodical Months; and is therefore 354 Days, 8 Hours, 48 Minutes, and 36 Seconds.

The *Civil* Lunar Year is also of two kinds, *viz. moveable* and *fixed*; the moveable civil Lunar Year consists of 12 civil Lunar Months, which being but 354 Days, falls short of a Year by 11 Days, so that the beginning of this Year varies with respect to the Seasons, till it runs through them all, which it does in about 32 Years Time; and from hence it is denominated *Moveable*, or the *wandring Lunar Year*: As the *Turks* now make Use of the civil Lunar Month, so they also do of this civil Lunar Year.

In order to render this kind of Year *fixed*, that is, to prevent the Seasons from shifting from one Part of it to another, the *Greeks*, and the *Romans*, till *Julius Cæsar*'s Time, after every three Years added a Month of thirty-three

three Days, which they called *Embolimæus*, or the *Intercalary* Month.

But because this Method is not sufficient, (for the civil Lunar Years fall short of the Astronomical Solar Years above 11 Days,) others adapt such Intercalations to them as do in a manner render the beginning of them fixed to the same Part of the Solar Year. Hence these Years properly intercalated are called *fixed* Lunar Years, as also from their Dependency on the Motion both of the Sun and Moon, *Luni-Solar* Years. Years of this kind are used by the *Jews*, and the Clergy of the Church of *Rome* in their Ecclesiastical Affairs.

The civil Astronomical Year is also either *moveable* or *fixed*. The moveable one was in Use among the *Egyptians*, and is from thence called, the *Egyptian* Year. It consisted of 365 Days, and therefore fell short of the tropical Year, with which all the Seasons return, near six Hours, so that four such Years are less than four Astronomical Solar ones, by almost a whole Day, and therefore in 1460 Years, the beginning of the Year would *pass* through all the Seasons.

Although the *Romans* did not use this kind of Year, yet theirs, taking one Year with another, was of the same Length (*a*); *Julius*

(*a*) For having, as observed above, six Months of 29 Days each, and six of 30, and an *Intercalary* Month of 33 Days, at the End of every third Year; their Years consisted one with another of 365 Days.

Cæsar

Cæsar therefore to *fix* this Year, when he rectified the Calendar, and threw out the civil Lunar Months, to make up the six Hours which this Year wants of the true Astronomical Solar Year, and with which only the Seasons keep pace, ordered that every fourth Year should have an intercalary Day, *i. e.* that a Day should be added to the Calendar, over and above those it contained the other three, or that every fourth Year should contain 366 Days. Hence it was that, as we observed above, *February* came to have 29 Days every fourth Year. But he did not add it to the End of the Month, but ordered that the sixth of the Calends of *March*, which according to our way of Reckoning is the 24th of *February*, should be reckoned twice. Upon which Account that Year was wont to be called *Bissextilis*, and is still retained by us under the name of *Leap* Year. From hence, Years reckoned after this Manner are called *Julian* Years.

But however the Year was not rendered *fixed* by this means, for it was now too long by about eleven Minutes, and therefore varied from the Sun about a Day in 131 Years. So that from the Time of the *Nicene* Council, (in which the Rule was established for the Time of celebrating *Easter*) to the Year 1582, in which Pope *Gregory* the 13th undertook a farther Reformation of the Calendar, there was found to have rose a Variation of 10 Days, the vernal Equinox being observed to fall on the

the 11th of *March*, which at the Time of the *Nicene* Council, fell on the 21st. The Pope therefore in order to bring back the Equinoxes to the Time of the Year they fell upon at the Time of that Council, took those ten Days out of the Calendar by ordering the 5th of *October* 1582, to be reckon'd the 15th, which necessarily removed the following Equinox from the 11th to the 21st of *March*. And to prevent the Seasons of the Year from going backwards for the future, he ordered that every hundredth Year, which according to the Form instituted by *Julius Cæsar* was to be a *Bissextile* or *Leap* Year, should be a common one, and consist only of 365 Days, but that since too much would be taken away by this Means, every fourth hundred Year should remain *Bissextile*.

This Method being established by Pope *Gregory*, is called the *Gregorian* Account, or the *New Stile*, as being new in comparison of that established before by *Julius Cæsar*, which is now called the *Old*: It is observed in all Places where the Papal Authority is acknowledged; and also, towards the End of the last Century it was received by many of the reformed People in *Germany*. In *Great-Britain* and *Ireland*, and some reformed Nations of the Northern Parts of *Europe*, the old *Julian* Form is still retained. But above an hundred Years being now lapsed since the abovementioned Reformation of the Calendar, and

the hundredth Year not being made a common one, as in the *Gregorian* Method; the *Gregorian* Account differs now from the *Julian* one Day more than it did at its first Institution; so that that Day which according to the *Old Stile* is the first of the Month, is the 12th of the same according to the *New*.

As the Form of the Year in different Nations is various, so also is the Beginning of it. Thus the *Jews* begin their Ecclesiastical Year with the *new* Moon of that Month whose *full* Moon happens next after the vernal Equinox. The Church of *Rome*, with the *Sunday* that falls upon the said *full* Moon, or that happens next after it; that is, with the Feast of the Resurrection of our Lord. The *Grecians* began theirs with the new Moon that happened next after the Summer Solstice. And the *Romans* anciently began theirs with the new Moon next after the Winter Solstice. The *Venetians*, *Florentines*, *Pisans* in *Italy*, and the Inhabitants of *Triers* or *Treves* in *Germany*, make the *vernal* Equinox the Beginning of their Year. The ancient Clergy made the 25th of *March*, or the Feast of the Blessed Virgin, the Beginning of the Year; which Method the Church of *England* still retains; and according to our Law the Year begins here too, though in Things that do not require a *legal Date*, we, with the neighbouring Nations, look upon the first Day of *January* to be the Beginning of the Year.

The

The *Persians* still retain the *Egyptian* Year, and the Period in which the first Day of it passes through all the Seasons of the Astronomical Solar Year, *viz.* 1460 Years, they call the *Sothiacal Period*, or *great Dog-Star Year*; as being reckoned to begin in that Year in which the *Dog-Star* (or *Sothis*) rises Heliacally on the first Day of the Month *Thoth*, which is the first Day of their Year.

A *Cycle* is a continual and successive Revolution of a certain Number of Years. The Cycles are four, *viz.* the Cycle of the Sun, the Cycle of the Moon, the Cycle of *Dionysius*, the Cycle of the Indiction.

In order to apprehend the Foundation and Nature of the Cycle of the Sun, it must be observed that to each Day of the Year is assigned in the Calendar, some one of the first seven Letters of the Alphabet A, B, C, D, E, F, G, the first Letter A being affixed to the first Day of *January*, to the second B, and so to G. After which the Letters are repeated again in the same Order, A falling to the 8th, B to the 9th, *&c.* By which means, because the common Year contains 52 Weeks and one Day over, the last Day of *December* has also the Letter A affixed to it. The Letters being appropriated in this manner to each Day of the Year, it follows, that whichever of the Letters belongs to any particular Day in the first Week of *January*, that Letter will belong to the same

ſame all the Year long. But ſince the ſame Letter, *viz.* A, is affixed to the laſt, which is affixed to the firſt Day of the Year, it is obvious, that at the End of one Year and beginning of the next, two ſucceeding Days will have the ſame Letter A; ſo that if *Monday* had an A the laſt Year, *Tueſday* has it this, and *Monday* the G: And therefore the Letters will move backwards with reſpect to the Days of the Week; for Inſtance, that Day which has G one Year, will have F the next, after that E, then D, and ſo on to A; and conſequently the ſame Day will run through the ſeven Letters in ſeven Years Time. But before the ſeven Years are expired, a Leap Year will intervene, having one Day more than common, which Day is added at the 24th of *February* (*a*), and has the ſame Letter that the 24th has, ſo that in this Caſe, as there was a Change or Regreſſion of the Letters, with reſpect to the Days of the Week, upon the Account of two Letters coming together at the End of one Year and the beginning of the next, there will be another like Change in *February* alſo; and therefore, as in the former Caſe, the ſame Letter would have run through all the Days of the Week in ſeven Years; this ſecond Change happening once in four

(*a*) Some ſay it is added after the 28th, and that it has the ſame Letter with that. From this Uncertainty there ariſes a Difficulty in fixing the Day of the Celebration of the Feſtival of St. *Matthias*, which has occaſioned a great Diſpute among the Learned.

Years,

Years, the Courſe of the Letters is ſo oft interrupted, that it does not become the ſame again, till after four times 7, or 28 Years.

Thus, ſuppoſe a Leap Year begins with a *Sunday*, it will end with a *Monday*; in which Caſe, A and G will be the *Dominical* or *Sunday* Letters, becauſe A is appropriated to the firſt of *January*, and G to the 30th of *December*: But ſuch another Year cannot return till after 28 Years, as any one that tries will readily find; the Conſequence of which is, that thoſe two Letters cannot be *Sunday* Letters again in one Year, till after that ſpace of Time. And becauſe the ſame *Sunday* Letters do return after every 28 Years, that Term is called the *Cycle of the Sunday Letter*, or otherwiſe, though not ſo properly, *the Cycle of the Sun* (*a*).

The Cycle of the Moon is a Revolution of 19 Years; after which Time, as was anciently ſuppoſed, the new and full Moons would fall upon the ſame Days of the Year again (*b*).

This Cycle is therefore of great Note in the Chriſtian Church, a Method founded upon this Suppoſition, being eſtabliſhed by the Fathers of

(*a*) The ninth Year before our Saviour's Birth, was the firſt of this Cycle; from whence may eaſily be known what Year of it the preſent is. The firſt of this Cycle is always Leap Year, and has for its Dominical Letters G and F. And the Letter A is, as already obſerved, always affixed to the firſt Day of *January*. From which *Data* may at any Time be computed what is the *Sunday* Letter for the preſent Year, and from thence what Day of the Week each Day of the Year falls on.

(*b*) This is alſo called the *Metonic* Cycle from *Meton*, the Author of it.

the *Nicene* Council, for finding the Time of celebrating *Easter*, and the other moveable Feasts of the Church for any Time to come.

In order to understand which, we must observe, that as the Christian Passover, or Festival of *Easter*, succeeded the *Jewish* one, the Time of its Celebration was regulated by that upon which the *Jewish* Passover was at its Institution, by God himself appointed to be kept, which was on the fourteenth Day of the first Month, according to the manner of reckoning among the *Jews*, *Exod.* 12. Now the *Jewish* Months being Lunar ones, each Month began upon the Day of the new Moon, or however at the Time of its *Heliacal* rising, and so the full Moon fell upon the fourteenth of each Month. And further, that Month was called the first of the Year, whose full Moon or fourteenth Day, either fell upon the vernal Equinox, or was the first that succeeded it. And at the Time of the *Nicene* Council, when the Rule for finding *Easter* was drawn up, the Equinox fell, or was thought to fall, on the 21st of *March*. But because our Saviour's Resurrection, which the Festival of *Easter* is designed to commemorate, happened on a *Sunday*; it was ordered that it should not be kept on the full Moon, but the *Sunday* following. Accordingly the Rule then established for finding *Easter*, and which is still made use of by the Church, runs thus; *Easter-Day is always the first* Sunday *after the first full Moon, which happens next after the One and twentieth Day of* March.

March. *And if the full Moon happens upon a* Sunday, Eaſter-Day *is the* Sunday *after*.

The Foundation of the Rule being thus explained, it remains now to ſhew the Meaning of it, which is not ſo obvious as at firſt Sight it appears to be.

It is to be obſerved then, that in order to find on what Days of the Year the new and full Moons would happen for the Time to come, the ancient Method was to obſerve on what Day of each Month the new Moons fell in each Year of the Moon's Cycle, and to the ſaid Days reſpectively to ſet the Number of the Year in their Calendar. Thus, obſerving that the new Moons in the firſt Year of the Cycle fall on *January* the 23d, *February* 21, *March* 23, *&c.* to thoſe Days they affixed the Number 1. And in like manner obſerving, that, in the ſecond Year, they fell on *January* 12, *February* 10, *March* 12, *&c.* to each of them they put the Number 2; and ſo for each Year of the Cycle. And on Account of their great Uſefulneſs, or becauſe they were wrote in Letters of Gold, the Number thus ſet to the Days on which the new Moons fell, were called the *Golden Numbers* (*a*).

The Numbers being thus affixed in the Calendar, it is only looking for the Golden Number of any Year, and over againſt it you have the Day of each Month of that Year, on which the new Moon happens.

(*a*) In the Table for finding *Eaſter* for ever in ſome Common Prayer-Books, they are ſtiled Prime Numbers, probably as being placed in the Prime or firſt Column of the Calendar.

 Thus,

Thus, at the Time of the *Nicene* Council the Time of the new Moons might be accurately enough found, but not at this Time, becaufe the new Moons do not happen at the fame Time of the Year every 19 Years, but fall fhort an Hour and half, which in 304 Years comes to a Day, fo that now the new and full Moons fall almoft 5 Days fooner than they are fhewn to do by the Golden Numbers, affixed to the Calendar.

However, as no proper Authority has interven'd to alter the Method of finding the *Eafter* full Moon by the Golden Numbers, eftablifhed at the *Nicene* Council, that Method is ftill retained in the Church. And therefore to underftand the abovementioned Rule aright, it muft be fuppofed, that by the full Moon is meant the Time of the full Moon as found by the Golden Numbers affixed to the Calendar in the Common Prayer-Book, and not the true full Moon, as found in an Almanack, or by Aftronomical Obfervation (*a*).

The Calendar Year confifting of 365 Days, and each Revolution of the Moon being 29 Days and an half, it follows that there will be 12 Revolutions of the Moon in each Year and 11 Days over; fo that on whatever Year the new Moon happens on the firft Day of

(*a*) The firft Year of Chrift was in the 2d of this Cycle.

By the abovementioned Method the true new Moon may at any Time be found, if we deduct five Days in Confideration of the abovementioned Deficiency in the Moon's Cycle, and fuppofe the new Moon to fall five Days fooner than that Method fhews it to do.

the

the Year, it will happen also that Year eleven Days before the End of it; the next Year 22 Days before the End of it; the next (33, that is, casting out 30, as is the Way, for one intire Lunation) 3 Days before the End of it; and so on. These Numbers are the *Epact*. Their Use is to find the Moon's Age at any Time. You add together the *Epact* for the Year, and the Number of the Months from *March* inclusive (for observe, it is then the Year is supposed to begin in this Account) and you have the Age of the Moon when the Month begins; add therefore the Day of the Month to this Sum, and you have the Age of the Moon at that Day. But this Way of reckoning is very gross, and will err from the Truth a whole Day sometimes.

By multiplying the Cycle of the Sun and Moon together, we have a third Period, containing the Properties of both the other two, so that at the End of this Cycle not only the Days of the Week have the same Letters belonging to them, and fall on the same Days of the Month, but the new and full Moons also fall on the same Days of the Year, so that the Time of celebrating *Easter* and the other Festivals of the Church, return to the very same Days of the Year. From whence this is called *the great Paschal Cycle*, and sometimes from its Author, the *Dionysian Cycle* (*a*). It

(*a*) This is also by some called the *Victorian* Period.

consists of 532 Years, and the first Year of it was 457 Years before Christ.

The *Cycle of the Indiction* is a Revolution of 15 Years, made use of by both the *Greeks* and *Romans*, probably upon some political Account. It has no Relation to the Motions of the Heavenly Bodies. The Nativity of our Saviour fell upon the 4th of this Cycle.

By multiplying the three Cycles of the Sun, Moon, and Indiction together, arises a Term of 7980 Years, called the *Julian Period*; which as it began several hundreds of Years before the Creation, and is not now ended, and in all probability will not, so long as the World stands, is of singular Use in Chronology, as taking in all Events, both past, present, and to come.

This Period was invented by *Joseph Scaliger*, and began 4713 Years before Christ.

An *Æra*, otherwise called an *Epoch*, is a Continuation of Time beginning from some certain Point, as from a Root, and continually proceeding forwards without beginning again.

That which is of principal Note among Christians, is the Æra of Christ.

The Author of this Æra was *Dionysius Exiguus*, who flourished about 500 Years after Christ. He began it on the Annunciation of the Virgin *Mary*, or the 25th of *March*; the 25th of *March*, as he supposed, immediately preceding the Nativity of our Lord: but it is now generally thought that our Saviour was

born

born the *December* before that; wherefore others, beginning the Æra from the Nativity it ſelf, or the 1ſt of *January* following it, reckon almoſt a quarter of a Year before thoſe who adhere to the *Dionyſian* Account.

The *Engliſh* and *Iriſh* ſtill adhere to the *Dionyſian* Account in their eccleſiaſtical and civil Affairs, which all the reſt of the World, and even they themſelves in common Account, have laid aſide for that which begins at *January* the firſt. This Æra is frequently reckoned backwards, as well as forwards: Thus, it is uſual to ſay an Event happened ſo many Years before Chriſt.

The firſt Year of this Æra, agreeably to what was obſerved above, anſwers to the 4714th of the *Julian* Period.

There are ſeveral *Epochs* or Æra's of leſs Note, which I ſhall but juſt mention; this Chapter having already run out to a much greater Length, than I at firſt intended.

The Æra of the *Creation* according to the *Greek* Church, and which is now of Uſe in ſome Eaſtern Nations, begins before the *Julian* Period, *viz.* 5508 Years before Chriſt. Tho' others, among whom are the greateſt Aſtronomers, begin it ſomething leſs than 4000 Years before our Saviour's Birth.

The Æra of the *Deſtruction of Troy*, according to *Dionyſius Halicarnaſſeus* and *Diodorus Siculus*, begins 1181 Years before Chriſt.

The Æra of the *Olympiads*, or *Olympic Games* of Uſe among the *Greeks*, 776 before Chriſt.

The Æra *Urbis conditæ*; or *of the Building of Rome*; made Use of by the *Romans*, according to *Varro*, began 753 Years before Christ; according to the *Fasti Capitolini*, 752.

The Æra of *Nabonassar*, made Use of by the *Chaldeans* and *Egyptians*, famous among Astronomers, as consisting of *Egyptian* Years, which are disturbed by no Intercalation. It begins 747 Years before Christ.

The Æra of the *Death of Alexander the Great*, 324.

The Æra of the *City of Antioch*, 49.

The Æra of the *Julian Reformation of the Calendar*, 45.

The Æra *Actiaca*, 30.

The *Dioclesian Æra*, 284 Years after Christ.

The Æra of the *Hegira*, or *Flight of Mahomet*, used by the *Turks* and *Arabs*, 622 after Christ.

The Æra of *Yesdegird*, or *Persian Æra*, 632 after Christ.

N. B. A Year in the Æra's of the Death of *Alexander*, and of *Nabonassar*, consists but of 365 Days; and a Year of that of the *Hegira* is only 354.

As the Subject of this Chapter, though founded in *Astronomy*, relates principally to the Science of *Chronology*, of which this *Compendium* is not designed to treat; I have little more than thrown a few Definitions together, leaving it to those who would see more on the Subject, to consult such Authors as have wrote more

more largely thereon; as *Gregory*, *Keil*, *Wells*, *Wheatly* on the Common-Prayer, *Holder* on Time, and many others.

SECT. III.

Of the physical Cause of the Motion of the Heavenly Bodies.

CHAP. XVIII.

Of the Forces *necessary to retain revolving* Bodies *in* circular *and* other Orbits.

And first of Bodies revolving in circular Orbits.

IN order that a Body may move round in the Circumference of a Circle, it is requisite, that as soon as the Body begins to move, some Power or Force continually act upon it, in such manner as to make it bend its Course every Moment towards the Center of the Circle; because, as is obvious to conceive, it would otherwise only move right forwards (*a*); for in Considerations of this kind, we do not suppose the revolving Body to be affected even by its own *Weight*, or any other moving Cause whatever, except that whereby its Motion is made circular: And because the Circumference of a

(*a*) See Part I. Chap. IV.

Circle

Circle is every where equally diſtant from the Center thereof, it is obvious there muſt be the moſt *exact Adjuſtment* imaginable between the Power that retains the Body in the Circle, whatever that Power be, and the Endeavour the Body has to move right on in a ſtraight Line: For in Caſe the former be an *over-balance* for the latter, it will bring the Body *nearer* and *nearer* to the Center of the Circle it ſhould deſcribe; and on the other hand, if it be too *weak*, it permits it to move *farther* and *farther* off. So that the Body, inſtead of a Circle, will deſcribe ſome other Figure, which will be of this or that Form, according as the Force the Body moves with, compared with the Power which bends its Courſe, is greater or leſs; and not only ſo, but as the latter (which is ſometimes the Caſe) acts more or leſs powerfully, as the revolving Body comes nearer or goes farther from the Point it revolves about. And here we may have recourſe to a Fact obvious to every one, which will in ſome meaſure illuſtrate what we have been ſpeaking of. Let a Ball be hung up by a String: This Ball, when drawn a little to one Side, will endeavour to return back to the loweſt Point, as if there were ſome Power ſeated in that Point to draw it thither from all Sides. But the Body may have a Caſt given it ſideways in ſuch Manner, as that by Virtue of the Caſt, and by Virtue of its Tendency to the loweſt Point together, it ſhall deſcribe a Circle parallel to the Horizon; or inſtead of that it may be made to deſcribe other

other Figures, as Ellipses suppose, which will be of various Forms according to the Force and Direction of the Cast given to the Ball. And if we suppose several Balls hung up, which when removed to *equal* Distances from their lowest Points, should tend thither again with *unequal* Forces, and these Forces urging them by *different* Laws and in *different* Manners, the Figures described by them, when thrown sideways with this or that Force and Direction, will be proportionably *different* one from another. Now in this Supposition, the Cast which is given to the Ball sideways, corresponds with the Endeavour a Body revolving in the Circumference of a Circle or other Figure, has to move right on continually, and so to leave the Center it ought to revolve about; and the Endeavour, Struggle, or Tendency the Ball, when drawn to one Side, has to move towards the lowest Point, or rather the Power which is, as it were, seated in that lowest Point, and occasions the abovementioned Endeavour or Tendency in the Ball to move thither, corresponds with the Power we supposed above necessary to retain a Body in the Circumference of the Circle or other Figure it describes, and which by bending the Course of the Body inwards, that is, towards the Center of the Figure, prevents it from going off in a straight Line.

We are therefore, in the next Place, to enquire what Degree of Force or retaining Power is necessary to bend the Course of a Body in such Manner that it shall describe this or that Orbit

Orbit (*a*); but ſince the Planets both primary and ſecondary, as alſo the Comets, deſcribe Orbits either circular or elliptical, as obſerved above, we will confine ourſelves chiefly to the Conſideration of the Forces neceſſary to retain revolving Bodies in ſuch Orbits.

This Subject has been already touched upon in the firſt Part of this *Compendium* (*b*), under the Title of *Centripetal and Centrifugal Forces*; but little more is taken Notice of there, than what was requiſite to ſhew the Poſſibility of ſuch a Motion as we are ſpeaking of, and to explain the Terms, which are theſe that follow, *viz.* *Centripetal* Force, or that whereby the revolving Body is prevented from going on in a right Line: The *Centrifugal* Force, or that by which it endeavours to go on in ſuch a Line. Theſe, like Action and Reaction, are equal in all Caſes, and therefore called by one common Name, *Central* Forces: That whereby the Body continues its Motion, is the *Projectile*

(*a*) For unleſs we can *meaſure* the Forces we aſſign, and ſay they are exactly equal to the Effects we ſuppoſe them to produce, it would be in vain to aſſign them at all. And this is what makes the Difference between true and falſe Philoſophy. The former by a mathematical Scrutiny, (as it were by a kind of Menſuration) finds that the Cauſes aſſigned are juſtly adapted to the Facts accounted for; while the other, by a conjectural kind of Method, aſſigns Cauſes which perhaps are plauſible at firſt Sight, but when meaſured by the Rules of Geometry, are found either too ſmall or too great for the Effects they are ſuppoſed to produce.

This was the Caſe of *Des Cartes*'s Vortices, the Doctrine of which captivated the whole Philoſophical World, and continued to do ſo, till they were weighed in that unerring Balance, and thereby diſcovered to be inadequate to the Purpoſes for which they were contrived.

(*b*) Chap. VIII.

Force; and the Time in which it revolves once round, is called the *Periodical Time*. The ſeveral Caſes proper to be conſidered, ſhall be comprized in the following Propoſitions.

PROPOSITION I.

When two or more Bodies revolve at *equal* Diſtances from the Center of the Circle they deſcribe, but with *unequal* Velocities, the central Forces neceſſary to retain them, will be to each other, as the Squares of their Velocities. That is, if one Body revolves *twice* as faſt as the other, it will require *four* times the retaining Force the other does; if with *three* times the Velocity, it will then require *nine* times the Force to retain it in its Orb, *&c.* (*a*)

N. B. We do not here conſider the Magnitude of the revolving Body, becauſe we ſuppoſe the Power at the Center, to act upon every Part of it alike, ſo that the larger it is, the more forcibly in Proportion it is acted upon; and therefore it is the ſame thing, whether it be large or ſmall.

(*a*) Let BDF Fig. 25. repreſent a Circle whoſe Center is S, draw ABC a Tangent to the ſame at the Point B, and let there be a Body moving from A towards C, and let it be ſuppoſed to be deſtitute of Weight, or whatever elſe might alter its Courſe. In this Caſe it would deſcribe the right Line ABC; but when it comes to B, let it begin to be acted upon by ſome central Force, or retaining Power ſeated in S, ſuch as ſhall *bend* its Courſe at B from a rectilineal to a circular one, and acting upon it afterwards in every Inſtant of Time in like manner, ſhall cauſe it to deſcribe the Circle BDFB. Let us then proceed to eſtimate the Quantity of Force exerted in producing one of theſe *Bendings*; in order to which let us conceive a Point of the Circle as D to

 be

be the very next to the Point B, though taken at some Distance in the Figure to avoid Confusion of Lines. Parallel to the Diameter BF draw the Line CD, and parallel to the Tangent BC the Line DG, draw also the Line BD, which will be the Diagonal of the Parallelogram CG. Now it is well known that supposing a Body would move from B to C, and by Virtue of some other Force acting upon it at the same Time, would move from B to G, it will neither move to C nor G, but to D; that is, its straight Course will be *bent* into a circular one at B, for that is what is meant by its moving from B to D when B and D are contiguous, as is here supposed. BG therefore is the Space over which the revolving Body would move by that Action of the central Force which causes the Bend at B, or any other Point of the Circle. And as BG is equal to the Space over which the revolving Body would move by Virtue of the central Force, it will always be proportional to the Force it self. The Value of that Line is therefore now to be determined. Join the Points D and F, then (by 31. El. 3.) the Triangle BDF will be rightangled at D, and consequently (by 8. and 6. El. 6.) BG : BD :: BD : BF, from whence $BG = \frac{BDq}{BF}$; but when B and D are contiguous (as is here supposed) the Difference between the Chord BD and the Arch BD vanishes, so that $BG = \frac{\text{the Arch } BDq}{BF}$ Having thus found a Quantity proportionable to the Act of the central Force requisite to retain a revolving Body in a Circle, we may proceed to establish the following fundamental *Lemma*.

LEMMA I.

The Space that a Body retained in a Circle, by Virtue of any central Force, would move over in a given Time, by Virtue of that Force only, is equal to the Square of the Arch described in that Time, divided by the Diameter of the Circle.

Dem. The Action of a central Force upon a revolving Body is analogous to that of Gravity upon falling Bodies, and therefore the Space a Body would move over by Virtue of that Force only, is as the Square of the Time that Action continues: As then the Square of the Time the Arch BD is described in (the Points B and D being contiguous as before) is to $\frac{BDq}{BF}$ which, by what was shewn above, is a Quantity equal to the Space the Body would move over towards the Center in that Time, so is the Square of the Time given to the Space the Body would move over by Virtue of that Force in that Time. If we then put t to express

expreſs the Time the Arch BD would be deſcribed in, T for the Time given, and S the Space the Body would move over towards the Center in that Time, we ſhall have this Proportion.

Viz.	1	$t^q : \frac{BD^q}{BF} :: T^q : S$
Let the Arch the Body would deſcribe in the Time T, be the Arch BN, then becauſe the Motion of the Body in the Circumference of the Circle is uniform, we have	2	$t : T :: BD : BN$
Squaring the laſt Proportion, we have	3	$t^q : T^q :: BD^q : BN^q$.
Altering the Poſition of the Means in the firſt Proportion, we have	4	$t^q : T^q :: \frac{BD^q}{BF} : S$
Comparing the third and fourth Step together	5	$BD^q : BN^q :: \frac{BD^q}{BF} : S$
Turning the fifth Step into an Equation	6	$S \times BD^q = \frac{BN^q \times BD^q}{BF}$
Dividing the laſt by BD^q	7	$S = \frac{BN^q}{BF}$ Q. E. D.

Coroll. From this *Lemma* it follows, that the central Force requiſite to retain a revolving Body in the Circumference of a Circle, muſt be ſuch as, ſuppoſing the projectile Motion ſtopped, would cauſe the Body in the Time it would otherwiſe deſcribe any Portion of that Circumference, to move towards the Center of the Circle over a Space or Diſtance of ſuch Length, as would ariſe from the Diviſion of the Square of that Portion of the Circumference by the Diameter of the Circle. And conſequently, it will in all Caſes be proportionable to the Square of the Arch the Body would deſcribe in a given Time divided by the Diameter of the Circle: Becauſe the Force exerted in any given Time is proportionable to the Space it cauſes a Body to move over in that Time.

To illuſtrate this, ſuppoſe the Diameter of the Circle 25 Feet, and the Arch BN 10 Feet, and that the revolving Body moves over thoſe 10 Feet in a Minute Then the Square of BN, which is 100, divided by the Diameter, or 25, gives 4 Feet: The central Force therefore at S, neceſſary to make the Body revolve in the Circle BNF at the Rate of 10 Feet in a Minute, muſt be ſuch, as ſuppoſing the Body left to itſelf at B, (that is, without any Motion at all towards C, or any Weight or

or Gravity of its own) would make it move over 4 Feet towards S in a Minute. Again, supposing Things as before, only that the Body revolves with but half its former Velocity, that is, that it describes but five Feet of the Circumference of the Circle in a Minute; then the Square of that 5 Feet, which is 25, divided by 25, the Diameter gives 1; so that the central Force in this Case must be such, that by Virtue thereof a Body destitute both of Weight and projectile Force, shall move over a Space 1 Foot long in a Minute. And conversely, if there be a Circle, whose Diameter is 25 Feet, and there be a Power lodged in the Center thereof capable of causing a Body to move over the Space of 4 Feet in a Minute; then in order that a Body may be made to revolve round in the Circumference of that Circle, it must be thrown forwards in a Direction at right Angles with the Radius of the Circle, with such Velocity as would carry it over the Space of 10 Feet in that Time. And if the central Force be capable of causing it to move over but 1 Foot in a Minute, then it must be thrown forwards with a Velocity that shall make it move at the Rate of only 5 Feet: Or, since the Velocity is uniform, and therefore the Arch described is proportional to the Time it is described in, instead of an Arch as BN to calculate with, we may take the whole Circumference, and say, that *the Space a Body would move over by Virtue of the Central Force alone, in the Time it describes the whole Circumference, is equal to that which would arise from the Division of the Square of that whole Circumference by the Diameter of the Circle.* And this is that *exact Adjustment* between the central and projectile Force we have been so long in Search of.

Suppose then we would calculate, how far a Stone would fall by its own Weight, in the Time in which, if thrown forwards with a sufficient Velocity, it would move quite round the Earth in a Circle. Let us call the Diameter of the Earth, that is, the Diameter of the Circle it describes, 1, then the Circumference thereof would be 3 nearly, the Square of that 3, is 9, that divided by the Diameter 1, gives 9; so that a Stone would fall by Virtue of its own Weight through a Space equal to about 9 Diameters of the Earth in the Time it would revolve once round it. But it is here to be observed, that the Stone must be supposed to be of the same Weight during its whole Fall, that it is of at first; and so in all other Cases, we suppose the Energy of the central Force to be the same at all Distances whatever, unless when the contrary is expressed.

We have hitherto been considering the Energy of the central Force, by comparing the Space it would cause a Body to move over in any certain Time, with the Space the Body ought to describe

describe along the Circumference of the Circle in the same Time: Let us now take it in another View, and compare the Velocity a Body ought to move with to describe a Circle, with the Velocity it would acquire in moving over some certain Space by Virtue of the Central Force only; from whence will arise a Proposition very useful in Matters of this kind; and which for Distinction Sake we will call

LEMMA II.

The Velocity a Body ought to revolve with in the Circumference of a Circle, is such as it would acquire in moving by Virtue of the central Force only, over half the Radius of that Circle.

To explain this, let the Body be supposed at Rest at B, and let it be acted upon by a central Force at S: Upon this the Body will proceed towards S with a Motion, which will increase every Moment (because as in the Case of falling Bodies we suppose the central Force acting upon it with the same Intenseness, whether the Body moves or not) and the Instant it gets half Way to the Center S, it will have acquired some certain Degree of Velocity; that's the Velocity it ought to move with along the Circumference of the Circle to render its projectile Force an exact Counterbalance to the centripetal one of the Power at S. Thus, suppose there were a Loadstone fixed at S, and a Piece of Steel at B, divested of its own Weight, so that it might be at Liberty to move freely towards the Loadstone, and to make the Instance parallel to the Case before us, let it be supposed that the Loadstone attracts the Steel with the same Degree of Force at all Distances, then whatever Velocity that Steel would have when it had got half way to the Loadstone, that's a Velocity with which, if the Steel were thrown from B along the Tangent BC, it would move round in the Path BDFB not approaching towards or receding from the Loadstone. But the Steel must be supposed to meet with no Resistance from the Air, for if it does it will continually lose somewhat of its Velocity, and so after some Revolutions would fall upon the Body of the Loadstone. Again, suppose it were required to find the Velocity a Ball ought to be thrown forwards with at a small Distance from the Surface of the Earth, that it should move quite round in a Circle, keeping the same Distance from the Surface of it all the Way. It must be such, as that Ball (upon a Supposition that Gravity acts with the same Degree of Force at all Distances from the Center of the Earth, as at the Surface) would acquire by falling half way from the Place it is thrown from to the Center of the Earth (for that is supposed to be the Seat of the Power that gives it its Weight, and thereby hinders it from moving forwards in the Direction it is thrown in). And that Velocity is easily calculated from

what was demonstrated, Part I. Chap. V. § 4, or 5, supposing as *Hugens* has determined by very accurate Experiments, that a Body near the Surface of the Earth falls $15\frac{1}{12}$ *Paris* Feet in the first Second of Time. But enough of this by Way of Explication, it is Time to demonstrate what is here affirmed.

Dem. Let the Diameter of the Circle be 16 Feet, in which Case half the Radius will be 4 Feet, then by *Coroll.* of *Lemma* 1. the Arch the Body must run over in the Time it would pass through those 4 Feet by Virtue of the central Force, must be such that the Square of it divided by the Diameter may be 4; that is, it must be 8 Feet, (because the Square of 8, which is 64 divided by 16 the Diameter, gives 4.) but if it describes an Arch of 8 Feet, in the Time it would fall through 4 Feet, it then moves with that very Velocity it might acquire by the Fall; because as has been demonstrated Part I. Chap. 5. § 7. if a Body moves uniformly with the Velocity it might acquire by a Fall, it will move over a Space equal to *twice* the Height of the Fall, in the Time thereof. Since then the Body in the Case before us, describes an Arch of the Circumference of 8 Feet in the Time it would fall through 4 Feet, it moves therein with a *Velocity* it would acquire by falling through 4 Feet, that is, half the Radius. Q. E. D. To avoid a Circumlocution in the midst of the Argument, I call the Space the Body would move over by Virtue of the central Force only, from its Analogy therewith, a *Fall*.

Universally thus, let the Diameter be equal to 4 times some certain Space, which Space let us call *d*; the Diameter then will be equal to 4 *d*, and half the Radius will be equal to the Space *d*, in which Case, the Arch the Body describes in the Time it would fall through the Space *d* must by the abovementioned Corollary, be such as that the Square of it divided by 4 Times *d* may be once *d*, that is, the Arch must be twice *d*, (because the Square of twice *d*, which is 4 times *dd* divided by the Diameter 4 times *d*, gives once *d*.) but as before, if the Body move uniformly through *twice* the Space *d*, in the time it would fall through *once* that Space, it then moves with that very Velocity it would acquire by falling through that Space, that is, *ex Hypoth.* through half the Length of the Radius. Q. E. D.

From what has been demonstrated above, we may easily learn the Relation which the central Force that is requisite to retain one Body in the Circle it describes, has to that which is required for the same, or another Body revolving in a larger or less Circle. Which Relation is expressed in the following *Lemma*.

LEMMA III.

The central Force whereby one Body is retained in the Circle it describes, is to that whereby a Body moving with a different Velocity

or

or in a different Circle might be retained herein, as the Square of the Velocity of the former divided by the Radius of the Circle it describes, to the Square of the Velocity of the latter divided by the Radius of the Circle which it describes.

Dem. By Corollary of Lemma 1, the Energy of the central Force is proportionable to the Square of the Arch the Body describes in a given Time, divided by the Diameter of the Circle. And Arches described in a given Time, are as the Velocities whereby they are described, and the Diameter of a Circle is as its Radius; consequently the Action of the central Force is proportionable to the Square of the Velocity the Body moves with divided by the Radius of the Circle it describes: The central Force therefore whereby one Body is retained in the Circle it describes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained therein, as the Square of the Velocity of the former divided by the Radius of the Circle it describes, to the Square of the Velocity of the latter, divided by the Radius of the Circle which it describes. Q. E. D.

Coroll. If we call the central Forces whereby the two revolving Bodies are retained in their respective Circles C and *c*, their Velocities V and *v*, and the Distances they revolve at, that is, the Radii of the Circles they describe R and *r*, we shall always have this Proportion, $C : c :: \frac{VV}{R} : \frac{vv}{r}$.

These Things which are the Foundation of what follows being established, we may now proceed to demonstrate each Proposition of this Chapter in the Order they lie in; the first of which is, That when two or more Bodies revolve at *equal* Distances from the Center, but with *unequal* Velocities, the central Forces necessary to retain them will be to each other as the Squares of their Velocities.

Dem. Let the Circle which one of the revolving Bodies is supposed to describe in this and the following Cases, be BDF, in Fig. 26. and that which the other describes be *b d f*, and let us call the Distances they revolve at from the common Center, that is, the Radii of the Circles they describe, R and *r*; the Velocities they revolve with V and *v*; and the central Forces necessary to retain them C and *c*. Then by Corollary of Lemma 3, we shall

have for the first step of this Demonstration,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
But by the Proposition R is equal to *r*, therefore	2	$C : c :: VV : vv$. Q. E. D.

Prop. II. When two or more Bodies move with *equal* Velocities, but at *unequal* Diſtances from the Center they revolve about, their central Forces muſt be inverſly as their Diſtances. That is, by how many times greater the Diſtance a Body revolves at is from the Center, ſo many times leſs Force will retain it (*b*).

Prop. III. When two or more Bodies perform their Revolutions in *equal* Times, but at *different* Diſtances from the Center they revolve about; the Forces requiſite to retain them in their Orbs will be to each other as the Diſtances they revolve at from the Center. For Inſtance, if one revolves at twice the Diſtance the other does, it will require a double Force to retain it; if it deſcribes a Circle three times as large as the other does, then a treble Force is requiſite, &c. (*c*)

(*b*) *Dem.* By Lemma 3, as before,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By the Propoſition V is equal to *v*, therefore,	2	$C : c :: \frac{1}{R} : \frac{1}{r}$ Q. E. D.

(*c*) In order to demonſtrate this the following Lemma will be of uſe.

LEMMA IV.

The periodical Time of a Body revolving in a Circle, is as the Diſtance it revolves at from the Center directly, *and as its Velocity* inverſly.

Demonſtration of the Lemma. The periodical Time of a revolving Body, *cæteris paribus*, depends on the Diſtance it revolves at from the Center, becauſe the greater that Diſtance is, the greater is the Circle it deſcribes, and ſo the longer it is in

coming

PROP. IV. When two or more Bodies revolving at *different* Diſtances from the Center, are retained by *equal* centripetal Forces, their Velocities will be ſuch, that their periodical Times ſhall be to each other as the Square Roots of their Diſtances. That is, if one revolves at four Times the Diſtance another does, it will perform a Revolution in twice the Time the other does; if at nine times the Diſtance, it will revolve in thrice the Time, &c. (*d*)

coming round. Again, *cæteris paribus*, the greater its Velocity is, the ſooner it comes round; and therefore the periodical Time is directly as the former, and inverſly as the latter: Or, as the former divided by the latter; and conſequently $T : t :: \frac{R}{V} : \frac{r}{v}$ Q. E. D.

Coroll. And therefore in comparing the periodical Times of two Bodies together, if we call their periodical Times T and *t*, the Diſtances they revolve at R and *r*; and their Velocities V and *v*, as before, we ſhall have this Proportion, *viz.* $T : t :: \frac{R}{V} : \frac{r}{v}$.

Demonſt. of the Propoſition. By Corollary of Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By Corollary of Lemma 4,	2	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying the 2d Step by the Ratio of V to *v*, we have,	3	$TV : tv :: R : r$
By the Propoſition, T is equal to *t*, therefore,	4	$V : v :: R : r$
Comparing the 1ſt and 4th Steps together,	5	$C : c :: \frac{VV}{V} : \frac{vv}{v}$
That is,	6	$C : c :: V : v$. Q. E. D.

(*d*) *Dem.* By Corollary of Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by $R : r$	2	$CR : cr :: VV : vv$
By the Propoſition, C is equal to *c*, therefore,	3	$R : r :: VV \;\; vv$

PROP. V. And in general, whatever be the Diſtances, the Velocities, or the periodical Times of the revolving Bodies, the retaining Forces will be to each other in a Ratio compounded of their Diſtances directly, and the Squares of their periodical Times inverſly. Thus for Inſtance, if one revolves at twice the Diſtance another does, and is three times as long in moving round, it will require $\frac{2}{9}$, that is, *two ninths* of the retaining Power the other does. For as, if the retaining Power was in a Ratio compounded of the Diſtance *directly* and *directly* alſo of the Square of the periodical Time, the latter Body would require *twice* the retaining Power the other would *multiplied by* 9, the Square of the periodical Time; ſo as the latter Ratio is an *inverſe* one, it requires twice that Power *divided by* 9; that is, $\frac{2}{9}$ or *two ninths* of it. For in compounding of Ratio's the Way is, if they are both *direct* to *multiply* them both together; if one be *direct* and the other *inverſe*, to divide the *direct* by the *inverſe*; if both are *inverſe*, to multiply them both together, and to divide *Unity* by

By Corollary of Lemma 4,	4	$T : t :: \frac{R}{V} : \frac{r}{v}$
Squaring the laſt Step,	5	$TT : tt :: \frac{RR}{VV} : \frac{rr}{vv}$
Comparing the 3d and 5th,	6	$TT : tt :: \frac{RR}{R} : \frac{rr}{r}$
That is,	7	$TT : tt :: R : r$
Extracting the Roots of the 7th Step,	8	$T : t :: \sqrt{R} : \sqrt{r}$ Q.E.D.

them:

them: Thus, if the retaining Power had been as the Distance *inversly* and *inversly* too as the Square of the periodical Time, then in the Case before us, the latter Body would have required a retaining Power, which should have been as the Number 1 divided by 2, multiplied by 9; that is as 1 divided by 18: That is, if we suppose 1 to express the central Force requisite for the former Body, the Fraction $\frac{1}{18}$ would have expressed that which would have been necessary for the latter; or in other Words, their retaining Powers would have been related to each other, as 1 is to $\frac{1}{18}$; that is, the former would have been 18 times greater than the latter (*e*).

PROP. VI. If several Bodies revolve at different Distances from one common Center, and the retaining Power lodged in that Center decreases as the Squares of the Distances increase; that is, if at a double Distance it be

(*e*) *Dem.* By Corollary of Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by R to *r*,	2	$CR : cr :: VV : vv$
By Corollary of Lemma 4,	3	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying the last by V to *v*,	4	$TV : tv :: R : r$
Dividing the last by T to *t*,	5	$V : v :: \frac{R}{T} : \frac{r}{t}$
Squaring the last,	6	$VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$
Comparing the 2d and 6th,	7	$CR : cr :: \frac{RR}{TT} : \frac{rr}{tt}$
Dividing the last by R to *r*,	8	$C : c :: \frac{R}{TT} : \frac{r}{tt}$ Q. E. D.

four times weaker; and at a treble Distance, 9 times weaker, &c. the Squares of the periodical Times of those Bodies will be to each other as the Cubes of their Distances from the common Center. That is, if there be two Bodies, whose Distances when *cubed*, that is, multiplied by themselves twice, are *double* or *treble*, &c. of each other, then their periodical times will be such, as that when *squared* only, that is, multiplied by themselves *once*, they shall be also *double*, or *treble*, &c. of each other (*f*).

(*f*) *Dem.* By Corollary of Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By the Proposition,	2	$C : c :: \frac{1}{RR} : \frac{1}{rr}$
Comparing the 1st and 2d Step,	3	$\frac{VV}{R} : \frac{vv}{r} :: \frac{1}{RR} : \frac{1}{rr}$
Multiplying by R to *r*,	4	$VV : vv :: \frac{1}{R} : \frac{1}{r}$
By Corollary of Lemma 4,	5	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying by V to *v*,	6	$TV : tv :: R : r$
Dividing by T to *t*,	7	$V : v :: \frac{R}{T} : \frac{r}{t}$
Squaring the last,	8	$VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$
Comparing the 4th and the 8th,	9	$\frac{1}{R} : \frac{1}{r} :: \frac{RR}{TT} : \frac{rr}{tt}$
Multiplying by TT to *tt*,	10	$\frac{TT}{R} : \frac{tt}{r} :: RR : rr$
Multiplying by R to *r*,	11	$TT : tt :: RRR : rrr$. Q.E.D.

And after like manner, we may easily demonstrate the seventh Corollary of the fourth Proposition of the first Book of Sir *Isaac Newton*'s *Principia*. Which because it is of more than ordinary Difficulty to Beginners, I here add. It is to this Effect.

When Bodies revolve in such Manner that their periodical Times are to each other as some Power (which call *n*) of their Distances,

Diſtances, the centripetal Forces requiſite to retain them will be to each other inverſly as twice that Power wanting one of their Diſtances; that is, inverſly as their Diſtances raiſed to the Power 2n—1.

Dem. By Corollary of Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by the Ratio of R to *r*,	2	$CR : cr :: VV : vv$
By the Suppoſition,	3	$T : t :: R^n : r^n$
By Corollary of Lemma 4,	4	$T : t :: \frac{R}{V} : \frac{r}{v}$
Comparing the 3d and 4th,	5	$R^n : r^n :: \frac{R}{V} : \frac{r}{v}$
Multiplying by V to *v*,	6	$VR^n : vr^n :: R : r$
Dividing by R^n to r^n,	7	$V : v :: \frac{R}{R^n} : \frac{r}{r^n}$
Dividing the Numerators in the laſt Step by R : *r*, gives 1 : 1, and dividing the Denominators $R^n : r^n$ by the ſame, brings them one Power lower, that is, to $R^{n-1} : r^{n-1}$; therefore the laſt Step is reducible to	8	$V : v :: \frac{1}{R^{n-1}} : \frac{1}{r^{n-1}}$
Squaring both Sides of the 8th Step, which in the Denominators R^{n-1} and r^{n-1} is done by doubling their Index n—1, we have	9	$VV : vv :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$
Comparing the 2d and 9th Steps,	10	$CR : cr :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$
Dividing by R to *r*, which in the Denominators R^{2n-2} and r^{2n-2} is done by adding + 1 to their Index 2n—2, by which means it becomes 2n—1, we have	11	$C : c :: \frac{1}{R^{2n-1}} : \frac{1}{r^{2n-1}}$ Q. E. D.

Scholium. The foregoing Demonſtrations would have proceeded with equal Succeſs, had we only taken the Antecedents, or the Conſequents in each Step; that is, only the large or only

the small Letters. Which Method, because I shall make use of it hereafter, I will here give a Specimen of. Let it then be required to demonstrate the first Proposition of this Chapter over again in this latter Method.

By Corollary of Lemma 3. The central Forces are as the Squares of the Velocities divided by the Radii of the Circles described, which put thus	1	$C : \frac{VV}{R}$
By the Proposition, the Circles described have one common Radius, call it 1. Then	2	$R = 1$
From the 1st and 2d Step compared, we have	3	$C : VV.$

That is, the central Forces of the revolving Bodies are to each other as the Squares of their Velocities, which is the Sense of the Proposition.

Again, let it be required to demonstrate the Corollary taken from Sir *Isaac Newton*'s *Principia*, over again in this Method.

By Corollary of Lemma 3, ——	1	$C : \frac{VV}{R}$
By the Supposition ——	2	$T : R^n$
By Corollary of Lemma 4, ——	3	$T : \frac{R}{V}$
Comparing the 2d and 3d Steps —	4	$\frac{R}{V} : R^n$
Multiplying by V ——	5	$R : VR^n$
Dividing both Sides by R, reduces R to 1, and brings R^n one Power lower, that is to R^{n-1}, so that we shall then have	6	$1 : VR^{n-1}$
Dividing by R^{n-1} ——	7	$\frac{1}{R^{n-1}} : V$
Squaring the last Step, which in the Quantity R^{n-1} is effected by doubling the Index $n-1$, we have	8	$\frac{1}{R^{2n-2}} : VV$
Multiplying the first Step by R ——	9	$CR : VV$
Comparing the two last ——	10	$CR : \frac{1}{R^{2n-2}}$
Dividing by R, which in the Quantity R^{2n-2} is done by adding one to the Index, we have	11	$C : \frac{1}{R^{2n-1}}$.

Q. E. D.

Secondly, Of the Forces *necessary to retain Bodies revolving in* other Orbs.

Prop. VII. If a Body be turned out of its rectilineal Course by Virtue of a central Force, which decreases as you go from the Seat thereof, as the Squares of the Distances increase; that is, which is inversly as the Square of the Distance, the Figure that Body shall describe, if not a Circle, will be a Parabola, an Ellipse or an Hyperbola, and one of the Foci of the Figure will be at the Seat of the retaining Power. That is, if there be not that exact Adjustment between the projectile Force of the Body and the central Power necessary to cause it to describe a Circle, it will then describe one of those other Figures, one of whose Foci will be where the Seat of the retaining Power is (*g*).

(*g*) In order to shew this it will be necessary to premise the following Lemma's.

LEMMA V.

The Velocity of a Body describing any Curve, (if its Course is regulated by a retaining Power) is in one Point of the Curve to its Velocity in any other Point thereof, inversly as a Perpendicular drawn from the Seat of the retaining Power to a Tangent of the Curve at the first Point, to a Perpendicular drawn from the same Place to a Tangent at the other Point. That is, in fewer Words, the Velocity of the Body is inversly as a Perpendicular let fall from the Seat of the retaining Power to a Tangent at the Point where the Body is. Thus, if the Body be at D, *a Point in Curve* DEF, *Fig.* 27, *and* PD *be a Tangent thereto, and* SP *a Perpendicular to that Tangent, drawn through* S *the Seat of the retaining Power, the Velocity of the Body at* D, *compared with its Velocity in other Parts of the Curve, will be inversly as* SP, *that is as the Quantity* $\frac{1}{SP}$.

Dem.

Dem. It has been demonstrated, Part I. Chap. VIII. that revolving Bodies describe equal Areas in equal Times; that is, if a Body describes the Curve Line DEF, and the Arches DE, EF, &c. be run over in equal Times, the Areas DES, EFS, &c. will be equal; which Areas if we suppose the Times as small as possible, may be considered as so many right-lined Triangles, because in that Case the Lines DE, EF, &c. loose their Curvity, and being produced are Tangents to the Curve. Produce the Line DE to P, and let fall the Perpendicular SP, then the Area of the Triangle DES, is had by multiplying its Base DE into half SP; and the Area of EFS is equal to its Base EF multiplied by half a Perpendicular drawn from S to EF produced, &c. But these Triangles being described in equal Times, are all equal, the longer therefore their Bases, the shorter their Perpendiculars. But the Bases being run over in equal Times by the revolving Body, they are as the Velocity of the Body wherewith they are described; consequently the greater the Velocity of the Body, the shorter is the Perpendicular to the Tangent where the Body is. Which is the Sense of the Lemma.

LEMMA VI.

Whatever Curve a revolving Body describes by Virtue of a centripetal Force, whether the Seat of that Force be within or without the Curve, the Action of that Force upon the Body when at any one Point of the Curve, must be to the same upon the Body, when at another Point of the Curve, directly as the Distance of the Body from the Seat of the retaining Power and inversly as the Cube of a Perpendicular let fall from the Seat of the retaining Power to a Tangent to the Curve at the Point where the Body is, multiplied by the Radius of a Circle of equal Curvature with the Figure at that Point. That is, if AX *in Fig.* 28, *be the Curve described, and* MDN, *whose Center is* C, *be a Circle of equal Curvature with the Figure at the Point* D, *and* PD *be a Tangent to that Point, and* SP *a Perpendicular thereto drawn through* S *the Seat of the retaining Power, the central Force requisite to act upon the Body at* D *must be as* $\frac{DS}{SP^3 \times CD}$, *that is, if we put* R *for the Radius* CD, *as* $\frac{DS}{SP^3 \times R}$.

Dem. By Lemma 5, the Velocity of the revolving Body when at D is as $\frac{1}{SP}$, calling then that Velocity V,

we have ——— ———	1	$V : \frac{1}{SP}$
Squaring both Sides ——— ———	2	$VV : \frac{1}{SP^q}$

By

By Corollary of Lemma 3, the central Force neceſſary to retain the Body in that Circle; (or, which is the ſame thing, at the Point D in the Curve ADX, becauſe they are both of equal Curvature there) would, if placed at C, be as $\frac{VV}{R}$. But whereas it is placed at S it acts obliquely to DC, and therefore to produce the ſame Effect, muſt act more forcibly in the Proportion of SD to DT, ſuppoſing ST perpendicular to DC; that is, becauſe SP is equal to DT, in the Proportion of SD to SP.

But by the Golden Rule	3	$SP : SD :: \frac{VV}{R} : \frac{VV \times SD}{SP \times R}$
Conſequently the central Force neceſſary if placed not at C but at S, is expreſſible by	4	$\frac{VV \times SD}{SP \times R}$
Putting then C for the Action of the central Force neceſſary to be placed at S, we have	5	$C : \frac{VV \times SD}{SP \times R}$ or, which is the ſame Thing, $VV \times \frac{SD}{SP \times R}$
Comparing the 2d and 4th Steps together	6	$C : \frac{1}{SPq} \times \frac{SD}{SP \times R}$
That is,	7	$C : \frac{SD}{SP^3 \times R}$. Q. E. D.

LEMMA VII.

Let AD *Fig.* 29 *be a Parabola, whoſe Axis is* AG, *and its Focus* S, *and let* PDF *be a Tangent at the Point* D, *through the Focus* S *and Point of Contact* D *draw the Lines* PS *and* DG *both perpendicular to the Tangent* PD, *then will* PS *be equal to half* DG.

Dem. Produce DP till it meets the Axis in X, and draw the Diameter DO, and join the Points D and S: Then becauſe DO, as being a Diameter is parallel to GX, the Angle FDO is equal to DXS; 'tis alſo equal to the Angle XDS (Miln. Conic. Sect. Part IV. Prop. 4.) the Triangle XSD is therefore an Iſoſceles one, and SP being perpendicular to the Baſe XD, XP is equal to half XD; but PS and DG being parallel, the Triangles XPS and XDG are ſimilar, and therefore PS is alſo equal to half DG. Q. E. D.

LEMMA VIII.

Let AD *Fig.* 30, *be a Portion of a Parabola, an Ellipse or an Hyperbola, one of whose Foci is* S, *its longer Axis* AB, *and its Parameter or Latus rectum* L; *and let* PF *be a Tangent at the Point* D, *join the Points* D *and* S, *through the latter of which draw the Line* SP *perpendicular to the Tangent, and through the former the Line* DG *perpendicular also to the Tangent, and crossing the longer Axis in* G, *then will* DG^3 *be equal to* $\frac{L^3 \times SD^3}{8SP^3}$.

Dem. Through the Point G draw the Line GT perpendicular to DS, then will the right-angled Triangles DGT and DSP be similar, because the Angles DSP and SDG are alternate, consequently

consequently	1	DG : DT :: SD : SP
But by Miln. Conic. Sect. Part IV. Prop. 6.	2	$DT = \frac{1}{2}L$
From the 1st and 2d Steps compared together	3	$DG : \frac{1}{2}L :: SD : SP$
Turning the 3d Step into an Equation	4	$DG = \frac{L \times SD}{2SP}$
Cubing the last Step	5	$DG^3 = \frac{L^3 \times SD^3}{8SP^3}$. Q.E.D.

LEMMA IX.

Things remaining as before, produce DG *to* C *in Fig.* 31, 32, 33, *so that the Line* DC *may be equal to* $\frac{L \times SD^3}{2SP^3}$, *then will a Circle whose Center is* C, *and Radius* CD, *be of the same Curvature, and coincident with the Figure* AD *at the Point* D, *whether it be a Parabola, an Ellipse or an Hyperbola.*

Demonstration.

I. *For the Parabola.* The same Lines being drawn as in the foregoing Figure, draw also through D, Fig. 31, the Diameter DY, and take a Portion of it as DI equal in Length to the Parameter of that Diameter, and let fall the Perpendicular IH meeting DG produced in H, and let L be the Latus rectum or

Parameter of the Figure, then by the 3d Step of Lemma 8,	1	$DG : \frac{1}{2}L :: SD : SP$
The Angle HDI wants IDF to make it a right one, the Angle PSD wants SDP to make it a right one, but IDF and SDP are equal, Miln. Conic. Sect. Part IV. Prop. 4, therefore HDI and PSD are so also, the Triangles therefore HDI and SDP are similar; consequently	2	SD : SP :: DH : DI.

From

From the 1st and 2d Steps compared together	3	$DG : \frac{1}{2} L :: DH : DI$
By Miln. Conic. Sect. Part IV. Prop. 10. Coroll.	4	$DI = 4DS$
From the 3d and 4th Step compared	5	$DG : \frac{1}{2} L :: DH : 4DS$
Multiplying the Antecedents of the first Step by 4,	6	$4DG : \frac{1}{2} L :: 4SD : SP$
Compounding the 5th and 6th Steps, that is multiplying them together	7	$4DG^2 : \frac{LL}{4} :: DH : SP$
Lemma 7,	8	$\frac{1}{2} DG = SP$
Comparing the 7th and 8th Steps	9	$4DG^2 : \frac{LL}{4} :: DH : \frac{1}{2} DG$
Turning the 9th Step into an Equation	10	$2DG^3 = \frac{LL \times DH}{4}$
Taking half the 10th Step	11	$DG^3 = \frac{LL \times DH}{8}$
By Lemma 8,	12	$DG^3 = \frac{L^3 \times SD^3}{8SP^3}$
Comparing the 11th and 12th Steps	13	$\frac{DH \times LL}{8} = \frac{LLL \times SD^3}{8SP^3}$
Dividing the 13th Step by $\frac{LL}{8}$	14	$DH = \frac{L \times SD^3}{SP^3}$
Halving the 14th Step	15	$\frac{1}{2} DH = \frac{L \times SD^3}{2SP^3}$
But by Supposition	16	$CD = \frac{L \times SD^3}{2SP^3}$
Comparing the two last Steps	17	$CD = \frac{1}{2} DH$

But DH being perpendicular to PF, and the Angle at I a right one, it is obvious a Circle whose Diameter is DH, (and consequently by the last Step, whose Radius is DC,) will pass through the Point I, 31. El. 3, and will have the Line PF for a Tangent to it at D; but by Miln. Conic Sect. Part III. Prop. 11. Coroll. 1. such a Circle will be of equal Curvature with the Parabola at the Point D; and the Circle and the Parabola will be also coincident with each other at that Point, because the Line PF is a Tangent common to both. Q. E. D.

II. *For*

II. *For the Ellipsis and Hyperbola.* Let AD in Fig. 32, be a Portion of an Ellipsis, in Fig. 33, a Portion of an Hyperbola, one of whose Foci is S, its Latus rectum L, and it longer Axis AB; let PF be a Tangent at the Point D, and draw the Lines SP, SD, and DG as in Fig. 30; draw also the Diameter D*d* and produce it to I, so that DI may be equal to the Parameter of that Diameter; let fall the Perpendicular IH meeting DG produced in H; through the Center E draw the Semiaxis EQ, and the Diameter K*k* conjugate to D*d*; and to the longer Axis AB, the Ordinates DM and KN; and in the Hyperbola produce EK and HD till they meet in R. Then by the known Property of the Ellipsis and Hyperbola De L'Hospit. Liv. 2. Prop. 2.

and Liv. 3. Prop. 2.	1	$EQ^q : EA^q :: DM^q : AM \times MB$
Simson's Conic. Sect. Lib. 2. Prop. 19. and Lib. 3. Prop. 43.	2	$AM \times MB = EN^q$
Comparing the 1st and 2d Steps together, and extracting their Roots	3	$EQ : EA :: DM : EN$

The Diameter K*k* being conjugate to D*d* is by the Definition of such a Diameter, parallel to the Tangent PF, and consequently DH being perpendicular to that Tangent, the Triangle GER is a right-angled one, that Triangle therefore and the right-angled one KEN are similar as having in the Ellipse, their Angles at E vertical; in the Hyperbola, common: The right-angled Triangles GER and DGM are also similar, as having their Angles at G in the Ellipse, vertical; in the Hyperbola, common: And consequently the Triangles KEN and DGM are similar in each Figure.

Therefore	4	$DM : EN :: DG : EK$
Comparing the 3d and 4th Steps together	5	$EQ : EA :: DG : EK$
The right-angled Triangles DER and DHI in the Ellipse are similar as having their Angles at D common, in the Hyperbola as having their Angles at D vertical, consequently	6	$DE : DR :: DH : DI$

By

By the Definition of the Parameter DI, for the Ellipse De L'Hospit. Lib. 2. Definit. 13. for the Hyperbola Lib. 3. Definit. 15.	7	$DE : EK :: 2EK : DI$
The Extremes of the two last Steps being the same, the Product of the Means of the one must be equal to the Product of the Means of the other, that is	8	$2EK^2 = DH \times DR$
Multiplying the 8th Step by EK	9	$2EK^3 = DH \times DR \times EK$
By Simson's Con. Sect. Lib. 2. Prop. 20. for the Ellipse, Lib. 3. Prop. 45. for the Hyperbola	10	$DR \times EK = EA \times EQ$
From the 9 and 10	11	$2EK^3 = DH \times EA \times EQ$
Dividing the 11 by DH	12	$\frac{2EK^3}{DH} = EA \times EQ$
By the Definit. of a principal Parameter	13	$\frac{1}{2}L : EQ :: EQ : EA$
Wherefore from the last by Definit. 10. Elem. 5.	14	$\frac{1}{2}L : EA :: EQ^2 : EA^2$
Multiplying the two last together,	15	$\frac{LL}{4} : EQ \times EA :: EQ^3 : EA^3$
Comparing the 5th and 15th together, we have	16	$\frac{LL}{4} : EQ \times EA :: DG^3 : EK^3$
Multiplying the two last Terms of the 16th Step by $\frac{2}{DH}$, it becomes	17	$\frac{LL}{4} : EQ \times EA :: \frac{2DG^3}{DH} : \frac{2EK^3}{DH}$
Comparing the 12th Step and the 17th together	18	$\frac{LL}{4} : EQ \times EA :: \frac{2DG^3}{DH} : EA \times EQ$

 The

The Consequents in the 18th Step being the same, the Antecedents are equal, that is	19	$\frac{2\,DG^2}{DH} = \frac{LL}{4}$
Multiplying by DH	20	$2\,DG^2 = \frac{LL \times DH}{4}$.

But the last Step here is the same with the 10th in the Demonstration for the Parabola, from whence therefore the same Conclusion is to be drawn here as there: for the Proposition in Milm. Conic. Sect. referred to at the End of that Demonstration, relates also to the Ellipse and the Hyperbola.

Demonstration of the Proposition above, to which this Note refers.

Let ADX, Fig. 28, be a Portion of the Figure the revolving Body describes, whether it be a *Parabola*, an *Ellipse*, or an *Hyperbola*; and let L be the Latus Rectum or Parameter of the Figure; let the Seat of the retaining Power be at S, one of the Foci thereof, and let the revolving Body be supposed at D; draw the Line PD a Tangent to the Curve at that Point, and let fall the Perpendicular SP, Perpendicular to the Tangent draw the Line DC, which let us suppose to be the Radius of a Circle as MDN, of the same Curvature, that the Figure ADX is at the Point D. Then it is obvious, that the same Force which placed at S would retain the revolving Body in the Circumference of the Circle when it came at D, will retain it in the other Figure, at that Point, they being at that Place one and the same Curve. But by Lemma 6, such Force is as $\frac{DS}{SP^2 \times R}$. Calling then the retaining Power C, we shall have this Proportion, *viz.*

	1	$C : \frac{DS}{SP^2 \times R}$.

And since the Circle MDN is of the same Curvature and coincident with the Figure ADX, its Radius DC is by Lemma 5, equal to the Quantity $\frac{L \times SD^3}{2\,SP^3}$, putting then R for the Radius as before we shall have this Equation

	2	$R = \frac{L \times SD^3}{2\,SP^3}$
Comparing the 1st and 2d Step together, we shall have this Proportion	3	$C : \frac{DS}{SP^2 \times \frac{L \times SD^3}{2\,SP^3}}$
Expunging SP^2 as being both Multiplicator and Divisor	4	$C : \frac{DS}{\frac{1}{2} L \times SD^3}$

Dividing

That a Body ſhall deſcribe Figures ſo different, when the Power that directs its Courſe is the ſame, and acts by one and the ſame Law, is owing to the Degree of Velocity the Body moves with. Thus ſuppoſing the central Power placed at S (in Fig. 34.) and that the revolving Body ſets off at D in the Direction DB perpendicular to the Line DS, with ſuch Velocity as it would obtain in falling by Virtue of that central Power only, half way from D to S, it will then, by Lemma 2, deſcribe a Circle as DKM, whoſe Center is the Point S. If it ſets off from D in the ſame Direction as before, but with a leſs Velocity, it then revolves within that Circle deſcribing an Ellipſe, as DLN, one of whoſe *Foci* is at S, the Seat of the retaining Power, and the other between that and the Point D, as at F. If it ſets off from D with ſomewhat greater Velocity than what it would acquire by the above-mentioned Fall, it will ſtill deſcribe an Ellipſe, as DEO, one of whoſe *Foci* ſhall be at S as before, but

Dividing all by DS	5	$C : \frac{1}{\frac{1}{2}L \times DSq}$
But $\frac{1}{2}$L is the ſame, whatever Point of the Figure deſcribed the revolving Body is at, and may therefore be conſidered as Unity, conſequently,	6	$C : \frac{1}{DSq}$

That is, the retaining Power, if it be inverſly as the Square of the Diſtance, and ſeated in the *Focus* of the Figure, will direct the Courſe of the revolving Body in ſuch Manner, as that it ſhall deſcribe the ſaid Figure, be it a Parabola, an Ellipſe or an Hyperbola. Q. E. D.

the other ſhall be beyond it as at G. If it ſets off with a Velocity which is greater than that it would acquire by the above-mentioned Fall, in the Proportion of $\sqrt{2}$ to 1, that is the Square Root of 2 to 1, it ſtill deſcribes a Figure, one of whoſe *Foci* is S as before, but the other, as G, goes off to an infinite Diſtance; that is, the Figure deſcribed becomes a Parabola, as DPQ, whoſe Focus is S. If it ſets off with a greater Velocity than this laſt, the Seat of the retaining Power remains ſtill at S, but the other *Focus* G goes yet farther off; that is, it comes on on the other Side the Point D as at H, and the Figure deſcribed becomes an Hyperbola, as DRT, whoſe *Foci* are S and H.

From hence it is obſervable, that of the four Figures there are but two wherein an exact Adjuſtment is requiſite between the retaining Power and the projectile Force, *viz.* the Circle and the Parabola. In the former of which, the Velocity muſt be ſuch as the Body would acquire by falling half way to S, in the latter it muſt exceed that in the Proportion of the Square Root of 2 to 1; that is, in the Proportion of ſomewhat leſs that $1\frac{1}{2}$ to 1. Thus for Inſtance, if one Degree of Velocity is requiſite to cauſe a Body ſetting off at D, to revolve in a Circle about the Point S, as a Center; it will require about one Degree and an half to make the Body ſetting off at the ſame Place, to deſcribe a Parabola by Virtue of the ſame retaining Power ſeated in the ſame Point S.

Again,

Again, the ſame Figures may be deſcribed, though the Seat of the retaining Power be in the other *Foci*, F, G, or H; for Inſtance, if it be at F, in order that the Body may deſcribe the Ellipſe DLN, it muſt ſet off at D with the ſame Velocity it had in the former Caſe when it came to N: If it be at G, it will deſcribe the Ellipſe DEO, ſetting off at D with the Velocity it had before at O: And if G, the Seat of the retaining Power be at an infinite Diſtance, or in other Words (becauſe to ſay a determinate Point is at an infinite Diſtance, is a contradiction in Terms) if the retaining Power act in ſuch manner as it would do if the Seat of it was at an infinite Diſtance; that is, if it act upon the revolving Body in Lines parallel to DG; and with the ſame Degree of Force, whether the Body be at D, at P, or at Q, &c. it will deſcribe the Parabola DPQ with whatever Velocity it ſets off with from D. For to deſcribe a Circle round the Point G at an infinite Diſtance; it muſt ſet off with an infinite Velocity, becauſe, as obſerved above, it muſt ſet off with the Velocity it would acquire by falling half way to that Point; now a Body will deſcribe an Ellipſe round a given Point with any Degree of Velocity leſs than it will deſcribe a Circle with round the ſame, as it was obſerved that the Ellipſe DLN was deſcribed with leſs Velocity than the Circle DKM, but a Parabola is no other than an Ellipſe, one of whoſe *Foci* is at an infinite

Distance, the Body will therefore describe a Parabola when the Seat of the retaining Power is at an infinite Distance, with any Degree of Velocity less than an infinite one.

This is the Reason, that Projectiles upon the Surface of the Earth are said to describe Parabola's, because on Account of the great Distance of the Center of the Earth, compared with the Height Bodies can be thrown to, Gravity does as to Sense, act upon them uniformly and in parallel Lines.

Again, when the Point G the Seat of the retaining Power goes off at an infinite Distance one way; it immediately, or indeed strictly speaking at the same Instant, becomes the Point H at an infinite Distance the other way; so that those two considered as at an infinite Distance, are as it were one and the same Point; and consequently it is the same thing whether the retaining Power be at G an infinite Distance to the *Right*, or at H an infinite Distance to the *Left*, and therefore the Figure will still be a Parabola, and may also be described with any Degree of Velocity: only if it be at H, an infinite Distance to the left, the Power must be negative in respect of what it was before; that is, it must be of the *repulsive* Kind, as it is obvious it ought to be, to cause the revolving Body to describe a Figure whose Convexity is turned towards the Seat of the retaining Power.

If

If the Seat of the retaining Power H comes nearer, so as to be at a finite Distance from the other Focus S, the Power must still be negative, or repulsive; and the Figure described will be an Hyperbola, whose Foci are H and S. And the Hyperbola in this Case may be described whatever be the Velocity the Body sets off with; for the central Force being of the repulsive kind, the Figure described will necessarily be convex towards the Seat thereof, that is, towards one of its Foci; but none of the Sections of the Cone have one of their Foci on the convex Side of the Figure, and at a finite Distance from its Vertex, except the Hyperbola (*b*).

Hence

(*b*) In order to evince the Truth of what is here affirmed, the following Lemma will be of Use.

LEMMA X.

Let the Curve Line DPQ *in Fig.* 35. *represent a conic Section, one of whose* Foci *is* S, *and let the Circle* DKR *be of the same Curvature therewith at its Vertex* D, *then if the Curve be a Parabola;* DR *the Diameter of the Circle will be equal to, if it be an Ellipse, it will be less; if an Hyperbola it will be greater than four times the Distance* DS.

Demonst. of the Lemma. (By Milnes Conic. Sect. Part III. Prop. 2. Coroll. 5.) DR the Diameter of the Circle is equal to the Parameter of the Axis of the Curve, whether it be a Parabola, Ellipse or Hyperbola, which let us call, L.

Therefore	1	L = DR
1. But in the Parabola, by the Definition of the Parameter, (De L'Hospit. Con. Sect. Livr. I. Def. 5.)	2	L = 4 DS
Comparing 1st and 2d Step	3	DR = 4 DS. Q. E. D.

 2. If

2. If the Curve be an Ellipse, let F be its other Focus, then will DF + DS be equal to its longer Axis, De L'Hosp. Con. Sect. Liv. 2. Cor. 2. of the Definitions; call its shorter Axis X. Then from the Definition of the Parameter (De L'Hosp. Con. Sect. Liv. II. Def. 8.) we shall have this Proportion	4	$L : X :: X : DS + DF$
Turning the last Step into an Equation	5	$L = \frac{X^q}{DS + DF}$
(By De L'Hospit. Con. Sect. Liv. 2. Corol. 4. of the Definitions) the Square of half X is equal to DS × DF, that is	6	$\frac{X^q}{4} = DS \times DF$
Consequently	7	$X^q = 4DS \times DF$
Comparing the 5th and 7th Step	8	$L = \frac{4DS \times DF}{DS + DF}$
Turning this Equation into a Proportion	9	$L : 4DS :: DF : DF + DS$
Comparing the 1st and the 9th Step	10	$DR : 4DS :: DF : DF + DS$

But DF is less than DF + DS, therefore DR is less than 4 DS. Q. E. D.

3. If the Curve be an Hyperbola, let H be its other Focus, then will DH — DS be equal to its longer Axis, by De L'Hosp. Con. Sect. Liv. 3. Corol. 2. of the Definitions: And call as before its shorter Axis X; then from the Definition of the Parameter (De L'Hospit. Conic. Sect. Liv. 3. Definit. 8.) we shall

have this Proportion	11	$L : X :: X : DH - DS$
Turning the last Step into an Equation	12	$L = \frac{X^q}{DH - DS}$
(By De L'Hospit. Con. Sect. Liv. 3. Coroll. 4. of the Definitions) the Square of half X is equal to DS × DH, that is	13	$\frac{X^q}{4} = DS \times DH$
Therefore	14	$X^q = 4DS \times DH$

Comparing

Fig. 26

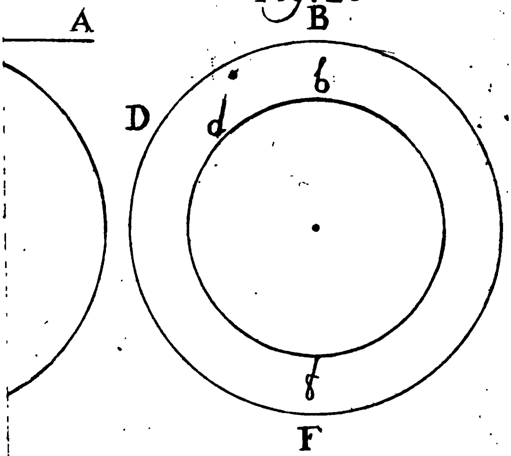
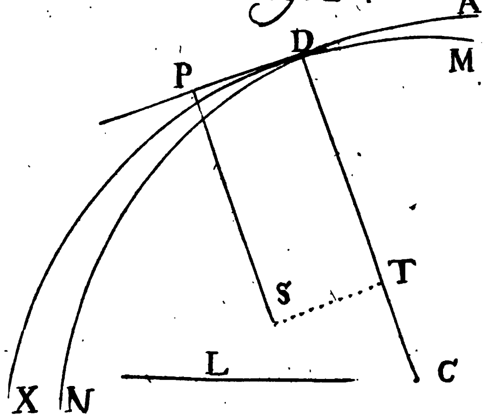


Fig. 28.



Comparing the 11th and 13th Steps	14	$L = \frac{4DS \times DH}{DH - DS}$
Turning this into a Proportion	15	$L : 4DS :: DH : DH - DS$
Comparing the 1st and 14th Steps	16	$DR : 4DS :: DH : DH - DS$

But DH is greater than DH — DS, therefore DR is greater also than 4 DS. Q. E. D. *This being premised, we may proceed to confirm what was laid down in the Text above.*

1. Let the Curve DPQ be a Parabola, and the Circle DKR being of the same Curvature with the Parabola at the Point D, as supposed above, the Velocity a Body ought to set off with from D, to describe the Circle, is the same with that with which it ought to set off, to describe the Parabola: but the Velocity it ought to set off with to describe the Circle, is by Lemma 2. such as it would acquire by falling to S, because DS being (by Lemma 10.) a Quarter of DR, is equal to half the Radius of the Circle; call this Space 2: but the Velocity it ought to set off with to describe a Circle, whose Center is S, is such as it would acquire by falling through half DS, which Space (because the whole Line DS is called 2) must be called one. Now the Velocities Bodies acquire by falling through certain Spaces are to each other as the Square Roots of those Spaces (by Part I. Chap. 5. § 6.) the abovementioned Velocities therefore are to each other as $\sqrt{2}$ to 1. That is, the Velocity a Body ought to set off with from D (the Vertex of the Figure, and in a Direction perpendicular to the Axis DS, which I would always have supposed) in order to revolve in a Parabola whose Focus is S, is to that it ought to have at the same Place (and in the same Direction) to describe a Circle whose Center is S, as $\sqrt{2}$ to 1.

2. Had the Figure DPQ been an Ellipse, a Quarter of DR the Diameter of a Circle of the same Curvature therewith, had been (by Lemma 10.) less than DS; and so the Velocity the revolving Body must have had to describe that Circle, or (which is the same Thing, because of their equal Curvatures) the Ellipse would have been acquirable by falling through a less Space than DS, and therefore would have been a less Velocity. That is, it would have exceeded the Velocity requisite for a Circle, whose Center is S the Seat of the retaining Power, in a less Proportion than that of $\sqrt{2}$ to 1.

3. If the Figure DPQ be an Hyperbola, a Quarter of DR is bigger than DS (by Lemma 10.) and so the Space a Body must fall through to get a competent Velocity for that Curve, will be greater than what it must fall through to obtain a competent Velocity for the Parabola; The Velocity therefore requisite for the

Hence we may observe, that, supposing the Motion of a revolving Body be so adjusted to the Power by which it is retained, that it shall describe a *circular* Orb, and its Velocity be afterwards altered by some Accident, and thereby the Adjustment thereof to the central Force destroyed, that Body shall not immediately fall to the Center: the Form of its Orbit only will be altered, and from being circular, will become elliptical, parabolical, or hyperbolical, according to the Alteration made in its Velocity. Let its Velocity be diminished in any Degree whatever, or let it be increased, but so that it still may bear a less Proportion to that which is requisite to make the Body describe a Circle than the Square Root of 2 does to 1, and the Orbit will become an Ellipse; if its Velocity be so far increased as to bear the same Proportion to that which is necessary to make it describe a Circle, as the Square Root of 2 does to 1, its Orbit becomes a Parabola; if its Velocity be still farther increased, the Orbit becomes an Hyperbola. Hence we have a probable Reason why the Orbits of the Planets are now elliptical; for supposing them to have been circular at first, as it is not unlikely they were, yet upon the first Disturbance in their Motions, whether from their

the Hyperbola exceeds the Velocity required for a Circle whose Center is S, in a greater Proportion than that of $\sqrt{2}$ to 1. From which all that is asserted above in the Text to which this Note refers, except what is there otherwise accounted for, may be collected.

mutual

mutual Action upon each other, the Resistance of the Medium they revolve in, the Access of a Comet, or any other Cause whatever; and whether their Motion be increased or decreased thereby, provided it be not increased in a Degree beyond the Proportion above laid down, their Orbits would immediately become elliptical. Should the Motion of any of them be increased so as to come up to, or exceed the Degree abovementioned, its Orbit would accordingly be changed into a Parabola, or an Hyperbola, and the Planet would go off, never to return.

PROP. VIII. If the Force of the central Power decreases as the Square of the Distance increases, and several Bodies revolving about the same, describe Orbits that are *elliptical*, the Squares of the periodical Times of those Bodies will be to each other as the Cubes of their middle Distances from the Seat of that Power (*k*).

(*k*) To demonstrate this Proposition, let the following Lemma be premised.

LEMMA XI.

The Area a revolving Body describes, is as its Velocity multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is.

Demonst. of the Lemma. Let AB in Fig. 36. be the Curve described, DC a Portion of the Curve described in the least Time possible, then will DC represent the Velocity of the Body; produce DC to P, then will DP be a Tangent to the Curve: From S the Seat of the retaining Power let fall the Perpendicular SP, draw DS and CS; then will DCS be the Area the Body describes, but DC by reason of its shortness may be considered as a straight Line, the Area DCS is therefore a Triangle, which is had by multiplying the Base DC into half the Perpendicular SP, therefore

therefore it is proportionable to DC multiplied by the whole Line SP, that is, to the Velocity of the revolving Body multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is. Q. E. D.

Dem. of the Proposition. Let ADB and *adb* in Fig. 37 and 38. represent two Ellipses, whose principal Parameters are L and L, and let S be one of the Foci of the one, and *s* one of those of the other, and let the Ellipses be supposed to lie in such manner upon one another, that the Points S and *s* may be one and the same Point, which let us suppose to be the Seat of the retaining Power; and let the revolving Bodies be at D and *d*, through which draw the Tangents PD and *pd*, the Perpendiculars PS and DC, and *ps* and *dc*, and the Lines DS and *ds*; and let DC and *dc* be Radii of Curvature to the Points D and *d*; and let the Force of the retaining Power at the Distance SD be called F, and at the Distance *sd* be called *f*, and let the Velocities of the revolving Bodies when at the Points D and *d* be called V and *v*. Then because by the Proposition the Force of the central or retaining Power is supposed to decrease as the Squares of the Distances SD and *sd* increase, that is, to be reciprocally as the Squares of those Distances, we shall have for the first Step of the following Process, this Analogy, *viz.* | 1 | $F : f :: \frac{1}{SD^q} : \frac{1}{sd^q}$

Parallel to the Tangents draw the Lines SH and *sh*, and let the Force of the retaining Power which acts upon the Body at D in the Direction DS be resolved into two others, *viz.* DP and DH, the former along the Tangent DP, the latter along the Radius of Curvature DC, then it is obvious that it is by this latter Force only that the revolving Body is retained in the Circle whose Radius is DC, that is, in the Ellipse at the Point D, the other Force along the Tangent only accelerates the Body when moving towards A, and retards it if moving towards B. To compute the Quantity of Force in the Direction DC, say as DS is to DH so is the oblique Force F, or instead thereof $\frac{1}{SD^q}$ (see Step the 1st) to a fourth Number, which fourth Number by the Golden Rule will be $\frac{DH}{SD^3}$, or (because SP is equal to DH) $\frac{SP}{SD^3}$; and in like manner we shall have $\frac{sp}{sd^3}$ for the Force retaining the other Body in the Circle whose Center is *c*; but by Corollary of Lemma 3. the Force necessary to retain a Body in a Circle whose Radius is CD is expressible by the Square of its Velocity divided by the Radius of the Circle it describes, that is, in the present

Case

Case by $\frac{V^q}{1\,CD}$. And in like manner the Force retaining the other

Body in its Circle, will be $\frac{v^q}{cd}$, consequently	2	$\frac{SP}{SD^3} : \frac{sp}{sd^3} :: \frac{V^q}{CD} : \frac{v^q}{cd}$
Multiplying the last Step by CD : *cd*	3	$\frac{SP}{SD^3} \times CD : \frac{sp}{sd^3} \times cd :: V^q : v^q$
Since CD is supposed to be the Radius of a Circle of equal Curvature with the Figure, we shall have by Lemma 9. these Equations, *viz.*	4	$CD = \frac{L \times SD^3}{2\,SP^3}$, and $cd = \frac{l \times sd^3}{2\,sp^3}$
Comparing the 3d and 4th Steps	5	$\frac{SP}{SD^3} \times \frac{L \times SD^3}{2SP^3} : \frac{sp}{sd^3} \times \frac{l \times sd^3}{2sp^3} :: V^q : v^q$
That is, by reducing the last Step to lower Terms	6	$\frac{L}{SP^q} : \frac{l}{sp^q} :: V^q : v^q$
Extracting the Square Root of each Term	7	$\frac{\sqrt{L}}{SP} : \frac{\sqrt{l}}{sp} :: V : v$
Multiplying by SP : *sp*	8	$\sqrt{L} : \sqrt{l} :: V \times SP : v \times sp$
But by Lemma 11. the Area one Body describes is to that which the other describes in the same time, as V × SP to *v* × *sp*, which Areas call A and *a*, and we shall have this Proportion, *viz.*	9	$A : a :: V \times SP : v \times sp$
Comparing the 8th and 9th Steps	10	$A : a :: \sqrt{L} : \sqrt{l}$

Now the Area of the whole Ellipse ADB is to that of the other *adb*, as AB the longer Axis multiplied by QN the shorter, to $ab \times qn$, those Quantities therefore may be put for the Areas themselves: If then we call the Time in which the Areas A and *a* are described in, one, (as one Hour or one Minute suppose) and the Times the whole Ellipses are described in T and

and t; we shall have (since the Area a revolving Body describes is proportional to the Time it is described in) the following

Analogies, *viz.*	11	$1 : T :: A : AB \times QN$
And	12	$1 : t :: a : ab \times qn$
Comparing the 3 last Steps together we have the two following ones, *viz.*	13	$1 : T :: \sqrt{L} : AB \times QN$
	14	$1 : t :: \sqrt{l} : ab \times qn$
By De L'Hospit. Con. Sect. Liv. 2. Def. 8.	15	$AB : QN :: QN : L$
Turning the last Proportion into an Equation	16	$AB \times L = QN^2$
Extracting the Square Roots of each Side of the Equation	17	$\sqrt{AB} \times \sqrt{L} = QN$
And in like manner	18	$\sqrt{ab} \times \sqrt{l} = qn$
Comparing the 13th and 17th Steps together	19	$1 : T :: \sqrt{L} : AB \times \sqrt{AB} \times \sqrt{L}$
And comparing the 14th and 18th together	20	$1 : t :: \sqrt{l} : ab \times \sqrt{ab} \times \sqrt{l}$
Dividing the latter Part of the 19th Proportion by $\sqrt{L}$	21	$1 : T :: 1 : AB \times \sqrt{AB}$
Dividing the latter Part of the 20th Proportion by $\sqrt{l}$	22	$1 : t :: 1 : ab \times \sqrt{ab}$
Comparing the 21st and 22d	23	$T : t :: AB \times \sqrt{AB} : ab \times \sqrt{ab}$
Squaring each Term	24	$TT : tt :: AB^2 \times AB : ab^2 \times ab$
That is	25	$TT : tt :: AB^3 : ab^3$

But half the longer Axis AB is equal to the middle Distance SN, (De L'Hospit. Conic. Sect. Liv. 2. Coroll. 3. of the Definitions) therefore since Halves are proportionable to their

Wholes	26	$TT : tt :: SN^3 : sn^3$. Q. E. D.

PROP.

Prop. IX. If the retaining Power decreaſes ſomething faſter as you go from the Seat thereof (or which is the ſame thing, increaſes ſomething faſter as you come towards it) than in the Proportion mentioned in the laſt Propoſition, and the Figure the revolving Body deſcribes be not a Circle, the Axis of that Figure will turn the ſame Way the Body revolves; but if the ſaid Power decreaſes (or increaſes) ſomewhat ſlower than in that Proportion, the Axis of the Figure will turn the contrary Way. Thus, if a revolving Body as D, Fig. 39. paſſing from A towards B deſcribes the Figure ADB, whoſe Axis AB at preſent points towards M and N, and the Power whereby it is retained decreaſes faſter than the Square of the Diſtance increaſes, after a Number of Revolutions the Axis of the Figure will point towards O and P, and after that towards Q and R, &c. revolving round the ſame Way with the Body, and if the retaining Power decreaſes ſlower than in that Proportion, the Axis will turn the other Way (*i*).

(*i*) *Dem.* Let S in Fig. 40. repreſent the Seat of the retaining Power, and let a Body as D, deſcribe the Orb ADB paſſing from A towards B; and let another as N deſcribe a revolving Orb as MNO equal and ſimilar to the former, and let both the Orbits be deſcribed in the ſame Time; and let the Velocity wherewith the Orb MNO revolves be ſuch, that the Angle ASN ſhall always be proportionable to the Angle ASD. To explain this a little farther, ſuppoſe that when the Bodies N and D ſet out from M and A, thoſe two Points were then together at A; or that the Orbits did then coincide; but that N advances forwards in ſuch Manner, as that if the Angle ASN is at any

time

time double, treble, or in any other Proportion of the Angle ASD, it shall always be so: That is, that the Line SN shall always move faster or slower than the Line SD in some certain Proportion; and that the Point M shall follow or go from the Body N, so as that the Arch MN shall always be equal to the Arch AD. And let it farther be supposed that the Line SN is always equal to SD.

The first Inquiry which arises here is, whether this be a possible Case; that is, whether the Body N moving with the above-mentioned Restrictions, the Line SN can describe Areas proportionable to the Times, because as was shewn, Part I. Chap. 8. that is always requisite where Bodies revolve round a central Force, and are retained in their Orbits thereby. Let it then be considered in the first Place, that the Body D revolving in the quiescent Orb ADB may do so, its Motion being clogged with none of those Suppositions; secondly, that the Line NS is always equal to DS, and the Velocity wherewith its Extremity N moves forwards, is proportionable to that which D the Extremity of the other moves with; and consequently the Areas which one describes, (for the Areas depend solely on the Length and Velocity of the Lines that describe them) will be proportionable to those which the other describes; Since then those of the former may be proportionable to the Times they are described in, it is possible those of the latter may be so too; the Supposition therefore above laid down is not absurd.

The next Inquiry is, by what Law the Action of the central Force at S must decrease as we recede from it, that a Body as N may describe the revolving Orb MNO in the Manner above-mentioned. Or, which will answer our Purpose as well, how the Force necessary to retain the Body N describing the revolving Orb MNO, must differ from that which is requisite to retain the other Body D describing the quiescent Orb ADB equal and similar thereto.

In order to this, let DR, in Fig. 41. represent a Portion of the quiescent Orb described in the least Time possible, this then may be considered as a strait Line; and let the Motion of the Body along this Line be resolved into two, the one towards S along the Line DT, and another along the Line TR at right Angles therewith; then will DT represent the Velocity wherewith the Body descends towards S, and consequently the Action of the central Force whereby it is retained in the Arch DR; and TR will represent the Velocity by which it advances forwards in the mean time. Let it now be supposed that the Orb ADB advances forwards, while the Arch DR is describing, but not by Virtue of any additional Force in the Center, but by some extrinsic one, no matter what: That is, suppose that the Line SD turns round

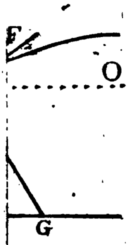
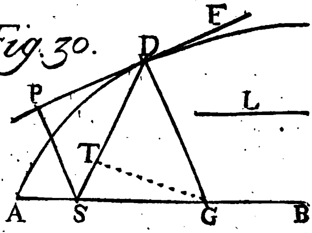


Fig. 30.



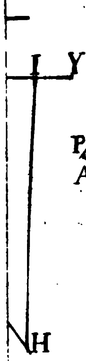
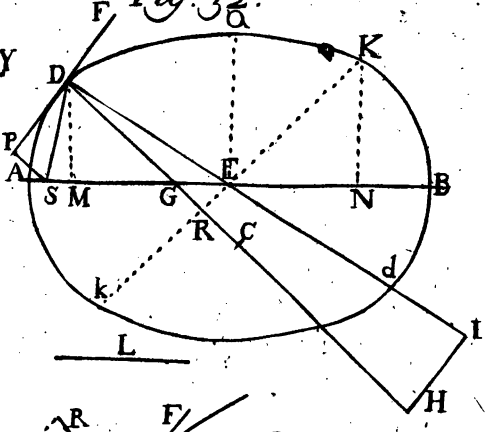
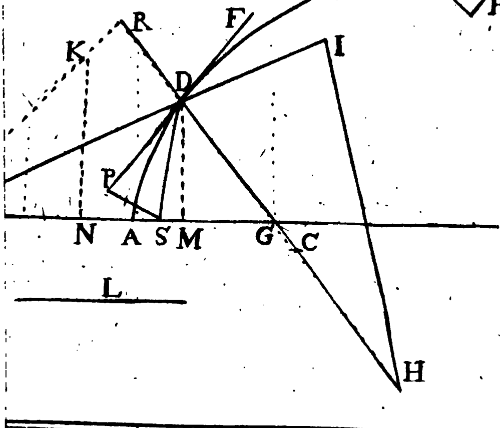


Fig. 32.





the Point S faster than it would otherwise have done, carrying the Body with it so much the faster; but that the Body descends towards S along that Line just as it did before: That is, in other Words, that the Line DT which represents the Descent of the Body, is of the same Length as before; but that the other Line TR which represents the Progress of the Body forwards, is longer than it was in the other Case. Let it then become TQ; in which Case, the Body moving over the Line DT and the Line TQ in the same Time, will in reality get to the Point Q: But observe, that the Point Q is farther from S than the Point R is, because the Angle at T is a right one; whereas the Law which we laid down above for the Motion of a Body describing a revolving Orb was such, that at the End of the Time in which DR would be described in the quiescent Orb, the Body in the revolving Orb, should be at the same Distance from S as if its Orb had not revolved; if then round the Point S, we describe a Circle as RFG, and through S draw the Line QG, N must be the Point the Body must come to. Since then the Body in describing the revolving Orb without any Addition to the central Force above what would cause it to describe the quiescent one, would come to Q; but with such an Addition as is necessary to make it revolve in the manner above laid down, it comes to N, it is obvious, that the Line QN being the Distance between those two Points will aptly represent that Addition. It remains then to get the Measure of that Line, which may be done in the following manner.

Produce QT to F, then (by 36. El. 3.) will the Rectangle $QN \times QG$ be equal to $QR \times QF$. Therefore

	1	$QN \times QG = QR \times QF$
Dividing by QG	2	$QN = \frac{QR \times QF}{QG}$

Let it be observed then in the first Place, that as the Motion of revolving Bodies is such that they describe equal Areas in equal Times, the longer the Line DS is (that is the nearer D is to A, suppose) the shorter the Line TR must be, and for the same Reason also the Line QT: That is, both TR and QT are reciprocally as SD, which let us put thus

	3	$TR : \frac{1}{SD}$
And	4	$QT : \frac{1}{SD}$
Observe secondly, that since RF is a Chord of a Circle, and ST perpendicular to it	5	$TR = TF$

From

From the Figure	6	$QR = QT - TR$
Because RT and TF are equal	7	$QF = QT + TR$

Since then by the 3d and 4th Steps, TR and QT are both reciprocally as SD, and by the 6th and 7th Steps, QF is equal to the Sum of those Lines, and QR is equal to their Difference, it is obvious that QF and QR are also each reciprocally as SD,

which put thus	8	$QF : \frac{1}{SD}$
And	9	$QR : \frac{1}{SD}$
Multiplying the 8th and 9th Steps together	10	$QR \times QF : \frac{1}{SD^q}$

Observe also that when the Points D, T, R, Q and N coincide, which must ever be supposed, because we are all this Time considering only what is done at one and the same Point of the Curve ADB, QN is nothing with respect to NG, therefore QG and NG are the same Line; therefore QG may be said to be as NG, or as half of it, *viz.* RS, or which will be the same Thing, DS; therefore instead of QG in the 2d Step, we may put DS, and

then it will stand thus	11	$QN = \frac{QR \times QF}{DS}$
The 11th Step without altering the Value of it, may be set thus	12	$QN = QR \times QF \times \frac{1}{DS}$
Comparing the 10th and 12th Steps together we have	13	$QN = \frac{1}{DS^q} \times \frac{1}{DS}$
That is	14	$QN = \frac{1}{DS^{3}}$.

That is, the Line QN, or the additional central Force requisite to cause a Body to move in a revolving Orb must be reciprocally as the Cube of the Distance of the revolving Body from the Seat of the retaining Power. Whereas then, when the centripetal Force decreases as the Square of the Distance increases, the revolving Body describes a Parabola, an Ellipse, or an Hyberbola; if to that centripetal Force be superadded another, that shall decrease as the Cube of the said Distance increases; those two Forces acting conjointly upon a revolving Body shall cause it to describe the same Figure as before, but the Axis thereof shall revolve the same Way that the Body does.

But observe here, that if a Force decreases faster in any Degree whatever than the Square of the Distance increases, but slower than the Cube, that Force is the Sum of two Forces, one of which

Thus it is the heavenly Bodies, *viz.* the Planets both primary and secondary, and also the Comets, perform their respective Revolutions. The Figures in which the primary Planets and the Comets revolve, are Ellipses, one of whose Foci are at the Sun. The Areas they describe by Lines drawn to the Center of the Sun, are in each proportionable to the Times in which they are described. The Squares of their periodical Times are to each other as the Cubes of their middle Distances from the Sun. The secondary Planets describe also Circles or

which decreases as the Square, the other as the Cube of the Distance increases. Thus for Instance, let the Distances be as 2 to 1, the Squares of them will be 4 to 1, their Cubes 8 to 1; and let the Forces be as 6 to 1; I say, this Proportion will arise from adding 4 to 1 to 8 to 1; for 4 to 1 added to 8 to 1, is 12 to 2, that is 6 to 1. Again, let the Forces be as 5 to 1; I say, this also will arise from adding 4 to 1 to 8 to 1; for instead of 4 to 1 let us take 12 to 3 which is the same Thing, this superadded to 8 to 1 makes 20 to 4, that is, 5 to 1. Again, let the Forces be as 7 to 1; this also is the Sum of 4 to 1 added to 8 to 1; for instead of 8 to 1 we may take 24 to 3, which added to 4 to 1 gives 28 to 4, or 7 to 1. And the same for Fractions: so that let a Proportion be what it will between 4 to 1 and 8 to 1, it may be considered as arising from 8 to 1 added to 4 to 1. When the retaining Power therefore by which a revolving Body is kept in its Orbit, decreases faster than the Square of the Distance increases, but not so fast as the Cube thereof does; it is a Power decreasing as the Square, having another Power decreasing as the Cube of the Distance superadded to it; and therefore as the Proposition in the Text asserts, if the retaining Power decreases something faster, &c. Q. E. D. As to the Figures turning the contrary Way, when the centripetal Force decreases somewhat slower than as the Square of the Distance increases; that is demonstrable after the same manner, *mutatis mutandis*; and therefore needs not be insisted on.

Ellipſes, one of whoſe Foci are in the Center of their primary ones. The Area that each deſcribes by Lines drawn to the Center of its Primary, is proportionable to the Time it is deſcribed in. The Squares of the periodical Times of the Secondaries belonging to the ſame Primary, are to each other as the Cubes of their Diſtances from their Primary. And above all, that very Force by which Stones or heavy Bodies fall to the Surface of the Earth with us, is at the Diſtance of the Moon, juſt ſufficient to retain her in her Orbit, ſuppoſing it to decreaſe as we go from the Center of the Earth in that Proportion, in which the Cauſe, that retains her and the reſt of the Planets in their Orbs, whatever it be, muſt do to make them elliptical (*l*). And farther, which alſo is a neceſſary

(*l*) To find out whether this be ſo or not, let us calculate what Velocity the Moon would acquire, were ſhe to fall half way to the Center of the Earth by Virtue of that Gravity, whereby heavy Bodies tend to the Earth; and compare it with the Velocity ſhe moves with in her Orbit; becauſe by Lemma 2 of this Chapter, if ſhe be retained in her Orb by that Gravity, thoſe Velocities ought to be the ſame. The mean Diſtance of the Moon from the Earth in round Numbers is 60 Semidiameters of the Earth, therefore the Force of Gravity at the Diſtance of the Moon is the Square of 60 times leſs than it is at the Surface of the Earth; therefore the Fall of a Body at that Diſtance in a Second of Time, will be ſo many times leſs than it is here; but the Fall of a Body here in a Second, is about 15 *Paris* Feet, conſequently at the Moon it is but ,004166 Feet. The Space then a Body would move over in a Second, with the Velocity acquired by that Fall, is, by Part I. Chap. 5. § 7. twice that Quantity, *viz.* ,008332 Feet, which Number therefore may be put to expreſs the Velocity a Body would acquire thereby. Now becauſe the Velocities Bodies acquire by falling, are as the ſquare Roots

necessary Consequence of what we are about to lay down, the Motion of the Planet *Saturn* is observed to be disturbed by *Jupiter*; and the Secondaries of *Jupiter* to be disturbed in their Motions upon the nearer Approach of *Saturn*; and the Course of the Moon is incessantly altered by the Action of the Sun, in such manner as to cause all those Irregularities mentioned in Chapter the 8th of this Part. All which, together with the Precession of the Equinoctial Points, the Nutation of the Poles of the Earth, and the Phænomena of the Tides, which naturally flow from it, make it extremely probable that there is a Virtue diffused about the Sun and Planets, not unlike that of Attraction, which decreases as the Squares of the Distances from the Centers of those Bodies increase, and acting upon Bodies in Proportion to the Quan-

Roots of the Spaces they fall through, Part I. Chap. 5. § 6. say, as the Square Root of ,004166, is to the Square Root of 590866170, which is half the Semidiameter of the Moon's Orbit, (supposing the Semidiameter of the Earth to be 19695539 Feet, which is *Cassini*'s Measure, and the Distance of the Moon from the Earth to be 60 Semidiameters above) so is ,008332 to 3135 Feet, which is therefore the Space the Moon would move over in a Second with the Velocity she would acquire by falling half way to the Center of the Earth. But this comes within an hundredth Part of her Velocity in her Orbit, as may easily be calculated from the Time she revolves in, *viz.* 27 Days, 7 Hours, and 43 Minutes, and the Semidiameter of her Orbit, which according to the Measures we have taken above, is 1181732340 Feet. That Force therefore by which Bodies fall to the Ground with us, is at the Distance of the Moon such as is requisite to direct her Course.

N. B. The Proportion of a *Paris* Foot to that of *England*, is as 367196 to 343800.

tity of Matter they contain, causes the Comets and primary Planets to revolve about the Sun, and the Secondaries about their respective primary ones, according to the Tenour of the Propositions laid down in this Chapter.

This being allowed, it will follow, that as the Sun attracts the Planets, and thereby retains them in their Orbs, they in like manner attract the Sun, though with Forces proportionable only to the Quantities of Matter they contain; so that strictly speaking, each primary Planet revolves not about the Center of the Sun, but about a Point which is the Center of Gravity between the Sun and that; and that the Sun moves also round that Point, and is always opposite to the Planet with respect thereto: And likewise that the Center of the solar System is not in any one Body, but in the common Center of Gravity of all the Bodies of which it consists. But then the Sun is so immensely large in respect of any one, or all of those Bodies put together, that that Center is very near the Center of the Sun. In like manner, the Moon does not revolve about the Center of the Earth as a Point at rest, but the Earth and Moon revolve each about the Center of Gravity common to them both, which Center of Gravity it is, and not the Center of the Earth, that describes the Orbit the Earth is commonly said to revolve in. And so of the other Planets which are attended by Secondaries.

The

The Irregularities in the Motion of the Planet *Saturn*, and those of the Secondaries of *Jupiter*, owing to the Cause abovementioned, are so exceeding small, that it shall suffice to have just mentioned them.

The more remarkable Effects of the disturbing Force of the Sun, are the lunar Irregularities, the Precession of the Equinoctial Points, the Nutation of the Poles of the Earth, and the ebbing and flowing of the Sea, which shall be particularly considered in the following Chapter.

CHAP. XIX.

Of the Lunar Irregularities, *the* Precession *of the* Equinoctial Points, *the* Nutation *of the* Poles *of the* Earth, *and the* ebbing *and* flowing *of the* Sea.

I. TO account for the lunar Irregularities, let S in Fig. 42. represent the Sun, T the Earth, and LMNO the Orbit of the Moon, and let the Moon be in one of its Quadratures at L, and let the Lines LS and TS be drawn. It is obvious, that the Tendency the Moon has towards the Sun is along the Line LS, and that which the Earth has, is along the Line TS: Let then the former of these be resolved into two others, the one along LA

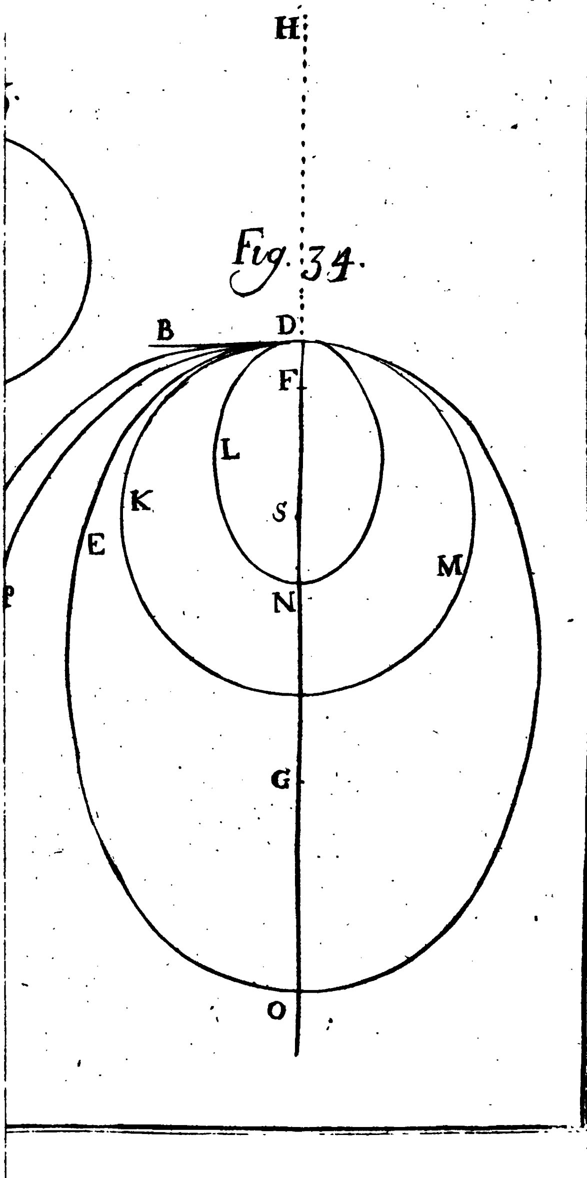
parallel and equal to TS, the other from L to T along the Line LT. The former of these Tendencies being parallel and equal to that whereby the Earth tends along the Line TS, alters not the Situation of the two Bodies L and T with respect to each other; that is, it disturbs not the Motion of the Body L; but the other along LT increases its Tendency towards T.

And this Increase will be to the Tendency the Moon has to A, which is the same the Earth has to S, as the Distance LT to LA, or TS. Or in other Words, the Gravity of the Moon towards the Earth in the Quadratures is augmented by the Action of the Sun; and that Augmentation is to the Tendency the Earth has to the Sun, as the Length of the Line LT, or the Distance of the Moon from the Earth, to TS the Distance of the Earth from the Sun.

So that the greater the Moon's Distance is from the Earth, the Distance of the Sun remaining the same, the greater will this increase of the Moon's Gravity towards the Earth be. But if the Distance of the Moon from the Earth remains the same, and the Distance of the Sun be augmented, this additional Increase will be the less in Proportion to the Cube of that Distance (*a*).

Let

(*a*) For, if TS be increased while LT remains the same, LT will be so much the less with respect to TS, that is the Increase will be diminished in Proportion to the Sun's Distance: But


H
Fig. 34.
B
D
F
L
K
S
E
M
P
N
G
O

Let now the Moon be in one of its Syzygies at M, then will the Tendency ſhe has to the Sun over and above what the Earth has, which is farther off at T, be to that which the Earth has, as the Difference of the Squares of SM and ST is to the Square of SM: but the Difference between the Squares of SM and ST bears nearly ſuch Proportion to the Square of SM as twice MT, that is MO, does to SM; becauſe the Difference between the Squares of two Numbers whoſe Difference is very ſmall with reſpect to either of them (as the Difference between SM and ST is with reſpect to the Diſtance of S) bears little more than double the Proportion to the Square of the leſſer Number, that the Difference between the Numbers themſelves bears to the leſſer Number (*m*). The

But when TS the Diſtance of the Sun is increaſed, the abſolute Force of the Sun, and therewith the abovementioned Increaſe, will be diminiſhed alſo in proportion to the Square of that Diſtance, conſequently taking in both the Accounts, it will upon the whole, be diminiſhed in Proportion to the Cube of that Diſtance.

(*m*) *Dem.* Let a be the leſſer Number, and $a+b$ the larger, and let their Difference b bear no ſenſible Proportion to the leſſer a.

Then the Square of the leſſer is ——	aa
The Square of the larger is ——	$aa+2ab+bb$
The Difference between theſe is ——	$2ab+bb$
And bb being rejected as inconſiderable, the Difference is only	$2ab$
Now the Proportion of $2ab$ to aa is (taking a out of each Term)	$2b$ to a

But $2b$ to a, is double the Proportion that once b has to a, and therefore the Difference between the Squares of the two Numbers bears little more than double the Proportion to the Square of the leſſer Number, that the Difference between the Numbers themſelves bears to the leſſer Number. Q. E. D.

Tendency

Tendency therefore the Moon when at M, has to the Sun, over and above what the Earth has, is to that which the Earth has, nearly as MO, or twice LT, to SM, or because of the Sun's great Distance, as twice LT to TS. Her Tendency therefore to the Earth is now diminished in that Proportion; But as was shewn above, it was augmented in the Quadratures in the Proportion only of LT to TS. The Diminution here is therefore nearly double of the Augmentation there.

And whereas that Augmentation, when the Distance of the Sun remains the same, was shewn to increase with the Distance of the Moon; but when the Distance of the Moon remains the same, to decrease with the Cube of the Sun's Distance; this Diminution being always nearly double of that, will do the same.

When the Moon is in the other Syzygy at O, she is attracted towards the Sun less than the Earth is by the Difference of the Squares of SO and ST; which as to the Effect, is the same Thing as though the Earth was not attracted at all towards S, and the Moon were attracted the contrary Way, so that her Tendency to the Earth is here also diminished, as well as when she was at M, and almost in the same Degree; for on Account of the Sun's great Distance, the Difference between the Squares of SO and ST is nearly the same as between ST and SM.

Or because this Way of accounting for the Diminution of the Gravity of the Moon towards

wards the Earth in the Syzygies may not be sufficiently clear, it may be considered otherwise, thus. The annual Course of the Moon round the Sun being performed in the same Time that the Earth's is, she ought to be retained in that Course by the same Force that the Earth is, whereas when she comes to M, the Action of the Sun upon her is greater than it is upon the Earth, by the Difference of the Squares of SM and ST; and when she is at O, it is less than it is upon the Earth by the Difference between the Squares of ST and SO: So that in the former Case she is drawn too much towards the Sun, and in the latter too little; and therefore in both Cases her Tendency towards the Earth is diminished; and almost in the same Degree; because, as was observed above, the Difference of the abovementioned Squares is nearly the same in either Case.

Let the Moon be in a Point of her Orbit between the Quadrature and the Syzygy, as at L in Fig. 43. Then being nearer the Sun than the Earth is, she will be attracted with a stronger Force: Let it be expressed by LS produced to D 'till LD be of such Length, that TS being put to express the Action of the Sun upon the Earth, LD may be long enough to express the stronger Force of the Sun upon the Moon: And let LD be resolved into two others, one of which let be LA equal and parallel to TS, then will the other be AD, or its equal and parallel LG. This LG

is

is the only disturbing Force upon the Moon at L, the other LA being parallel and equal to TS, affects the Moon just as the Sun does the Earth; and so alters not their Situations with respect to each other. Let then, to avoid a Confusion of Lines, this Figure with the Line LG, be removed to the 44th. This Force LG may be resolved into LI and LH, the one a Tangent to the Orbit of the Moon, and the other perpendicular thereto: The former accelerates the Motion of the Moon when going from the Quadrature at Q to the Syzygy at B; and will retard it when going from B to R. The other when H falls upon TL produced, as in this Figure, diminishes the Tendency of the Moon towards the Earth, and when it falls between L and T, it augments it.

When the Moon is at L between the Quadrature R and the opposite Syzygy O, in Fig. 45. the Tendency of the Moon towards the Sun is less than that of the Earth; it may then be expressed by LD a Line shorter than TS, and is resolvable into LA a Line equal and parallel to TS and DA, or its equal and parallel LG. Which LG is the only disturbing Force, and may, as in Fig. 44. be resolved into two others, one of which shall draw the Moon towards O, the other to or from T, as the Case may happen. So that in the first Place, the nearer the Moon is to its Syzygies, the greater will be its Velocity; and the nearer it is to the Quadratures, the slower it will move; because

because one of the Forces into which LG is resolvable (as LI in Fig. 44.) accelerates its Motion from the Quadratures to the Syzygies; and retards it as much from thence to the Quadratures. *Which is the first Irregularity* (*n*).

2. When the Moon is in the Quadratures as at L or N, or in the Syzygies as at M or O, see Fig. 42. the disturbing Force is directed to or from the Center of the Earth; and therefore when the Moon is passing those Points it is no Impediment to her describing Areas proportionable to the Times; but when she is at L in Fig. 44, or 45, where the disturbing Force is expressed by LG, only one of the Lines into which it is resolved, as LH, points to or from the Center of the Earth, the other, as LI, pointing another Way, prevents her describing Areas proportionable to the Times. So that it is only in the Quadratures and Conjunctions, that the Areas are proportionable to the Times. *Which is the second Irregularity.*

3. The Motion of the Moon being accelerated during her Progress from the Quadratures to the Syzygies, and retarded from thence to the Quadratures, her Motion in the Syzygies is too quick, in the Quadratures too slow: add to this, her Tendency to the Earth is in the former Situation too small, in the latter too large: Both which conspire to render her Orbit more curved in the Quadratures than in the Syzygies, so that she runs off farther from

(*n*) See the Lunar Irregularities enumerated in Chap. VIII.

the

the Earth in the Quadratures, and comes nearer in the Syzygies than ſhe would otherwiſe do, deſcribing an Orbit, one of whoſe Axes, *viz.* that which paſſes through the Quadratures, is longer than that which paſſes through the Syzygies. *Which is a third Irregularity.*

4. The Gravity of the Moon towards the Earth in the Syzygies being about twice as much diminiſhed by the Action of the Sun, as it is augmented in the Quadratures; if we take a whole Revolution together, it may be conſidered as diminiſhed only. In the Perihelion therefore, at which Time the Earth and Moon are neareſt the Sun, it will be diminiſhed the moſt of all; that Diminution (as ſhewn above) being inverſly as the Cube of the Sun's Diſtance, and ſo the Gravity or Tendency of the Moon to the Earth will be the leaſt. On which Account ſhe will run out into a greater Orbit; and ſo her periodical Time will be greater, than when the Earth is in its Aphelion. *Which is a fourth Irregularity.*

5. When the Moon is in the Quadratures, the Action of the Sun (as ſhewn above) increaſes the Tendency of the Moon to the Earth in Proportion to her Diſtance from thence, this Force ſuperadded to the Action of the Earth upon the Moon, which decreaſes as the Square of the Diſtance increaſes, occaſions that Force to decreaſe as you go from the Earth, *ſlower* than it would otherwiſe do; or which is the ſame Thing to increaſe ſlower as you go the

other way. But by Prop. 9. of the foregoing Chapter, when a retaining Power decreases or increases slower than the Square of the Distance increases or decreases, and the Planet describes an Ellipse, the *Linea Apsidum* of that Planet will go backwards; the *Linea Apsidum* therefore of the Lunar Orbit, when she is in the Quadratures, goes backwards: When the Moon is in the Syzygies the Action of the Sun diminishes her Tendency towards the Earth, and thereby make it decrease as you go from, or increase as you go to the Earth, too fast; and so the *Linea Apsidum* at that Time goes forwards. But the Diminution in the Syzygies exceeds the Augmentation in the Quadratures, and so the *Linea Apsidum* goes farther forwards than backwards every Time, till at Length it revolves quite round according to the Order of the Signs. *Which is a fifth Irregularity.*

6. When the Gravity by which a Planet is retained in an Orbit that is excentrical, decreases or increases *too fast*, the Planet when going off from the Seat of the retaining Power, that is, towards its upper *Apsis*, will go off too far; and when it is coming to its lower *Apsis*, it will approach too near; and so the Excentricity of its Orbit will be increased. When its Gravity decreases or increases *too slow*, the Planet will not in the former Case go off far enough, nor come so near to the Seat of the retaining Power in the latter, as it ought to do; in this Case therefore, the Excentricity of its Orbit

Orbit will be diminiſhed. But the Tendency of the Moon to the Earth when in the Syzygies, decreaſes or increaſes too faſt, the Excentricity of her Orbit is therefore at that time the greateſt; and on the contrary it is the leaſt when ſhe is in the Quadratures. And if we compare ſeveral Revolutions of the Moon together, we ſhall find, then when the *Linea Apſidum* is in the Quadratures, the Excentricity of the Lunar Orbit will be the leaſt of all; becauſe in that Situation of the *Linea Apſidum*, the Difference between the Tendency the Moon has to the Earth in one of the Apſes, and that which it has in the oppoſite one, is the leaſt of all: Whereas when the *Linea Apſidum* is in the Syzygies, that Difference will be the greateſt; and therefore the lunar Excentricity will be ſo too." *Which is a ſixth Irregularity.*

7. We have hitherto been conſidering ſuch Irregularities in the Courſe of the Moon as would happen if its Orbit were coincident with the Plane of the Ecliptic: But as it is not ſo, there will ariſe others, in order to account for which, it will be proper to premiſe the following Conſiderations.

Firſt, That when the Line of the Nodes is in the Syzygies, the Plane of the Moon's Orbit paſſes through the Center of the Sun as well as through that of the Earth, and ſo the Moon in that Situation of the Nodes, is not drawn

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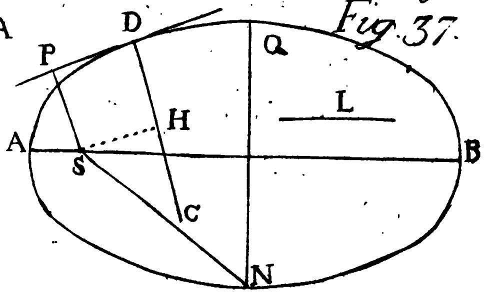
Fig. 37.



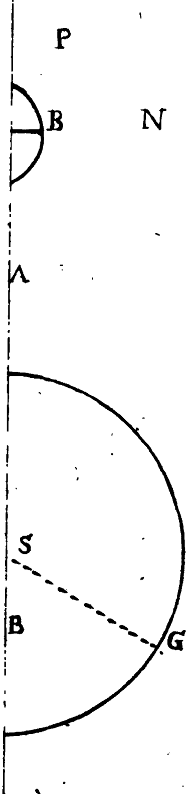


Fig. 38.

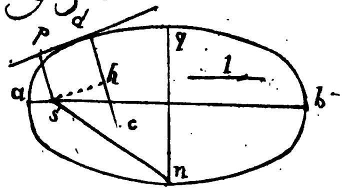
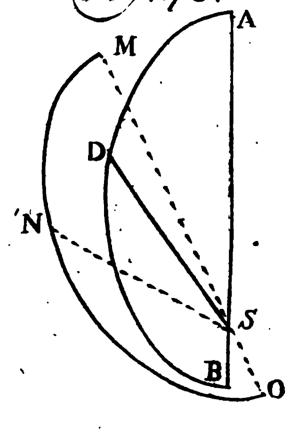


Fig. 40.



drawn out of the Plane of her Orbit by the Sun.

Secondly, That when the Line of the Nodes is in any other Situation, and the Moon not in one of the Nodes, ſhe is continually drawn out of the Plane of her own Orbit, on that Side on which the Sun lies. For inſtance, if the Plane of her Orbit produced paſſes above the Sun, the Sun draws her downwards; if on the contrary, the Plane of her Orbit produced paſſes below the Sun, it draws her upwards.

From theſe two Conſiderations it follows, that when the Line of the Nodes is not in the Syzygies, and the Moon having paſſed either of the Nodes, has got out of the Plane of the Ecliptic on either Side of it, the Action of the Sun occaſions the Moon to return back to the Plane of the Ecliptic ſooner than ſhe otherwiſe would do; but where the Moon enters that Plane, there is the next Node; ſo that each Node does as it were come towards the Moon, meeting her Part of the Way. And the nearer the Line of the Nodes is to the Quadratures, the greater is this Effect, becauſe in that Caſe the Sun is the fartheſt of all from the Plane of the Lunar Orbit produced. So that the Line of the Nodes goes backwards the faſteſt of all, when it is in the Quadratures; and not at all in the Syzygies. *Which is the ſeventh Irregularity.*

8. Again, when the Nodes are in the Quadratures, and the Moon has lately paſſed one

of them, and is approaching that Syzygy that is next the Sun, the Action of the Sun upon the Moon prevents her ascending so far; that is, departing so far from the Ecliptic as she otherwise would do; and so diminishes the Inclination of her Orbit to the Ecliptic: And as she goes on to the next Quadrature, by hastening her Descent to the Ecliptic, it occasions the Moon to cross it in a larger Angle than she would otherwise do; and so increases the Inclination of the Orbit as much as it diminished it before. And, for the same Reason, while the Moon passes from that Quadrature to the opposite Syzygy, the Action of the Sun decreases the Inclination of her Orbit, and increases it again in her Passage from thence to the next Quadrature. All which needs no farther Illustration, unless the following Instance may be of Use. If you toss a Stone up into the Air, the Action of the Earth upon the Stone prevents it from rising so high as it would otherwise do; and if a Stone be thrown down obliquely, the same Action by bending its Course towards the Earth all the Way, makes it strike the Earth in a larger Angle than it would otherwise do.

When the Nodes are in the Syzygies, the Inclination of the lunar Orbit to the Plane of the Ecliptic is neither increased nor diminished: The Sun being then in the Plane of the Moon's Orbit produced.

But while the Nodes are passing from the Syzygies to the Quadratures, the Inclination of

of the Moon's Orbit to the Plane of the Ecliptic, is diminiſhed in every Revolution of the Moon; and while they are paſſing from thence to the Syzygies, it is continually increaſing. So that the Inclination of the lunar Orbit is the greateſt of all when the Nodes are in the Syzygies, and leaſt when they are in the Quadratures. *Which is an eighth Irregularity.* But this requires a particular Explanation.

Let then S in Fig. 46. repreſent the Sun, NFDG the Plane of the Ecliptic, QR the Quadratures, and MO the Syzygies. And let NLDI be the Orbit of the Moon; and ſuppoſe the Nodes at N and D in the middle between the Syzygies and the Quadratures. Farther, let there be a Point H in the Ecliptic oppoſite to the Point S, and let the Orbit NLDI be ſo inclined to the Plane of the Ecliptic, that if it were extended every Way, it would paſs above S and below H. Then becauſe when the Moon is nearer the Sun than the Earth is, ſhe is attracted towards the Sun more than the Earth is; and when ſhe is farther off, the Earth is attracted more than ſhe is, in which caſe ſhe is therefore as it were attracted the other way; let us imagine a Sun at S, and another at H; and let it be remembered that the Orbit of the Moon produced, paſſes above S and below H: And let the Moon be aſcending from N towards L. Then the Attraction here being towards S, and the Orbit produced being above S, it is obvious that the Moon will

not riſe ſo high as L, but will paſs to A, a Point between L and F, deſcribing the Curve NA, ſo that the Inclination of the lunar Orbit is perpetually diminiſhed, while the Moon is paſſing over 90 Degrees from the Node N, it being a quarter of a Circle from N to A. To avoid Confuſion in the Figure, let us ſuppoſe that the Moon came to L. In going from thence to the next Quadrature at B, which is 45 Degrees, the Attraction of S prevails ſtill, becauſe the Moon is as yet nearer to S than the Earth is; and therefore as the Orbit produced is above S, and the Moon going downwards, the Attraction of S haſtens her Deſcent, and ſo makes her deſcribe the Curve LC inſtead of LB, which if produced would make with the Plane of the Ecliptic, a larger Angle than her Orbit LB does at D; in going over this 45 Degrees therefore the Inclination of her Orbit is increaſed. Let us now ſuppoſe her going from B towards D; the Attraction here lies towards H, becauſe ſhe is now paſt the Quadrature; and ſhe is tending to a Point below H; H therefore attracts her upwards, making her deſcribe the Curve BE, inſtead of BD, which is about 45 Degrees more, and makes a leſs Angle with the Plane of the Ecliptic, than BD does: Not that E falls beyond D, it only happens ſo in the Figure, becauſe we ſuppoſed the Moon to move from L and B inſtead of A and C. Upon the whole therefore, while ſhe moves from N to D, the Inclination of her

Orbit

Orbit is diminiſhed during three Parts out of four of her Paſſage. In like manner it is diminiſhed by the Attraction of H, while ſhe deſcends from D to I, and augmented by the ſame Attraction in going from thence to K, and diminiſhed again between K and N. Add to this, that while the Moon moves from N to L, or from D to I, the diſturbing Force, whether of S or H, is much more conſiderable than it is when ſhe is between L and D, or I and N; becauſe in the former Caſe, the Difference between her Diſtance from the Sun, and that of the Earth from the ſame, is greater than it is in the latter. While the Nodes therefore are between M and R, and O and Q; that is, while they are paſſing from the Syzygies to the Quadratures, the Inclination of the lunar Orbit is continually diminiſhing.

For though we have ſuppoſed the Nodes equally diſtant from the Quadratures and Syzygies, the like Effects will happen, though different in Degree, when they are nearer to the one than to the other; as is eaſy to imagine.

Let now the Nodes be in the mid Way, between R and O, and Q and M, ſee Fig. 47. and let the lunar Orbit produced paſs above S and below H as before, and let the Moon be aſcending from the Node D towards B. She being there in the Power of H, and moving in a Plane which if produced paſſes below H, will be attracted upwards thereby, ſo as to deſcribe the

Line DC instead of DB; by which means the Inclination of her Orbit will be increased. In going from thence to I, she is in the Power of S, which lies below her Orbit produced, and so her Ascent will be diminished, and she will go to A, instead of I; whereby the Inclination of her Orbit will be lessened; and afterwards as she descends towards N she will be attracted downwards all the Way coming to E instead of N; by which means the Inclination of her Orbit is again increased. So that upon the whole it is increased three Parts out of four of her Passage from Node to Node; for the like will hold in her Passage through the other Part of her Orbit, and as well when the Nodes are not in the middle between the Quadratures and the Syzygies, as when they are, except in Point of Degree. And for the Reason mentioned in the other Case, the Force which augments the Inclination of the Orbit, is superior for the Time being, to that which diminishes it. While the Nodes therefore are passing from the Quadratures to the Syzygies, the Inclination of the Moon's Orbit to the Plane of the Ecliptic is continually increasing. Which is what remained to be made out.

All these Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion, because as was observed above, the Effect of the Sun's Action whereby they are produced, is inversly as the Cube of its Distance from the Earth. They are also greater when

when the Moon is in Conjunction with the Sun, than in Opposition for the same Reason; for the Earth and Moon taken together, are nearer the Sun in the former Situation of the Moon, than they are in the latter.

II. Let us now imagine that the Circle NLDI is a solid Ring like that of *Saturn*, and that it moves round its Center T the same Way the Moon does round the Earth; it is obvious that every Point of this Ring, will endeavour to put on the same Motion that we have shewn the Moon to do: That is, that every Point in its Passage from N to L, will endeavour to move in the Line NA, (see Fig. 46.) every Point between L and B, will endeavour to describe the Curve LC, and every one between B and D, the Curve BE; and the like for Fig. 47. So that the Ring as to the Motion of its Nodes, and its Inclination to the Plane in which its Center moves, will be affected in the same manner that the Orbit of the Moon is; and therefore its Nodes, when in the Syzygies, will stand still, and its Inclination to the Plane of the Ecliptic will be the greatest: In all other Situations the Nodes will go backwards, and fastest of all when in the Quadratures, at which Time the Inclination of the Ring will be the least.

Let us now suppose that there is a Redundancy of Matter surrounding the Earth in the equatoreal Parts thereof; or in other Words, that the Earth is an oblate Spheroid, having

its equatoreal Diameters longer than that which paſſes through the Poles, as we ſhall ſhew it to be in the next Chapter. This Redundancy of Matter, will, like a Ring ſurrounding the Earth and fixed to it, endeavour to put on the abovementioned Motions, and thereby communicate them to the Earth itſelf. The equinoctial Points therefore which anſwer to the Nodes of the Ring, when they are in the Syzygies, that is, at the Equinoxes, will ſtand ſtill, and the Inclination of the Equator to the Plane of the Ecliptic will be the greateſt; in all other Situations they will go backwards, and faſteſt of all at the Summer and Winter Solſtices, at which Times they are as it were in the Quadratures with the Sun; and then the Inclination of the abovementioned Plane will be the leaſt.

III. From hence it follows, that the Axis of the Earth, being perpendicular to the Plane of the Equator, changes therewith its Inclination to the Plane of the Ecliptic twice in every Revolution of the Earth about the Sun. For inſtance, it increaſes while the Earth is moving from the Solſtitial to the Equinoctial, and diminiſhes as much in its Paſſage from the Equinoctial to the Solſtitial Points. Which Phænomenon is called the Nutation of the Poles.

IV. Another Phænomenon and of the ſame kind with the lunar Irregularities, is the ebbing and flowing of the Sea; only, as thoſe ariſe from

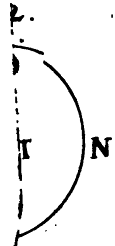
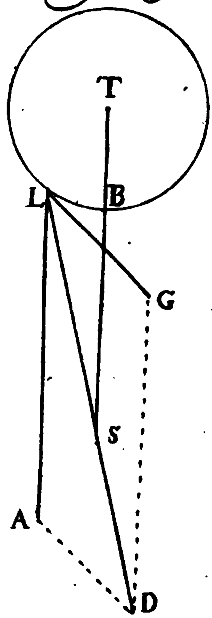


Fig. 43.



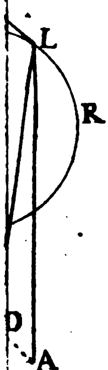
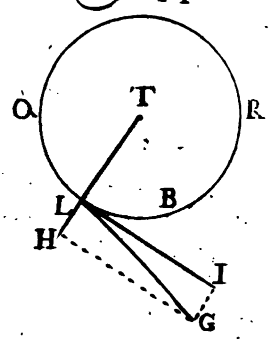


Fig. 44.



from the Action of the Sun upon the Moon, this is owing to the Influence both of the Sun and Moon upon the Waters of the Ocean; and is to be accounted for upon the same Principles, after the following Manner.

Let the Point L in Fig. 48. represent the Moon, MNOP, the Earth, whose Center is C. And let G be the common Center of Gravity of the Moon and the Earth. Which latter let us suppose surrounded with Water to a great Depth. Then, according to what was observed at the End of the last Chapter, these two Bodies continually revolve about the Point G, the Point M describing the Circle AB; the Point C, the Circle NP; and the Point O, the Circle EF; and all in the same periodical Time; consequently by Proposition the third of the foregoing Chapter, the Forces they require to retain them in those Circles ought to be to each other as their Distances from the Point G: That is, as GM, GC and GO. Consequently the Point O, which for Distinction sake we will call the *Nadir*, requires a greater Force than the Center C; and the Center, a greater Force than the Point M, which we will call the *Zenith*. Now these Points are retained in those Circles by the Moon at L, consequently the *Nadir* which requires the most, is attracted the least, as being farthest off; and the *Zenith* which requires the least, being the nearest, is attracted the most; that is, the *Nadir* is attracted too

little,

little, and the *Zenith* too much: The obvious Consequence of which is, that the Water both in the *Zenith* and *Nadir*, will endeavour to leave the Center C; or, in other Words, will lose part of its Weight. But the Water at N and P will have its Weight augmented, just as the Tendency of the Moon at L (in Fig. 42.) towards T was shewn to be augmented by the Action of the Sun at S. So that the Water at N and P will be heavier than an equal Quantity at M or O. And consequently the Surface of the Waters at N and P will subside, and that at M and O will rise, 'till the Equilibrium be restored. On which Account, the Form of the Earth, or rather the Sea, will become an oblong Spheroid or Oval, as represented by NKMH, in Fig. 49. whose longer Axis produced passes through the Moon at L. As therefore the Moon turns round the Earth once a Day, this Oval of Waters turns with her, occasioning thereby the two Floods and Ebbs observable in each 25 Hours: or to speak more accurately, the Oval of Waters keeps pace with the Moon in her Monthly Course, while the Earth in the mean Time, by its Rotation about its Axis, carries each part of its Surface from Ebb to Flood, and from Flood to Ebb continually.

And as the Moon thus raises the Water in one Place, and depresses it in another, the Sun does the same; but in a much less Degree, on Account of the small Proportion the Semidiameter

Semidiameter of the Earth bears to the vaſt Diſtance of the Sun; for, as was ſhewn of the Moon, the Force of the Sun whereby it diſturbs her Motion, was proportionable to the Relation the Diſtance of the Moon from the Earth bears to that of the Sun from the ſame, which in the Caſe before us, is the Relation the Semidiameter of the Earth bears to the Diſtance of the Sun, which Relation is very ſmall.

When the Moon is in Conjunction or Oppoſition with the Sun, the Tides which each of them endeavours to raiſe are in the ſame Place, which is the Reaſon they are ſo large at thoſe Times. Whereas when the Moon is in the firſt or laſt Quarter, the Sun being in the Meridian when the Moon is in the Horizon, depreſſes the Water where the Moon raiſes it, on which Account, the Tides are then *(cæteris paribus)* the leaſt of all.

On the full and new Moons, which happen about the Equinoxes, at which Time the Luminaries are both in the Equator or near it, the Tides are the greateſt of all, on the three following Accounts; in the firſt Place, the two Eminences of Water are at the greateſt Diſtance from the Poles, and ſo the Difference between Ebb and Flood is more ſenſible; for if thoſe Eminences were at the Poles, it is obvious we ſhould not perceive any Tide at all: Secondly, the equatoreal Diameter of the Earth produced paſſes through the Moon, which Diameter is longer

longer than others, and ſo there is a greater Diſproportion between the Diſtances of the Zenith, Center and Nadir, from the Center of Gravity of the Earth and Moon, than at other Times: Thirdly, the Water riſing higher in the open Seas, it ruſhes to the Shores with greater Force, where being ſtopped, it riſes higher ſtill; for it not only riſes at the Shores in Proportion to the Height it riſes to in the open Seas, but alſo according to the Velocity it flows with from thence againſt the Shore. The Reaſon why the Spring Tides, which happen a little before the vernal and after the autumnal Equinox, *viz.* in *February* and *October* are the greateſt, is becauſe the Sun is nearer the Earth in the Winter than in the Summer, and ſo the Tides, which otherwiſe would be the greateſt at the Equinoxes, are ſo, a little before the former, and as much after the latter.

We have hitherto conſidered the Tides in general; we muſt now ſee what happens as to Places of different Latitudes. All which will be eaſily underſtood by the help of the 49th Figure: In which let AFD repreſent the Earth whoſe Center is T, the Poles P and O, this the South, the other the North Pole; EQ the Equator, and the Circles FH and KD two Parallels of it, the laſt on the North Side of it, and the firſt on the South Side of it. Let the Fluid ſurrounding the Earth, form itſelf into an oblong Spheroid, whoſe longer Axis HK produced,

produced, paſſes through the Moon at L. The right Lines TK or TH will repreſent the greateſt Height of the Water, reckoning from the Center, and ſuppoſing NM perpendicular to KH, TN or TM will denote the leaſt, and will repreſent the Height of the Water in all Parts of the Globe through which that Circle NM paſſes. The right Lines TE, TF, TQ and TD, ſuppoſing them drawn, will ſhew the Height of the Water in the reſpective Places E, F, Q and D. Let us now conſider a Place, which by the diurnal Motion of the Earth, deſcribes the Parallel KD: When this Place is at K, the Height of the Water TK, is the greateſt; that is, it is high Tide or Flood when the Moon L is in the Meridian; but afterwards in the ſame Place, the Height of the Water is the leaſt, when the Place is come to X, and again it is Flood when the Place is come to D. But becauſe TK is greater than TD, in the preſent Caſe when the Moon is on the North Side the Equator, the Height of the Sea will be greater, when the Moon is in that Part of the Meridian which is above the Horizon, than when it is in that which is below it. Likewiſe TH is greater than TF, and therefore in a Place which lies under the Parallel FH, or on the other Side the *Line*, the greateſt Height of Water that happens when the Moon is on that Side, is alſo when ſhe is in that Part of the Meridian that is above the Horizon of that Place. Hence it is, that the

Moon

Moon in the Northern Signs makes the greateſt Tides on our Side the *Line*, when ſhe is above the Earth; and in the Southern Signs when ſhe is below it. But on Account of that libratory Motion of the Waters, by which they thus riſe and fall alternately, and which would continue ſome Time, although the Sun and Moon ſhould ceaſe to act, the Difference between thoſe Tides which happen when the Moon is above and below the Horizon, is not ſo great as it would otherwiſe be; and the higheſt Spring Tides are not exactly in the new and full Moons, but happen generally three or four Tides after them, and ſometimes later: Becauſe when the Luminaries come to act more forcibly than ordinarily, as being in Conjunction or Oppoſition, the Waters will librate backwards and forwards ſeveral Times, before they arrive at their greateſt Height.

Things would be thus, if the Globe of the Earth were wholly covered with Water of a ſufficient Depth, but the Continents which ſtop the Tide; the Straights, and the Shoalneſs of the Sea in ſome Places, which are Impediments to the free Courſe of the Water, cauſe many Exceptions to what has been laid down; and in particular, that even in the open Ocean the Time of high Water is not, when the Moon comes to the Meridian, but always ſome Hours after it. But to be particular in ſuch Circumſtances is not the Deſign of this Treatiſe. See more in *Philoſoph. Tranſact.* No. 226.

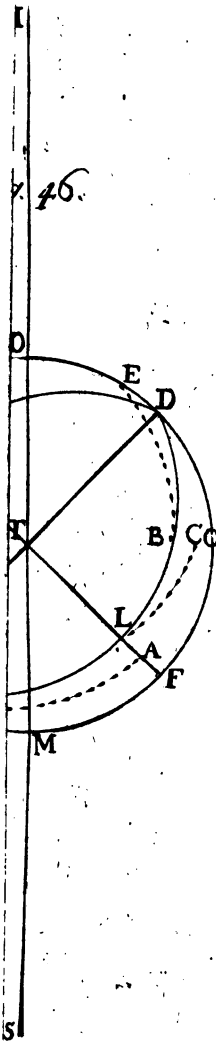
No. 226. However, there is one Thing, which becauſe I don't find it taken Notice of by others, I ſhall juſt mention; and that is, that at both the Summer and Winter Solſtices, there ought to be but one Tide in 24 Hours within the arctic and antarctic Circles; to ſhew this, let KD be the Tropic of Cancer, and let the Sun and Moon be perpendicular over a Point of that Tropic, as at L: Then as obſerved before, there will be an Ebb at N, but becauſe NPK is a quarter of a Circle, N will be a Point of the arctic Circle, and the high Water under that Circle will be at A, the Point oppoſite to N; ſo there will be but one Flood and one Ebb in 24 Hours Time: And as is obvious enough, the ſame will happen in all Places within that Circle, except at the Pole itſelf, where there is no Tide at all. The ſame Things will alſo happen at the other Solſtice, when the Luminaries are in the Tropic of Capricorn. And in thoſe Parallels which lie between the above-mentioned Circles and the Equator, each Ebb will be nearer the Time of the leſſer Flood, than it will be to that of the greater. Thus, in the Parallel KD, the Ebbs will be at X, and in the Parallel FH, at Y, which Points are nearer to F and D the leſſer Heights of the Water, than to H and K the greater ones. The Reaſon it does not happen exactly in this Manner, at leaſt not in Point of Degree, is that Riſing and Falling or Libration of Waters mentioned above, by which

which the Ebbs and Floods of each Place are rendered leſs unequal than they ought, according to the Theory, to be.

CHAP. XX.

Of the Figures *of the* Heavenly Bodies.

WHEREAS the Heavenly Bodies do not conſiſt wholly of ſolid Matter, but are in all probability partly fluid and partly ſolid, like our Earth, or at leaſt were ſo at firſt; thoſe which have no Motion about their Axes, if ſuch there be, will, from the mutual Gravity and Attraction of their Parts among themſelves, ſettle into a ſpherical Form. But as to ſuch as revolve about their Axes, all their Parts will endeavour to receed from the Axis of their Motion, and thereby the equatoreal Parts where the Motion is the quickeſt, will tend leſs towards the Center than the reſt; their Endeavour to fly off from the Axis about which they revolve, taking off part of their Tendency that Way; ſo that thoſe Parts will become lighter than ſuch are nearer the Polès. The polar Parts therefore will preſs in towards the Center, and raiſe the equatoreal Parts, till the Quantity of Matter in the latter is ſo far increaſed, as to compenſate for its Lightneſs, and an Equilibrium be reſtored. On which Account, the Form



...g. 46.

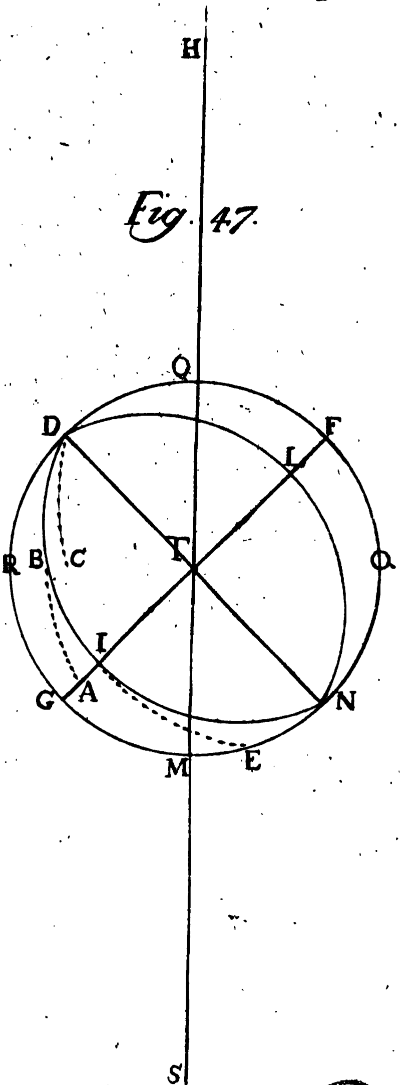


Fig. 47.

Form they will aſſume, will be that of an oblate Spheroid, whoſe ſhorter Axis paſſes through the Poles. And other Circumſtances remaining the ſame, the faſter the Bodies revolve, the more oblate or flat will their Form be; accordingly the Axis of the Planet *Jupiter*, which Planet turns round its Axis in leſs than 10 Hours, is, as appears from the Obſervations of Mr. *Flamſtead* and Monſ. *Caſſini*, no greater with reſpect to the Diameter of its Equator, than in the Proportion of ſeven to eight.

By Virtue only of the Rotation of the Earth about its Axis, the Weight of Bodies at the Equator is leſs than at the Poles, in the Proportion of 288 to 289; that is, a Body which at the Poles would weigh 289 Pounds, would, if carried to the Equator, loſe by Virtue of the centrifugal Force alone, one Pound of its Weight, and ſo weigh but 288 Pounds. From hence ariſes, as obſerved above, a ſpheroidical Form of the Earth, and from that ſpheroidical Form ariſes another Diminution of Gravity at the Equator, by which, if the Earth were homogenious throughout, Bodies at the Equator would loſe one Pound in 1121, and ſo on both Accounts taken together, the Gravity of Bodies at the Poles would be to the ſame at the Equator as 230 to 229 (*n*). From whence, if we ſuppoſe

(*n*) To calculate this, let us ſuppoſe the Semidiameter of the Earth to be 19695539 *Paris* Feet; which on account of the vaſt Diſtance on the Surface of the Earth *Caſſini* took in mea-

 ſuring

suring it, Sir *Isaac Newton* makes use of. Since then the Earth turns round its Axis in 23 Hours, 56 Minutes and 4 Seconds, a Body at the Equator moves through 1436,223 Feet in a Second: But the centrifugal Force of a Body revolving in a Circle is equal to the centripetal Force, which would be requisite to retain it in that Circle, and the Space a Body would fall through in a Second, by Virtue of that centripetal Force, is by Chap. XVIII. Lemma 1, equal to the Square of the Arch described in a Second, divided by the Diameter of the Circle, that is, in the present Case to ,0523 Feet, or 7,54064 Lines. Now the Space a Body falls through at *Paris*, by Virtue of the Gravity there, is 15 Feet, 1 Inch and $2\frac{1}{17}$ Lines, or 2174,055 Lines. Therefore the centrifugal Force at the Equator is to the Gravity at *Paris* as 7,54064 to 2174,055. But Bodies do not fall at *Paris*, by the whole Force of Gravity, or as they would do at the Poles of the Earth, because they are in some Measure prevented by the centrifugal Force there. We must therefore compute how much that centrifugal Force is, and add it to their Gravity at *Paris*, to find what it is at the Poles. Which may thus be performed.

Let EPQ in Fig. 50. represent the Earth, PP its Poles, EQ its equatoreal Diameter. Let A represent any Point between the Equator and the Poles, and parallel to the equatoreal Diameter EQ draw the Line AH, then centripetal and centrifugal Forces being equal as observed above in Chap. XVIII. and EC and AH being Radii of the Circles which the Points E and A describe, while the Earth revolves about its Axis PP, the centrifugal Force at E will by Prop. 3. of that Chapter be to that at A, as the Radius EC, or which is very nearly the same, as AC, to AH. But a Body at A is not thrown off by Virtue of this centrifugal Force directly from the Center of the Earth, but from H along the Line HA produced. Let then AB represent that Force, which because it does not tend directly from the Center, let it be resolved into two others, *viz.* BN and AN; the former perpendicular to the Radius, the latter coincident therewith: It is by this latter Force only that the Gravity of a Body at A is diminished. But the centrifugal Force at the Equator was shewn to be to that other Force along the Line AB, as AC to AH, and that other is to the Force along AN as AB to AN, which because of the similar Triangles ABN and ACH is also as AC to AH, therefore the centripetal Force at the Equator is to that at A, so far as it diminishes the Gravity of Bodies there, as ACq to AHq; that is, because AH is the Cosine of Latitude of the Place A, as the Square of the Radius to the Square of the Co-sine of the Latitude of the Place where the Body is. Now the Latitude of *Paris* is 48 Degrees and 50 Minutes,

ſuppoſe the Gravity of Bodies within the Earth to be directly as their Diſtance from the Center, as it was ſhewn to be in Part I. Chap. 3. § 8. thoſe Numbers will alſo expreſs the Relation between its polar and equatoreal Diameter (*o*). This

Minutes, the centrifugal Force therefore at the Equator is to that at *Paris*, as the Square of the Radius to the Square of the Coſine of that Latitude, that is, as 7,540 64 Lines to 3,267. Add this to 2174,055 the Gravity by which Bodies deſcend at *Paris*, and we have 2177,322 for the Gravity they fall with at the Poles: Which Number is to 7,540 the centrifugal Force at the Equator, as 289 to 1. So that the Gravity at the Poles is to the Gravity at the Equator, ſo far as it is diminiſhed in this latter Place by the Rotation of the Earth about its Axis, in the Proportion of 289 to 288. But this is not all: For it may be gathered, from what Sir *Iſaac Newton* has demonſtrated in his *Principia*, Book I. Sect. 13. where he treats of the attractive Forces of Bodies not ſpherical (though by a tedious and intricate Calculation too long to be inſerted here) that ſuppoſing the Earth to be an oblate Spheroid, ſuch as we ſhall determine by and by, and homogeneous throughout, a Body at the Poles even when the Earth is at Reſt, would be heavier than the ſame at the Equator in the Proportion of about 1121 to 1120. The Weight of a Body therefore at the Poles, when the Earth revolves about its Axis, is to the Weight of the ſame at the Equator in a Proportion compounded of 289 to 288, and 1121 to 1120; that is, in the Proportion of 230 to 229.

N. B. A *Line* is the 12th Part of an Inch.

(*o*) To ſhew this, call an equatoreal Column extended from the Surface to the Center, r; and a polar Column, x; and call the Force of Gravity at the Equator, p: then from what was juſt obſerved in the foregoing Note, ſuppoſing the Earth to be at Reſt, the Force of Gravity at the Poles will be $\frac{1121p}{1120}$, becauſe as $1120 : 1121 :: p : \frac{1121p}{1120}$: and becauſe the Gravity of a Column if it be equally heavy in all Parts, is equal to the Force of Gravity multiplied by the Contents of the Column, the Weight of an equatoreal Column of Matter, if as heavy in all Parts as at the Surface, would, ſuppoſing the Earth to be at Reſt, be equal to p multiplied by r, that is, pr; and the Gravity of a

This is upon a Supposition that the Earth was at first fluid, or a Chaos, having its solid and fluid Parts confusedly mixed together; but if we suppose it at first partly fluid and partly dry, as it now is, since we find that the Land

polar Column would for the same Reason be $\frac{1121px}{1120}$; but whereas if we suppose the Force of Attraction inversly as the Squares of the Distances, the Gravity of Bodies within the Earth will decrease as we go to the Center, Part I. Chap. 3. § 8. where it terminates in nothing, the Weights of the above-mentioned Columns decreasing uniformly therewith, will be but half what we made them before, that is, $\frac{pr}{2}$ and $\frac{1121px}{1120 \times 2}$. Call the centrifugal Force at the Equator, n; then since the centrifugal Force decreases also as we approach the Center, and there terminates in Nothing, the centrifugal Force of a whole Column of Matter will be $\frac{nr}{2}$. Take this away from $\frac{pr}{2}$, the Weight of that Column when the Earth is at Rest, and the remainder $\frac{pr}{2} - \frac{nr}{2}$ will be the Weight of the same when it moves. But to preserve an Equilibrium of Parts, this Weight must be equal to $\frac{1121px}{1120 \times 2}$ which was shewn to be the Weight of a Column at the Poles.

Which gives us this Equation $\frac{pr}{2} - \frac{nr}{2} = \frac{1121px}{1120 \times 2}$

Multiplying by 2, we have $pr - nr = \frac{1121px}{1120}$

Multiplying by 1120, we have $\Big\}$ $1120pr - 1120nr = 1121px$

Which gives us this Equation $r : x :: 1121p : 1120p - 1120n$.

But as was determined in the foregoing Note, p is to n as 289 to 1. Putting therefore those Numbers for p and n, in the last Step, we shall have $\Big\}$ $r : x :: 230 : 229$.

That is, the equatoreal Semidiameter is to the polar one, as 230 : 229. Which was to be shewn.

is very nearly of the fame Figure with the Sea, except raifed a little to prevent its being overflowed, the Earth muft ftill be of the fame Form; for otherwife the major Part of the Water would flow towards the Equator, and fpread itfelf like an Inundation over all the Land in thofe Parts.

The fpheroidical Figure of the Earth is greatly confirmed by Obfervations made with Pendulum Clocks, at different Diftances from the Equator. Firft of all Monf. *Richer* in the Year 1672, when at the Ifland *Cayenne*, found that his Clock, which at *Paris*, kept true Time, now loft 2 Minutes and 28 Seconds every Day. Dr. *Halley* going to the Ifland St. *Helena* in the Year 1677, was obliged to fhorten the Pendulum of his Clock one eighth Part of an Inch. With many others, all which compared together make it appear that a Pendulum at the Equator that fwings Seconds, ought to be about one fixth Part of an Inch fhorter than at *Paris*; from whence it appears that the Difference between the Gravity at the Poles and at the Equator, is almoft twice as great as what arifes from the Rotation of the Earth about its Axis, and its fpheroidical Form put together (*p*). And therefore the Difference between the equatoreal and polar Diameter muft be fo too.

From

(*p*) For the Length of a Pendulum, fwinging Seconds is proportionable to the Space a Body would fall through in a Second; as being equal to an eighth Part of that Space, Part I. Chap. 6. Prop. 10. And the Space a Body would fall through in a Second, is proportionable to the Force by which it falls; confequently

From hence it is probable, that the Parts of the Earth which lie near the Center, may be denser than such as lie nearer the Surface. For on this Supposition, and not otherwise, is it accountable, that the Gravity at the Equator discoverable by Pendulums, and that which arises from the above-mentioned Causes, should be so different (*q*).

quently the Length of a Pendulum swinging Seconds under the Equator ought to be to the Length of one that shall swing Seconds under the Pole, as the Gravities in those Places are to each other: That is, as 229 to 230. The Length of a Pendulum therefore under the Equator, being 438½ Lines, (as being two Lines shorter than one at *Paris*, which is 3 Foot 8⅗ Lines) the Length of one at the Poles should be 440,4 Lines; that is, it ought to be but 1,9 Line longer than the former. Whereas it is found by the Observations above-mentioned, that the Difference between a Pendulum in the Latitude of *Paris*, and one under the Equator, is as much as that, or rather more; and consequently the Difference between one at the Equator and one at the Poles would be almost twice as much; there being almost as much Difference between a Pendulum at the Poles and at *Paris*, as between one at *Paris* and the Equator: And consequently the Diminution of Gravity is almost twice as much as that which arises from the Rotation of the Earth about its Axis, and its spheroidical Form put together.

(*q*) If the Earth be more dense at the Center than near the Surface, we may consider that Redundancy of Matter near the Center over and above what there would be there, was the Earth of uniform Density, apart, and as a separate Body from the rest, then upon Account of the spheroidical Form of the Earth, a Body at the Poles is nearer to this redundant Matter than at the Equator, therefore in removing a Body from the Poles to the Equator, its Gravity so far as it depends on the Attraction of this redundant Matter, that is, that Part of its Gravity which is owing to that Attraction, is diminished: The whole Gravity of the Body is less therefore at the Equator than at the Poles. As therefore the Pendulum discovers a greater Difference of Gravity than arises from the Rotation of the Earth about its Axis, and the spheroidical Form of it considered as homogeneous, put together, it is very probable that the Earth is more dense at the Center than at the Surface.

So that upon the whole, Bodies are lighter at the Equator than at the Poles, on a threefold Account. First, by the centrifugal Force there, in the Proportion of 289 to 288: Secondly, on Account of the spheroidical Form of the Earth, or its Flatneſs at the Poles, in the Proportion of 1121 to 1120; and laſtly on Account of the greater Denſity of the Earth at the Center, but in a Proportion not yet ſufficiently determined, for want of knowing the exact Length of a Pendulum that ſwings Seconds at the Poles.

The greater warmth of the Air near the Equator, increaſes the Length of a Pendulum by Rarefaction, on which Account alone, it would be neceſſary to ſhorten it at the Equator; but this, as may be gathered from the Obſervations of *Pitcairn* and *De La Hire*, cannot be between this Latitude and the Equator above one fortieth part of an Inch; for as *De la Hire* obſerved, there was but two third Parts of a Line Difference between an Iron Rod ſix Feet long, expoſed to the Froſt and the Summer's Sun. Beſides, this is allowed for above, in ſaying that a Pendulum ought to be ſhortened but one ſixth Part of an Inch.

It follows from hence, that the Tendency of heavy Bodies upon the Surface of the Earth, except at the Equator and the Poles, is not towards the Center of the Earth, but towards a Point between that and the Equator. Thus,

let

let EPQ in Fig. 50. represent the Earth, PP its Poles, EQ its equatoreal Diameter. The Tendency of a Body as A, upon the Surface of the Earth, between E and P, will not be towards C, but along the Line AD which crosses the equatoreal Diameter in a Point nearer the Body than the Point C is (*r*).

Another Consequence of the speroidical Form of the Earth is, that as you go from the polar Parts to the Equatoreal, the Degrees upon the Surface of the Earth grow less and less;

(*r*) To determine the Point D, say as BC is to AH; that is, as the Radius is to the Co-sine of the Angle of the Latitude of the Place A, so is the centrifugal Force at E, to a fourth Number, which will express the centrifugal Force at A in the Direction AB. Produce therefore the Line HA to B, till AB be of such Length, that it may bear the same Proportion to AC, that the Quantity last found has to Gravity upon the Surface of the Earth. Compleat the Parallelogram ABCD, and D will be the Point sought, and the Tendency of an heavy Body will be along the Line AD. Thus suppose it required to find the Direction in which heavy Bodies tend towards the Earth in at the Latitude of 51 Degrees and 46 Minutes. Since the centrifugal Force at the Equator, as shewn above, bears that Proportion to the Force of Gravity, which 1 does to 289, let us take those Numbers (or which will do as well, any other two that are proportionable to them) the former to express the centrifugal Force of a Body at the Equator, and the latter the Force of Gravity. Then say, as the Radius is to the Co-sine of 51 Degrees 46 Minutes, so is 1 to a fourth Number, which will be found ,618. This Quantity is therefore to Gravity as ,618 to 289. Therefore AB or its equal CD is to AC in that Proportion. But the Proportion between CD and AC being known, and the Angle at C the Latitude of the Place, *viz.* 51 Degrees 46 Minutes being given, the Angle CAD will be found about 5 Minutes, which is the Measure of the Deviation of the Line of Direction of heavy Bodies in that Latitude, from a Line drawn to the Center of the Earth.

that

that is, a Degree meaſured upon the Meridian near the Poles, contains more Miles than the ſame meaſured near the Equator. The Reaſon of this is, that, as appears by Inſpection of the Figure, an Arch near the Poles comes nearer to a ſtraight Line; that is, it is leſs curved than one near the Equator; the former is therefore an Arch of a larger Circle than the latter; but the larger the Circle, the larger are the Degrees meaſured upon its Circumference. For in going from P towards E, we are not to imagine ourſelves upon the Circumference of a Circle whoſe Center is C, but to paſs continually from one Arch to another, which Arches are Portions of different Circles, the Centers of which approach, as the Arches themſelves become more curved.

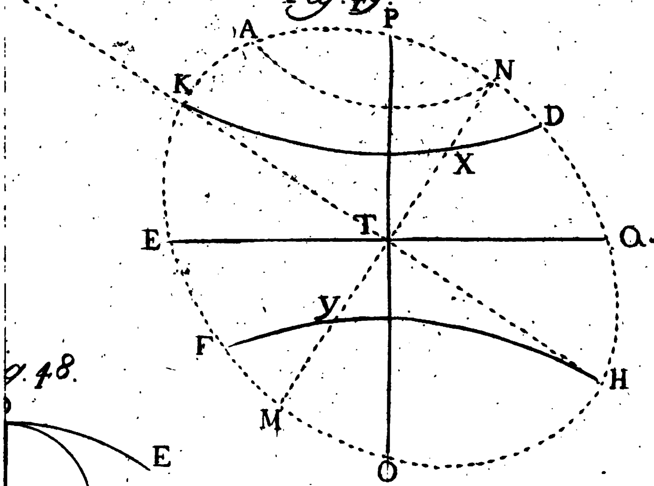
As the Waters of the Earth by Virtue of the Attraction of the Moon, and the Revolution of the Earth about the common Center of Gravity of that and the Moon, were ſhewn in Chapter the laſt, to put on the Form of an oblong Spheroid, whoſe Axis produced paſſes through the Moon; ſo in like manner, if we conſider the Moon as we have now done the Earth, we ſhall find, that as ſhe turns round her Axis in the ſame Time ſhe turns round the Earth, and therefore has nearly the ſame Side always towards the Earth, her Figure is that of an oblong Spheroid, whoſe longer Axis points to the Earth; thoſe Parts which lie next

the Earth being attracted too much, and also having too small a centrifugal Force; and those on the contrary which lie on the opposite Side of her being attracted too little, and having a centrifugal Force too great; which for the Reasons given in the above-mentioned Chapter, will necessarily give her that Form.

Her Revolution about her Axis would, as in the Earth, give her a contrary Figure, but it is so very slow, that it is without any sensible Effect.

FINIS.

Fig. 49.



g. 48.

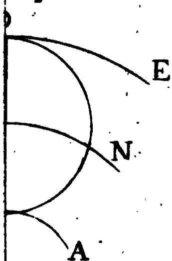
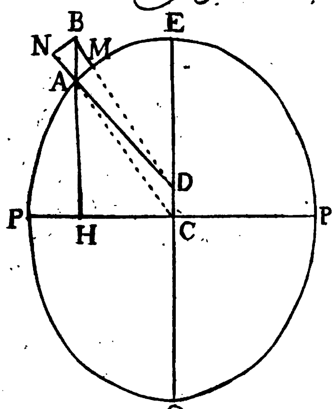


Fig. 50.



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